LAMINAR FLOW ANALYSIS OVER A FLAT PLATE BY COMPUTATIONAL FLUID DYNAMICS

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ABSTRACT

In this paper theoretical estimations of boundary layer thickness and heat transfer coefficient is examined using Computational Fluid Dynamics (CFD) for laminar air flow. The feasibility and accuracy of using CFD to calculate convective heat transfer coefficients is examined. A grid sensitivity analysis is performed for the CFD solutions, and it is used to determine the convective heat transfer coefficients. The coefficients are validated using analytical solution. In addition the local Nusselt number are obtained, which can be used in estimation of flow and heat transfer performance over a flat plate. The results tell us that for the laminar forced convection simulations the convective heat transfer coefficients differed from analytical values by 5%. The result also tells us that the boundary-layer thickness for laminar flow decreases with distance from the leading edge of the flat plate and increases with Reynolds number. The effect of Reynolds number, Prandtl number on flow is also investigated. These estimations can quickly give us the conclusion of dependences between the variables of interest.

KEYWORDS: *boundary layer thickness, laminar flow, forced convection, heat transfer, local Nusselt number.*

I. INTRODUCTION

Convective heat transfer studies are very important in processes involving high temperatures such as gas turbines, nuclear plants, thermal energy storage, etc. The classical problem i.e., fluid flow along a horizontal, stationary surface located in a uniform free stream was solved for the first time in 1908 by Blasius [1]; it is still a subject of current research [2, 3] and, moreover, further study regarding this subject can be seen in most papers [4,5]. Moreover, Bataller [6] presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow)and about a moving plate in a quiescent ambient fluid (Sakiadis flow). Aziz [7] investigated a similarity solution for laminar thermal boundary layer over a flat-plate with a convective surface boundary condition. Numerous studies such as Refs [8–10-16] considered different variations in temperature and heat flux at the plate.

Thompson [11] discusses solution of partial differential equations involved in areas such as Fluid Mechanics, Elasticity and Electromagnetic Field by using FEM. Details about fluid mechanics and heat transfer problems and their solution can be found in [12]. A more complex transient heat conduction equation is discussed in Winget and Hughes [13]. Similarly Johan et al. [14] and Jacob and Ebecken [15] develop step size selection schemes based on heuristic rules for compressible Navier-Stokes equations and structural dynamics problems respectively.

There have been a number of studies on the heat transfer characteristics of laminar air flow over flat plate. Most of the earlier studies were experimental in nature. Jackson [17] simulated the periodic behavior of a two-dimensional laminar flow past various shaped bodies, including flat plates aligned

over a range of angles of attack with respect to the incoming free-stream. Knisely [18] performed Strouhal number measurements of several rectangular cylinders with side ratios ranging from 0.04 to 1.0 and with angles of attack ranging from 0° to 90°. Lam [19] investigated the flow past an inclined flat plate at α=15°, using phased-averaged LDA measurements. The flow field around flat plates, characterized by sharp leading and trailing edges, was also investigated by Breuer and Jovicic [20]. Breuer et al. [21] simulated the flow over an 18° inclined plate, showing how the trailing edge vortices are able to dominate the wake features. Zhang et al. [22] studied the transition route from steady to chaotic state for a flow around an inclined flat plate. Kandula. Max [23] investigates Frost growth and densification in laminar flow over flat surfaces. William L. Oberkampf, Timothy G. Trucano [24] investigates verification and validation in computational fluid dynamics. S.L. Chernyshev, A.Ph. Kiselev, A.P. Kuryachii [25] discussed Laminar flow control research at TsAGI: Past and present.

Convective heat transfer or, simply, convection is the study of heat transport processes by the flow of fluids. Problems related to convective heat transfer rest on basic thermodynamics and fluid mechanics principles, which essentially involved with partial differential equations.

The convective heat coefficients can be predicted through an experiments or through computer modeling techniques that apply discretization schemes (finite element, control volume, etc) to simplify governing equations that normally would have no analytical solution. Experiments have the advantage of providing results tailored to a specific problem, but in order to properly measure boundary layer data expensive equipment such as a Particle Image Velocimetry (PIV) or Laser-Doppler Anemometry (LDA) is often required. Experiments are also generally time consuming to prepare and results inevitably include errors in accuracy. Computer modeling can be used to predict results, but the model must always be validated with experimental data in order to verify the accuracy of the solution. However, it will be shown that other techniques can also be employed to validate computer models that do not rely solely on experimental data.

This paper demonstrates how CFD can be used for the determination of the heat transfer process in the boundary layer for laminar air flow. The commercial CFD code Fluent 6.1.22 was used for all simulations. The coefficients are validated using analytical solutions. A grid sensitivity analysis is performed for the CFD solutions, and it is used to determine the grid independent solutions for the convective heat transfer coefficients. The effects of parameters such as Reynolds number and Prandtl number on flow over a flat plate is also analyzed. The local Nusselt number along the flat plate over a wide range of governing parameters (Re, Pr numbers) is also presented. Two different flow forms can occur in the boundary layer. The flow can be laminar or turbulent. We will deal only with laminar flow in this paper.

II. LAMINAR FLOW CFD SIMULATIONS

Heat transfer in the laminar regime will be simulated with CFD for laminar flow over a flat plate with constant wall temperature as shown below in Figure 1. The geometry is divided into discrete volumes using a structured grid. The number of cells in the grid impacts the solution, as is demonstrated later in the grid sensitivity analysis section. A typical mesh is shown below in Figure 2.
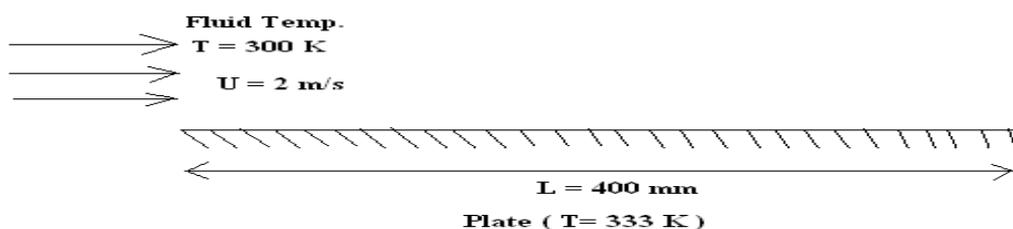

Figure1. Schematic representation of laminar flow over flat plate with constant wall temperature
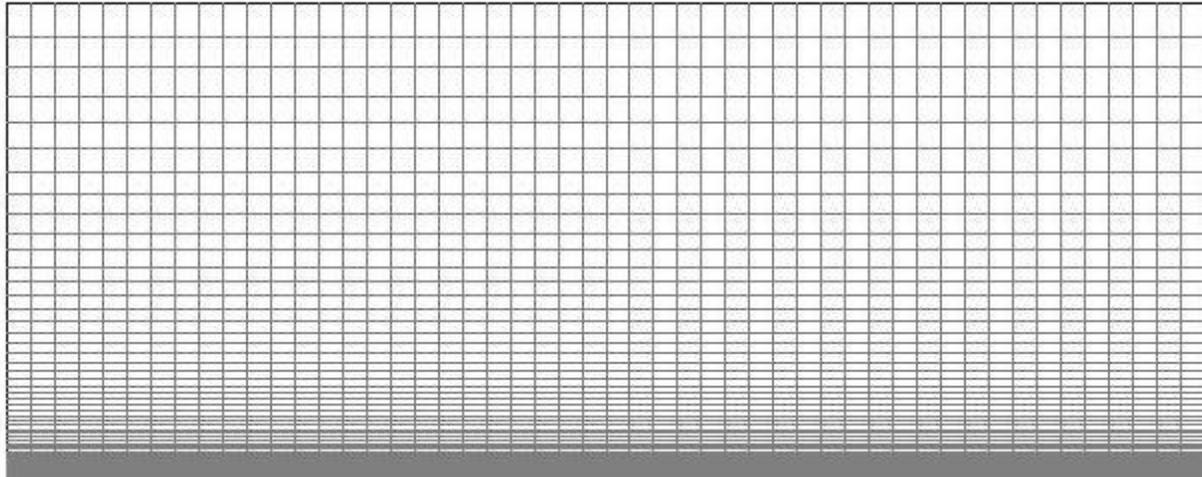
Figure2. Mesh used in laminar CFD simulations

The heat transfer coefficient may be obtained from analytically derived values of the Nusselt number. The values differ slightly based upon the heating conditions. It is found that the Nusselt number can be expressed as:

$$Nu = \frac{h\,L}{K} \tag{1}$$

Where L is the length of plate (m), k is the thermal conductivity of air (w/m k), h is the heat transfer coefficient (w/m2k) and Nu is the Nusselt number. Heat transfer coefficient for a flat plate can be determined by solving the conservation of mass, momentum, and energy equations (either approximately or numerically). They can also be measured experimentally.

The appropriate parameters may then be input to yield the following analytical values for h:

$$h = \frac{Nu\,K}{L} \qquad W/m^2\,K \tag{2}$$

Convective heat transfer between a moving fluid and a surface can be defined by the following relationship:

$$Q = h\,(T_s - T_f) \tag{3}$$

Where Q is the heat flux (W/m2), h is the convective heat transfer coefficient (W/m2 K), Ts is the surface temperature (K), and T f is the fluid reference temperature (K).

2.1 Reference Temperature

The equation for convective heat transfer requires a fluid reference temperature, previously designated T_f in Equation 1. The actual value used for T_f depends largely upon the geometry used in the problem. Correlations that describe convective heat transfer coefficients, such as the ones shown in Equation 1. The simulation result is validated with equation 2, as presented in the laminar case studies.

The solution procedure to determine the convective heat transfer coefficients for the reference temperatures is shown below in Table 1. By substituting the known boundary conditions (either the constant heat fluxes Q_w or the constant wall temperature T_w) and the data from the CFD simulation into Equation 3, the convective heat transfer coefficients can be calculated along the length of the plate.

2.2 Laminar CFD Simulation Results

The convective heat transfer coefficients for the constant wall temperature are presented in Figure 3. The results indicate that convective heat transfer coefficients differed from analytical values by 5%.

Table1. Convective heat transfer coefficient solution parameters

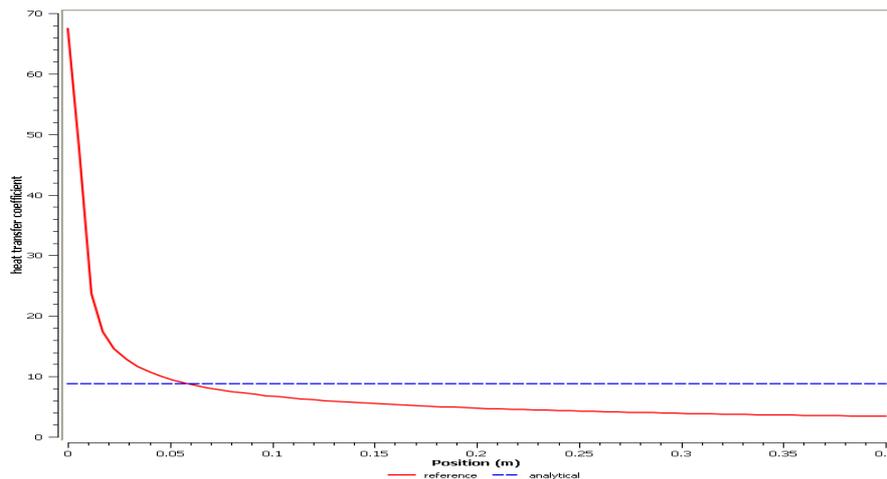|	Constant wall temperature
Q w (x)	Q from Fluent
T w (x)	T w = 333 K
T air (x)	T air = 273 k

Figure3. Convective heat transfer coefficients for constant wall temperature

Table2. Laminar flow heat transfer results

Temperature	Convective heat transfer coefficient, h from analytical values (W/m^{2}K)	Convective heat transfer coefficient, h from heat flux (Q)(obtained from fluent) W/m^{2}K	Heat flux (Q) from Fluent (W/m^2)	Heat flux (Q) from analytical values (w/m^2)
T w = 333 K T air = 273 k	8.77	9.28	121.81	115.76

III. GRID SENSITIVITY ANALYSIS

The determination of a mesh for the CFD calculations is not a trivial task. A mesh that is too coarse will result in large errors; an overly fine mesh will be costly in computing time. To demonstrate the impact of the mesh size on the calculation results, any CFD simulation should be accompanied by a grid sensitivity analysis. One such analysis is presented here. For the purposes of the grid sensitivity analysis, the convective heat transfer coefficients are calculated and compared for different grid densities. The process was repeated for CWT case to compare the grid dependency for same boundary conditions. Only the coefficients calculated from the air and wall temperature are part of this comparison.

Table3. Mesh information

	Φ h (3000)*	Φ2h (2000)	Φ4h (4200)	Φ8h (3600)
Number of cells in Y direction	50	40	30	20
Number of cells in X direction	60	110	140	180
Total Number of cells	3000	2000	4200	3600
Q(W/m^2)	121.81	125.85	93.72	94.08
h (w/m^{2}k)	9.28	9.53	7.1	7.127

* Original mesh

The initial grid used for the simulations had a total of 3000 cells. It was decided to proceed with several coarser grids and one finer mesh. The details of the different meshes are presented in Table 3. The notation h is adopted to describe the solution for the finest mesh. The subsequent meshes are all notated with respect to the finest mesh.

The convective heat transfer coefficients for the constant wall temperature case are plotted below in Figure 4
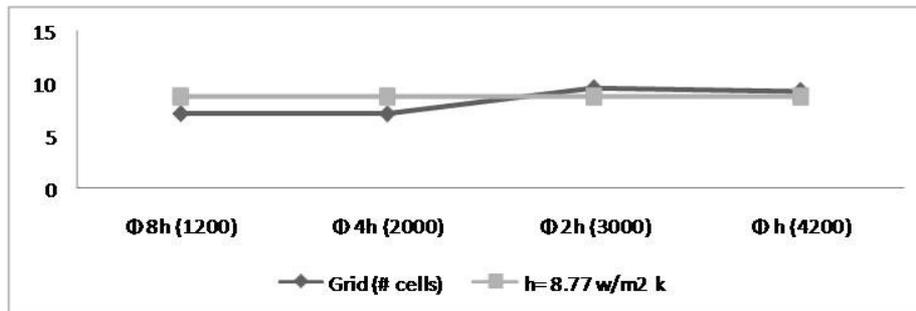

Figure4. Grid convergence of the heat transfer coefficient for constant heat flux

IV.   ESTIMATION OF BOUNDARY LAYER THICKNESS

If a flat plate is put in a flow with zero incidences, the flow at the plate surface is slowed down because of the friction. This region of slowed down flow becomes even larger, since more and more fluid particles are caught up by the retardation. The thickness of the boundary layer δ(x) is therefore a monotonically increasing function of x. The transition from boundary-layer flow to outer flow, at least in the case of laminar flow, takes place continuously. That means that a precise boundary cannot be given. It is often used in practice that the boundary is at the point where the velocity reaches a certain percentage of the outer velocity, e.g. 99% (δ 99).

Thickness of boundary-layer for a flat plate at a location x is given by:

$$\delta = \frac{5.48\ x}{\sqrt{Re_x}} \qquad (4)$$

It can be seen from figure5, the variation of boundary layer thickness up to x= 0.05m is linear and after that the variation of boundary layer thickness is not linear.
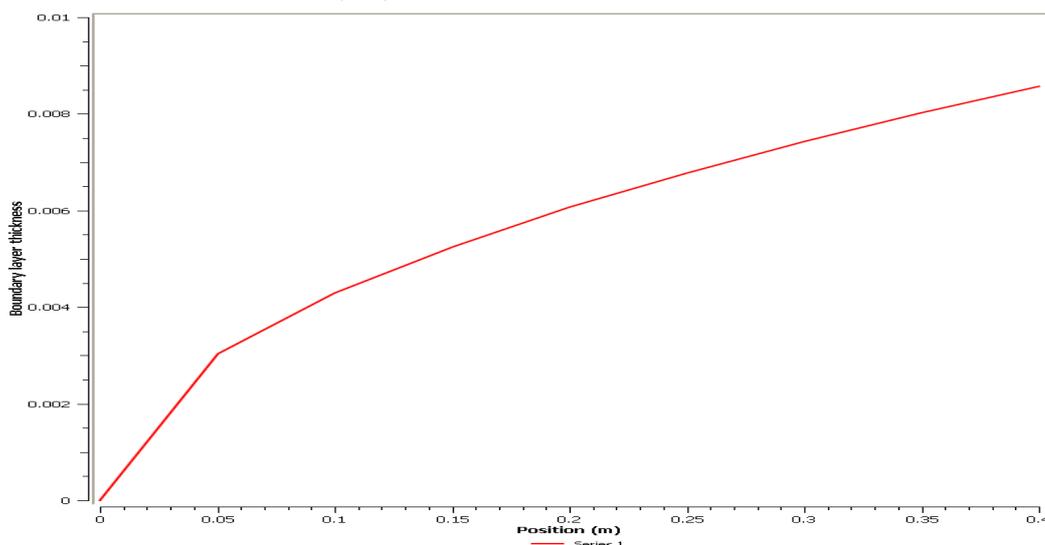

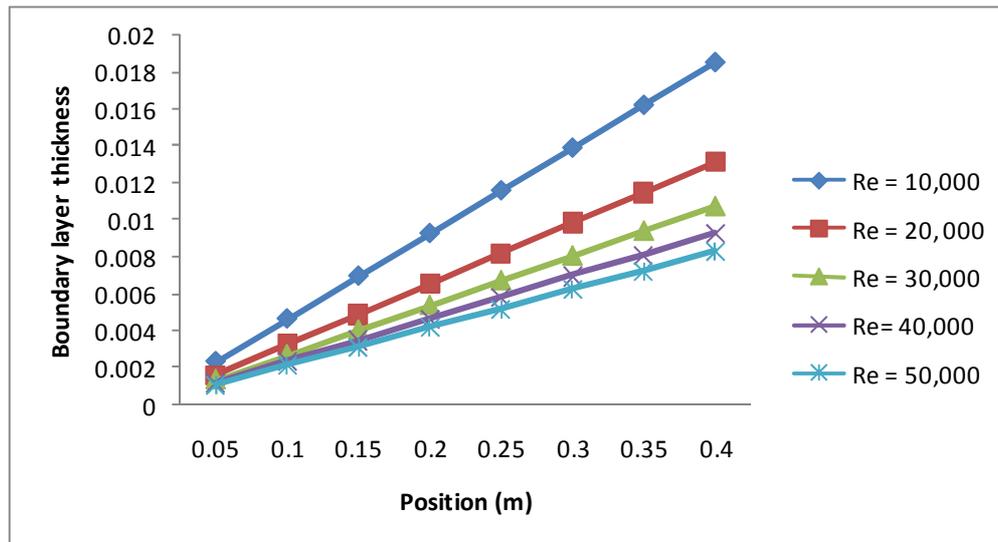**Figure5.** Variation of boundary-layer thickness along flat plate for Re = 50,000

Figure6. Variation of boundary-layer thickness for different values of Reynolds number

From above figure6, it is observed that boundary layer thickness is maximum for Reynolds number 10,000 and minimum for Reynolds number 50,000. It can be shown that with increasing Reynolds number thickness of boundary layer decreases.

4.1 Effect of Reynolds Number

The Reynolds number is a measure of the ratio of inertia forces to viscous forces. It can be used to characterize flow characteristics aver a flat plate. Values under 500,000 are classified as Laminar flow where values from 500,000 to 1,000,000 are deemed Turbulent flow. Is it important to distinguish between turbulent and non turbulent flow since the boundary layer thickness varies, as shown in Fig. (7)
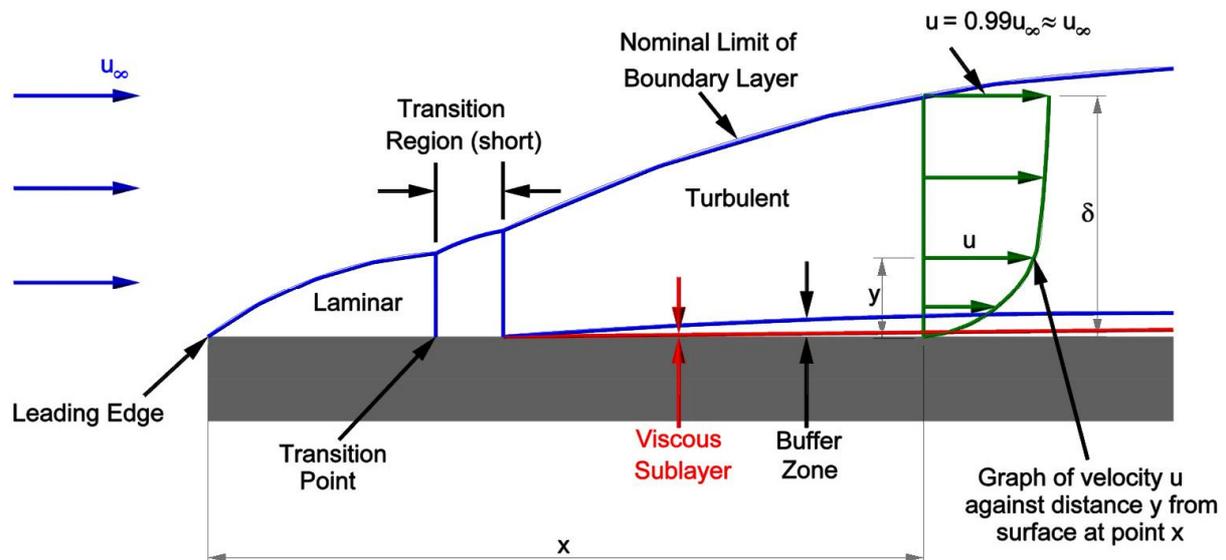

Figure7. Flow over a flat plate

Figure8. Show the effect of increasing Reynolds number. For different value of Reynolds number, local nusselt number increases. It is observed that the variation of nusselt number is linear till the Reynolds number increases to 5876. After that the variation of nusselt number with increasing Reynolds number does not remain linear. This change is clearly observed in figure8 below-
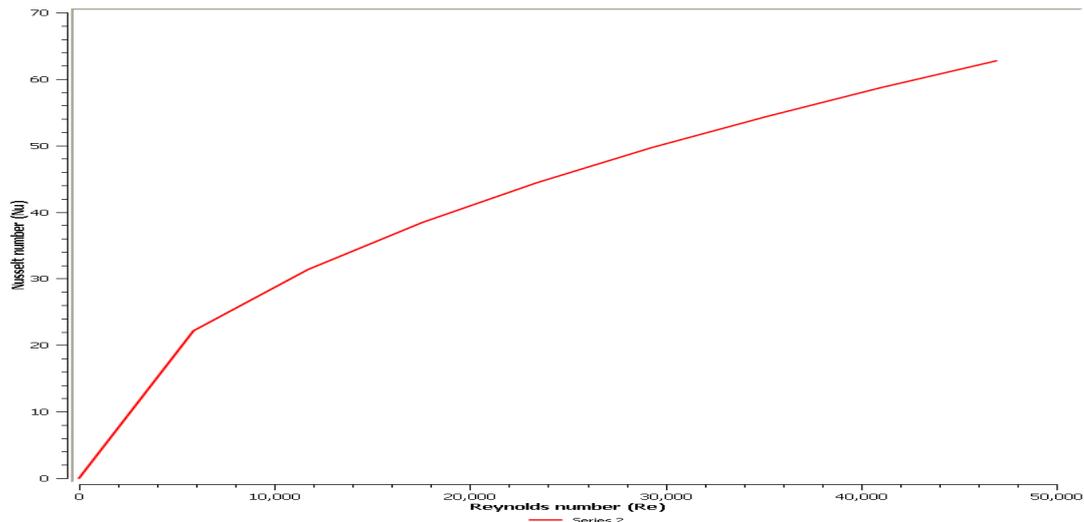
Figure8. Variation of Nusselt number with Reynolds number

4.2 Effect of Prandtl Number

Figure9 Shows the variation of local nusselt number with Reynolds number for different values of Prandtl number. For low Prandtl number, heat diffuses much faster than momentum flow and the velocity boundary layer is fully contained within the thermal boundary layer. On the other hand, for high Prandtl number, heat diffuses much slower than the momentum and the thermal boundary layer is contained within the velocity boundary layer.
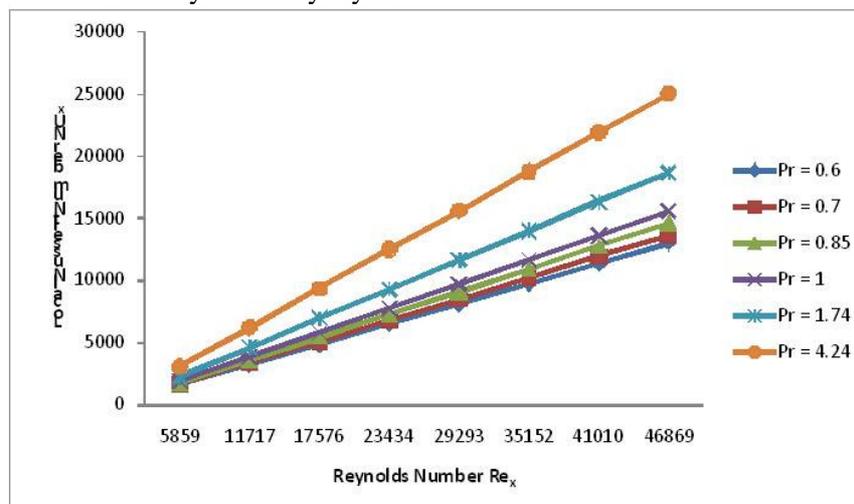

Figure9. Variation of Local nusselt number with Reynolds number for different values of Prandtl number

V.   RESULTS AND DISCUSSION

The results of the present work lead to the following conclusions:

First, a validation exercise was performed by comparing the computed convective heat transfer coefficients (h_c) for laminar air flow over flat plate by Computational Fluid Dynamics to analytical solutions. The CFD simulations were performed for constant wall temperature conditions. The CFD results showed a 5% error with the analytical solutions, indicating a performance of the CFD code, at least for the cases studied.

A grid sensitivity analysis was performed on the mesh for laminar air flows. The convective heat transfer coefficient (h_c) was calculated over a flat plate.

The boundary-layer thickness decreases with distance from the leading edge of the plate and increases with Reynolds number.

It is observed that the variation of nusselt number is linear till the Reynolds number increases to 5876. After that the variation of nusselt number with increasing Reynolds number does not remain linear. The effect of increasing Reynolds number and Prandtl number on flow tends to increase average and local nusselt number.

Computational Fluid Dynamics can be used to determine the convective heat transfer coefficients.

VI.   CONCLUSION

In this study, a two-dimensional forced convection laminar air flow over flat plate is analyzed using the finite element method through solving partial differential equations of the fluid flow. . The fluid flow is expressed by partial differential equation (Poisson's equation). While, heat transfer is analyzed using the energy equation. This paper demonstrated the use of CFD for heat flow in forced laminar air flow over flat plate. This paper also gives a few guidelines on the use of CFD to perform such calculations. As a result, the CFD models can be used with confidence for cases similar to the ones described here.

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