

# A STUDY ON THE BASIC CONTROL OF SPEED RATIO OF THE CVT SYSTEM USED FOR ELECTRIC VEHICLES

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## ABSTRACT

*In this paper, the basic control of the chain continuously variable transmission (CVT) used in electric vehicles is examined. First, transient power request of the vehicle is obtained for a specified velocity profile. Afterwards, the torque and angular speed of the electric motor are selected and then the speed ratio of the CVT is determined. This speed ratio is implemented in the control unit of the variator as a reference and it is controlled by considering the dynamics of the CVT with a feedback PI controller. Finally, the real velocity of the vehicle is obtained with the help of dynamic formula that includes overall system dynamics and it is compared with the reference velocity profile. The simulation results demonstrate that the comparison is in a good agreement and it implies that the developed model can be a reference for a complex control algorithm.*

**KEYWORDS:** *Electric Vehicle, Chain CVT, Control, Optimum Operation Line*

## I. INTRODUCTION

The harmful gas emission of internal combustion engines is one of the most critical reasons why the electric vehicle is getting wider in the last decade. The zero emission is the most attractive advantage of the EV considering the environmental problems. However, the electric motor (EM) has a limitation for providing the requested velocity of the vehicle. Thus, the researches show that a transmission system is necessary to maintain the optimum operation condition of the EM. There are several options of the transmission systems that are suitable for the EV and the continuously variable transmission (CVT) is one of the best solutions of them. The usage of the CVT in an EV was investigated in [1,2]. Besides the electric vehicle application, there are many works about the application of the CVT system for the hybrid vehicles [3,4].

The one of the key issues of the CVT is the precious and quick speed ratio control. In order to do that, the whole dynamics of the system must be known deeply. The transient or shifting dynamics of the belt/chain variator has been studied for last two decades with or without vehicle dynamics. About the pure dynamics of the shifting phenomena, the Carbone et. al. in [5] proposed a first order non-linear differential equation. Yildiz et. al. in [6] modelled the shifting dynamics of the chain CVT for the direct implementation in transmission control. They demonstrated that the whole dynamics of the variator can be represented by the first order differential equation and the input speed ratio and the torque load are the main parameters that affect the time response of the system. The dynamic behavior of the variator with a controller is also studied by many researchers [7-9].

The dynamics of CVT with the vehicle dynamics have been also studied in many works. Especially, the research has focused on the ratio control of the CVT system to keep the internal combustion engine in optimum operation area. Lee and Kim [10] studied the shift speed control of CVT to improve fuel economy of internal combustion engine. They claimed that the fuel consumption of the vehicle can be improved 2% compared with that of the conventional control. Adachi et. al. [11] developed a CVT controller for fuel efficiency of the engine. The results of their study show that the quick and precisely control of the CVT can contribute the fuel economy of the vehicle. Similar studies

about the fuel consumption performance and its improvement, several other works are available in the literature [12-14].

In order to investigate the dynamic performance of the CVT for an electric vehicle, a simple control of the chain continuously variable transmission (CVT) is studied in this paper. The power request of the EV is calculated for a certain velocity profile. The torque and angular speed of the electric motor are selected from the characteristic map of EM. This selection is made by focusing on the maximum efficiency area of the motor as the limitation of the CVT allows. Then, the speed ratio of the variator is determined. This calculation is implemented as a reference value and it is controlled with a feedback PI controller. Eventually, the real velocity of the vehicle is obtained from the dynamic equations and it is compared with the desired velocity profile. The comparison shows good agreement and the developed model can be useful for different control algorithm.

## II. DYNAMIC MODEL OF ELECTRIC VEHICLE COUPLED WITH A CHAIN CVT

In this section, the dynamic model of the electric vehicle coupled with a chain CVT is obtained. The differential form of the kinetic energy with simplicity gives the following formula:

$$P_m + P_v = \frac{dE_k}{dt} \tag{1}$$

where  $P_m$  and  $P_v$  are the power of electric motor and the resistance power of the vehicle respectively, and  $E_k$  is the kinetic energy of the vehicle. From the above equation, the spontaneous power request of the vehicle can be determined with the following equation:

$$P_m = \frac{1}{\eta_p} [(F_R v) + (M v \dot{v} + I_m \dot{\omega}_m \omega_m)] \tag{2}$$

where  $v$  is the velocity of the vehicle,  $\omega_m$  is the angular speed of the electric motor,  $M$  is the total weight of the vehicle,  $I_m$  is the mass moment of inertial of all rotating component on the motor shaft.  $F_R$  is the resistance force of the vehicle that equals:

$$F_R = (f_R M g \cos \alpha + M g \sin \alpha + 0.5 g_L C_W A v^2) \tag{3}$$

The terms of the Eq. 3 are the wheel, air, and gradient and acceleration resistance force respectively referring to the Figure 1a. The slope angle  $\alpha$  and the other parameters of the Eq. 3 are given in Table 1. In the calculations, the slope of the road is taken as zero.

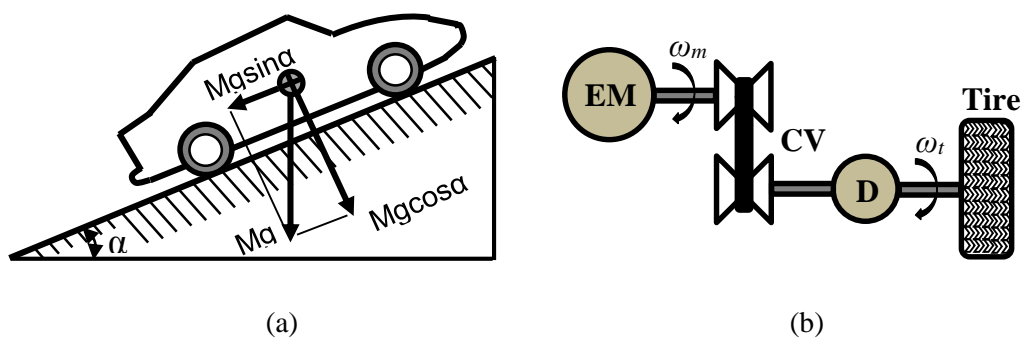


Figure 1: Scheme of the vehicle (a) and the powertrain (b)

The velocity of the vehicle and the angular speed of the motor can be combined in the following equation referring to the Figure 1:

$$v = \tau_{cvt} \tau_D R \omega_m \tag{4}$$

where  $\tau_{cvt}$  and  $\tau_D$  are the speed ratio of the CVT and the fixed speed reducer respectively, and R is the radius of the tire. These quantities are also described in Table 1.

The time derivative of the Eq. 4 yields:

$$\dot{v} = \dot{\tau}_{cvt} \tau_D R \omega_m + \tau_{cvt} \tau_D R \dot{\omega}_m \tag{5}$$

Substituting Eq. 5 into the Eq. 2, the following relation is obtained:

$$\dot{v} = \frac{\frac{T_m}{R \tau_{cvt} \tau_D} - (f_R M g \cos \alpha + M g \sin \alpha + 0.5 g_L C_W A v^2) + \frac{I_m v \dot{\tau}_{cvt}}{R^2 \tau_{cvt}^3 \tau_D^2}}{\frac{I_m}{R^2 \tau_{cvt}^2 \tau_D^2}} \tag{6}$$

It is clearly seen from Eq. 6 that the time derivative of the speed ratio of CVT affects the velocity profile of the vehicle. Thus, the dynamics of the variator must be considered in the mathematical model of the system. The whole dynamics of the chain CVT can be represented by the following first order non-linear differential equation adopted from [6]:

$$\dot{\tau}_{cvt} = \Delta_{DR} k \omega_{cvt} g \left[ \ln \left( \frac{S_{DR}}{S_{DN}} \right) - \ln \left( \frac{S_{DR}}{S_{DN}} \right)_{eq} \right] \tag{7}$$

where  $k = (1 + \cos^2 \beta_0) / \sin(2\beta_0)$ , and  $(S_{DR} / S_{DN})_{eq}$  is the clamping force ratio at the steady state condition. As it can be seen from Eq. 7, the time derivative of the speed ratio is a function of the input angular velocity, the elastic displacement of the input pulley, the actual value of the transmission ratio. The term  $g(\tau)$  is as follows:

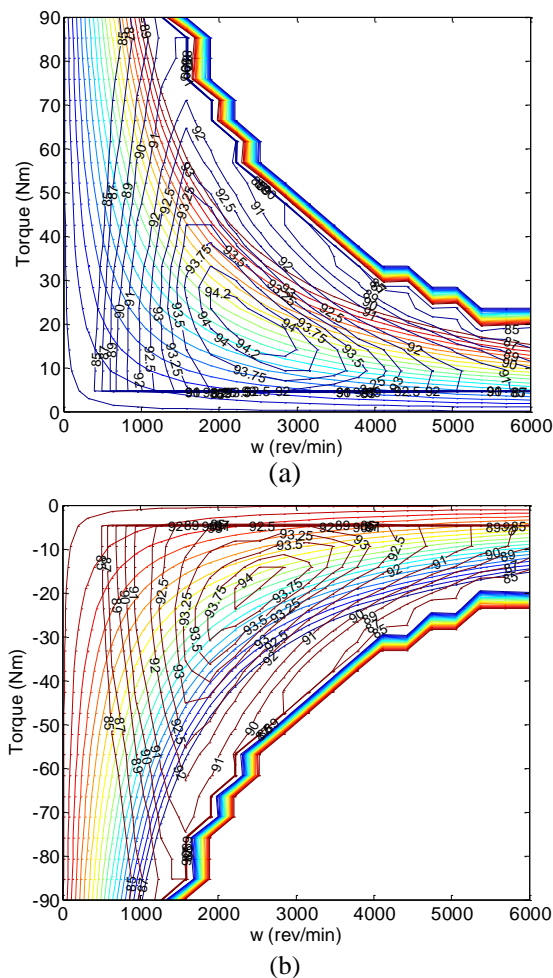
$$g(\tau) = \tau c \left[ \frac{\left\{ \pi + 2 \arcsin \left[ \rho \left( \frac{1-\tau}{\tau} \right) \right] \right\}}{\left\{ \pi - 2 \arcsin \left[ \rho \left( \frac{1-\tau}{\tau} \right) \right] \right\}} \right] \tag{8}$$

where  $\rho(\tau) = R/d$  and  $c \approx 0.81$ . The clamping force ratio at steady state condition seen in the Eq. 12 is calculated from the equilibrium equations given in [6]. This ratio is a function of speed ratio and the traction coefficient  $\mu_{tr}$  which is:

$$\mu_{tr} = \frac{\cos \beta_0}{2} \frac{T_{cvt}}{R_{cvt} S_{DR}} \tag{9}$$

where  $\beta_0$  is the sheave angle of the pulley of the CVT,  $T_{cvt}$  is the torque load on the CVT shaft,  $R_{cvt}$  is the pitch radius and  $S_{DR}$  is the clamping force of the drive pulley. A clamping forces ratio matrix is obtained by the way explained in [6] for using in the control algorithm. This matrix is a function of the traction coefficient and speed ratio. The contact radius of the chain of variator and the slope of the Eq. 7 called  $m(\tau)$  are also calculated as lookup tables for implementation in the controller design adopting from [6].

The maximum power of the electric motor considered in this paper is about 14 kW and the characteristic map is shown in Figure 2. The Figure 2a and the Figure 2b show the motor and generator modes of the motor respectively.



**Figure 2:** Electric motor efficiency maps: Motor mode (a), generator mode (b)

### III. CVT SPEED RATIO CONTROLLER DESIGN

In the developed model, the speed ratio is controlled by a PI feedback controller that is a well-known control algorithm for decades [15,16]. The dynamic behavior of the CVT is represented by the Eq. 7. In this controller, the initial condition of the angular velocity and the torque load of the drive pulley are given, and then the initial value of the clamping force ratio calculated with the help of the lookup table of the clamping force ratio obtained from the theoretical model [6]. For a constant clamping force of the driven pulley, the speed ratio is feed-backed shown in Figure 3. By changing the clamping force of the drive pulley, the desired speed ratio is obtained. In the model, the dynamics of the hydraulic unit is neglected since the valves are much faster than the variator.

In the model, the power request of the vehicle with time is determined and then, the angular velocity of motor is selected. This selection is made by staying on the optimum operation line of the motor map that is not shown in here. Afterwards, the speed ratio of the CVT is calculated with the help of the Eq. 4. However, since the CVT speed ratio varies in a range from 0.5 to 2, the angular velocity of the motor must be reselected considering this limitation. The updated angular speed of the motor corresponds to a new torque load to cover the same power request of the vehicle. Thus, the last angular speed and torque of the motor are determined, which enables one to find the necessary speed ratio of the CVT. This speed ratio is the instantaneous reference value of the PI-controller. The output

of the controller and its derivative are put into the Eq. 6, and the solution of this equation gives the real vehicle speed.

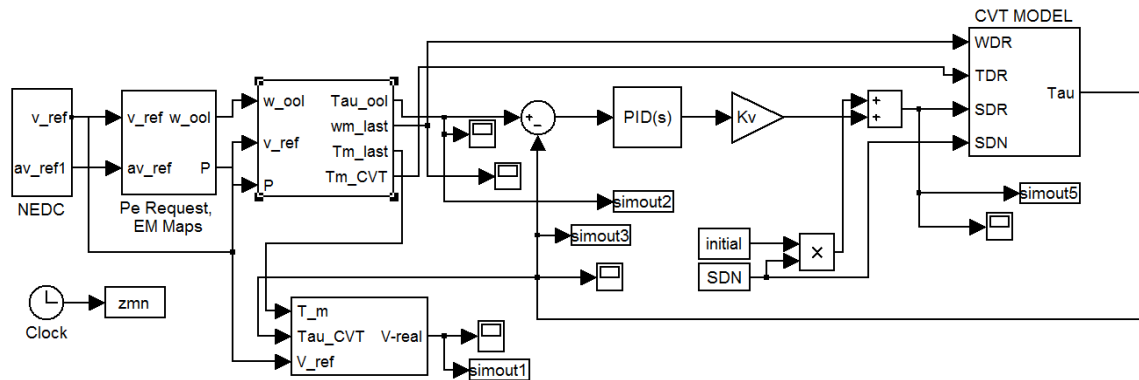


Figure 3: Configuration of the PI controller of the speed ratio of CVT for an electric vehicle

The error between the instantaneous reference value and the actual value of the speed ratio is as follows:

$$e(t) = \tau_{ref} - \tau(t) \tag{10}$$

The control variable is:

$$u(t) = K_p e(t) + \frac{1}{T_i} \int_0^t e(t) dt \tag{11}$$

where  $K_p$  and  $T_i$  are the proportional and integral time gains respectively. These coefficients are calculated by numerical experiments satisfying on the conditions of minimum settling time and minimum overshoot.  $K_p$  and  $T_i$  are determined 2.7 and 0.8. The other parameters used in the developed model are given in the Table 1.

Table 1. Vehicle parameters

Tire radius	
Frontal area of the vehicle	$A=1.407 \text{ [m}^2\text{]}$
Aerodynamic drag coefficient	$C_x=0.32$
Constant gear ratio	$\tau_D = 1:4.47$
Rolling friction coefficient	$f_R=0.01$
Mass moment of inertia	$I_m=0.0145 \text{ [kg.m}^2\text{]}$
The sheave angle of CVT	$\beta_0=11^\circ$
The distance between pulleys	$d=0.155 \text{ [m]}$
The clamping force of driven pulley	$S_{DN}=2.25 \text{ KN}$

#### IV. SIMULATION RESULTS AND DISCUSSION

In this part, the simulation results are given. First, the reference velocity of the vehicle which are the New European Driving Cycle (NEDC) is given in the Figure 4. The necessary power request of the vehicle as a function of time is given in the Figure 5.

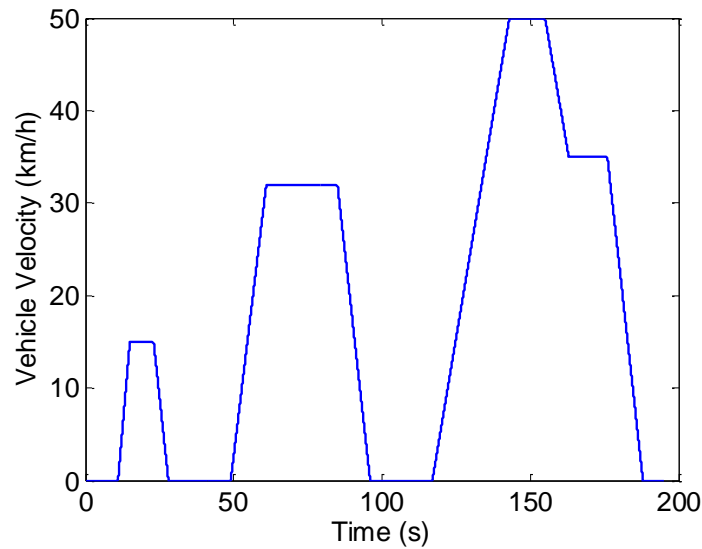


Figure 4: Reference velocity profile for the EV (NEDC)

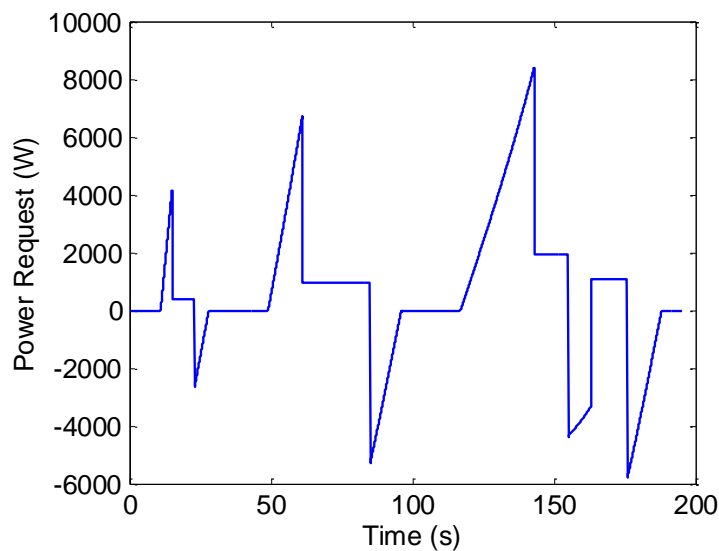
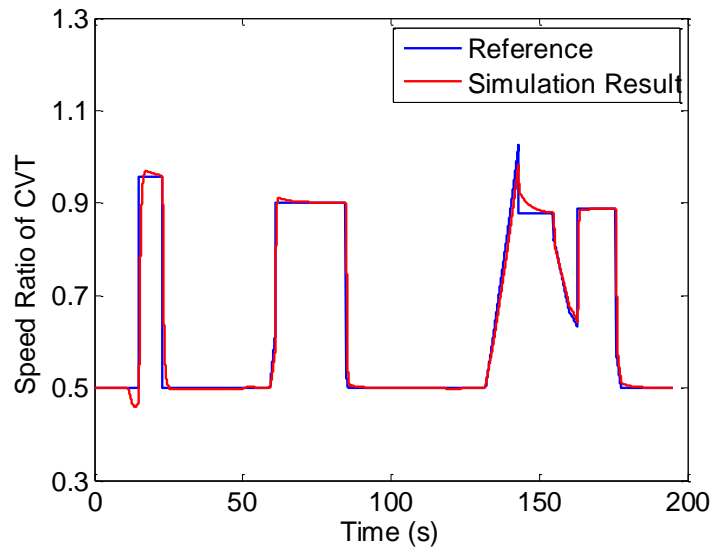


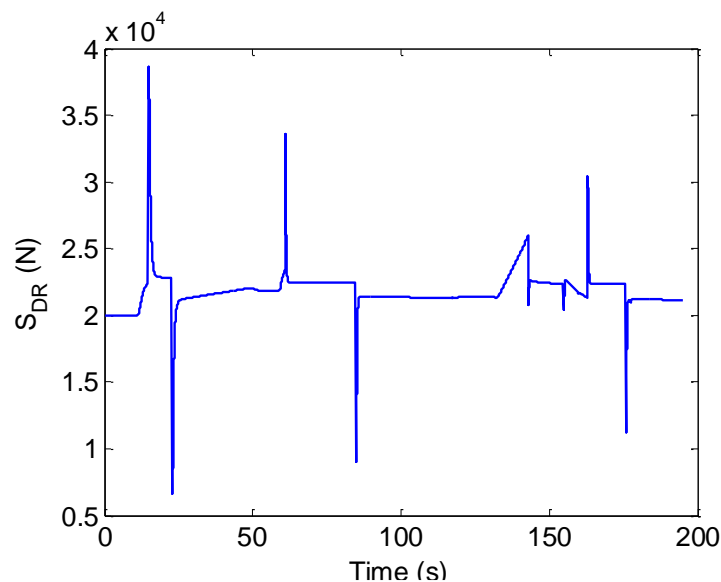
Figure 5: The necessary power request of EV

The necessary speed ratio of the CVT after the selection of the torque and the angular velocity of motor are given in Figure 6. This value is demonstrated as a reference speed ratio. The result of the controlled speed ratio is also given in the Figure 6. The comparison shows that the developed model for the control of the speed ratio considering the whole transient dynamics of CVT and vehicle is accurate. It is also observed from this figure that some delays exist between the reference and process of the speed ratio despite the overall comparison in good agreement. This small mismatching can be compensated with a PID gain schedule for different initial conditions of the CVT, which affect the shifting dynamics of the variator. Also, modern control technics such as fuzzy or model reference adaptive control can be applied to minimize the time delay. It should be noted that since the dynamics of the CVT is non-linear, the nonlinear control technics will be able to give better control results.

In figure 7, the variation of the clamping force of the drive pulley of the CVT is given for a constant clamping force of the driven pulley.



**Figure 6:** Simulation results of the reference and controlled speed ratio of CVT



**Figure 7:** The variation of the clamping force of the drive pulley of the CVT for the constant clamping force of driven side ( $S_{DN}=22600$  N)

In the Figure 8, the last simulation result that is the real vehicle speed considering the whole dynamics of the powertrain of the electric vehicle is given as a function of time. The comparison shows a good agreement between the simulation results and the reference velocity of the vehicle. It can be also seen from this figure that a time delay exists between the results. This is an expected case since the time delay already occurs between the desired and the final speed ratio of the CVT as shown in Figure 6. The dynamics of the system give a resistance to track the exact desired profile of the speed ratio and the velocity of the vehicle. However, this delay should be minimized by some control technic and algorithm, but these are out of the scope of this paper.

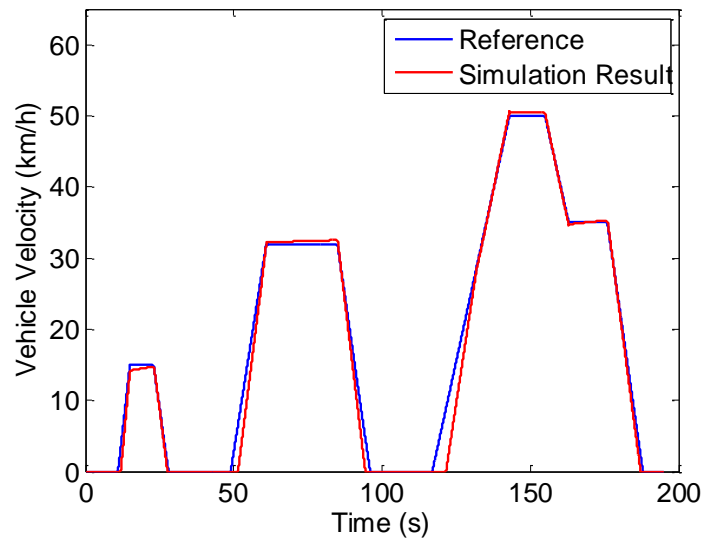


Figure 8: The simulation result of the real vehicle velocity and its comparison with the reference one

## V. CONCLUSION

In this paper, the control of the chain continuously variable transmission (CVT) for an electric vehicle is considered. The power request of the EV is obtained for a certain velocity profile and the torque and angular speed of the electric motor are selected from the characteristic map of EM. This calculation is checked whether the CVT ratio is in the limited value and the angular velocity of the motor are recalculated. This angular speed corresponds a new torque load for providing the same power request of the vehicle. Thus, the last angular speed and torque of the motor are determined and finally the last speed ratio of the CVT is determined. This speed ratio is implemented as a reference value and it is controlled with a feedback PI controller. The real velocity of the vehicle is obtained from the simulation and it is compared with the desired velocity profile. The comparison shows good agreement and thus, the proposed model can be a reference for different control algorithm.

## VI. FUTURE WORK

More future works can be done about the speed ratio controller. For instance, the parameters of the PI controller can be tuned for considering different initial conditions of the CVT. Furthermore, modern control technics such as 'fuzzy' or 'model reference adaptive control' can be applied since the dynamics of the CVT is non-linear. Also, the different driving cycle can be implemented to check the validity of the developed model. Moreover, the complex vehicle model can be considered with battery model and the controller can update for such a condition.

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