SCRUTINY TO THE NON-AXIALLY DEFORMATIONS OF AN ELASTIC FOUNDATION ON A CYLINDRICAL STRUCTURE WITH THE NONLINEAR EQUATIONS

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ABSTRACT

This paper is devoted to homogenization of partial differential operators to use in special structure that is a plate allied to an elastic foundation when it is situated through the basic loads (especially with the harmonic forces) with a Non-axially deformation of the cantilever. Furthermore, it contains the Equations of motion that they can be derived from degenerate Non-linear elliptic ones. Through the mentioned processes, there exists many excess works related to computing the bounded conditions for this special application form of study (when the deformation phenomenon has occurred). At the end of the article whole results of the study on a circular plate are debated and new ways assigned to them are discussed. Afterwards all the processes are formulised with the collection of contracting sequences and expanding sequences integrable functions that are intrinsically joints with the characteristic functions to expanding the behaviour of an elastic foundation. Thenceforth all the resultant functions are sets and compared with the other ones (without the loads). Sample pictures and analysis of the study were employed with the ANSYS software to obtain the better observations and conclusions.

KEYWORDS: *Intrinsically Joints, Contracting Sequences, Nonlinear Equations, Elastic Foundation, Differential Operators, Cantilever*

I. INTRODUCTION

Important classes of equations that they can actually be solved are the linear ones. It is noticeable the differences between a plate which is lie on an elastic foundation when is not loaded, and the same case with loaded, however, by axial forces. Therefore, p_r should be assumed equal to zero (that is related to the pressure on the internal wall of the elastic foundation), and it should be taken $\varphi = \pi / 4$ in the argument of $\rho e^{i\varphi}$. [1] In every rotation of a cylindrical plate that is attached to the foundation, it worthy to be mentioned the behaviour of the two sections that are rotating with one another; these facts are originally related to the place of adhesion. [2]

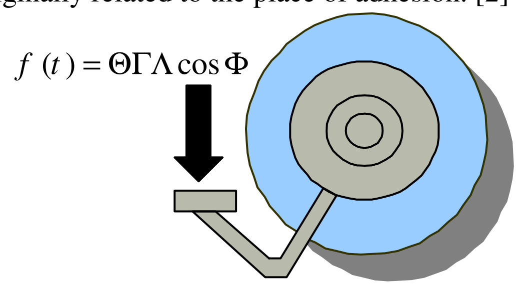


Figure.1: Schematics of the cylinder with the Harmonic forces - f(t) – that Occurred, which they caused the rotation of the cantilever

Table.1: components of the harmonic forces f(t)

Symbol	*Unit*	*Object*
Θ	1st Order	Coefficient
Γ	1st Order	Coefficient
Λ	1st Order	Coefficient
Φ	Degree	$(e^{i\theta})^{\circ}$

Figure.1 is the schematics of the cylinder with the cantilever attached to it, when the harmonic forces are acts on the plate of the cylinder and caused the deformation of the foundation. In fact attitude of the loading was demonstrated and it means the model of the structure, which is related to this study. [3]

II. RELATED WORK

As a matter of fact the primary process of the loading by the harmonic forces on the elastic foundation is very exquisite, because of the location of adhesion presumably going to be fractured. Last statement is originally related to the fraction phenomenon. Status of the cylinder location was demonstrated in Figure.1, and all the objects and units that we needs to analyse could be noticed from Table.1. Figure.4 shows us the status of the cylinder in the 2D and 3D mapping page on the x-y coordinates by the ANSYS software. [4] When the elastic foundation attached to the centre of the structure, fraction phenomenon is going to be increased in the adhesion location of the body and cause the deformation of the elastic part.

To the further describing the deformation of the cantilever, let assumed the loaded around the circle, edge freely supported and then setting uniform force p acting on it, it follows: [5, 6]

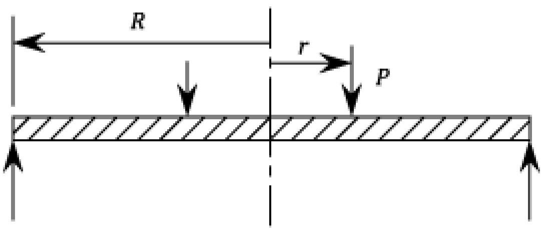


Figure.2: a divided plate into two regions, one for $x < r$ and the other for $x > r$
At $x = r$ the values of θ, y and M_{xy} must be the same for both regions

- If $x < r$, $w = 0$ and $P = 0$ then from the equation:

$$[1].\qquad \theta = \frac{C_1 x}{2} + \frac{C_2}{x}$$

And from the equation:

$$[2].\qquad y = \frac{C_1 x^2}{4} + C_2 Lnx + C_3$$

Since θ and y are not infinite at $x = 0$ then $C_2 = 0$ and since $y = 0$ when $x = 0$ and $C_3 = 0$ then above equations reduce to: $\theta = \frac{C_1 x}{2}$ and, $y = \frac{C_1 x^2}{4}$

- If $x > r$ and $w = 0$; then from the equation: [7]

$$3.\quad \theta = -\left(\frac{Px}{8\pi D}\right)(2Lnx - 1) + \frac{C_1^{'} x}{2} + \frac{C_2^{'}}{x}$$

And from the equation:

$$4.\quad y=-\left(\frac{Px^2}{8\pi D}\right)(Lnx-1)+\frac{C_1'x^2}{2}+C_2'Lnx+C_3'$$

Moreover equating the values of θ and M_{xy} at $x=r$ gives the following facts:

$$5.\quad -\left(\frac{\Pr}{8\pi D}\right)(2-1)+\frac{C_1'r}{2}+\frac{C_2'}{r}=\frac{C_1r}{2}$$

Then;[8]

$$6.\quad -\left(\frac{\Pr^2}{8\pi D}\right)(Lnr-1)+\frac{C_1'r^2}{4}+C_2'Lnr=\frac{C_1r^2}{4}$$

And;

$$7.\quad \left(\frac{P}{8\pi D}\right)\left[\left(1+\frac{1}{m}\right)2Lnr+1-\frac{1}{m}\right]+\left(\frac{C_1'}{2}\right)\left(1+\frac{1}{m}\right)-\left(\frac{C_2'}{r^2}\right)\left(1-\frac{1}{m}\right)=\left(\frac{C_1}{2}\right)\left(1+\frac{1}{m}\right)$$

When $M_{xy}=0$ at $x=R$ gives:

$$8.\quad \left(\frac{P}{8\pi D}\right)\left[\left(1+\frac{1}{m}\right)2LnR+1-\frac{1}{m}\right]+\left(\frac{C_1'}{2}\right)\left(1+\frac{1}{m}\right)-\left(\frac{C_2'}{R^2}\right)\left(1-\frac{1}{m}\right)=0$$

From equations 5 to 8, all the constants can be found to be:

- ✓ $C_1'=\frac{P}{4\pi D}\left[2+\frac{R^2-r^2}{R^2}\left(\frac{1-\frac{1}{m}}{1+\frac{1}{m}}\right)\right]$
- ✓ $C_2'=-\frac{\Pr^2}{8\pi D}$
- ✓ $C_3'=\frac{P\,r^2}{8\pi D}(Lnr-1)$

And the central deflection of the foundation is given by the value of y at $x=R$ and by swap equation number 4, reduces to: [9, 10]

$$9.\quad y=\left(\frac{P}{8\pi D}\right)\left[(R^2-r^2)\times\frac{\left(3+\frac{1}{m}\right)}{2\left(1+\frac{1}{m}\right)}-r^2Ln\frac{R}{r}\right]$$

• Then for $x>r$ we have:

$$10.\quad M_{xy}=\left(\frac{P}{8\pi}\right)\left[\left(1+\frac{1}{m}\right)2Lnx+\left(1+\frac{1}{m}\right)\times r^2\left(\frac{1}{x^2}-\frac{1}{R^2}\right)\right]$$

This has the max value when we set $x=r$;
Hence from the equations:

$$11.\ f_x=M_{xy}\times\frac{12\mu}{t^3}$$

$$12.\ f_z=M_{yz}\times\frac{12\mu}{t^3}$$

Those are originally sets for the Figure.3,

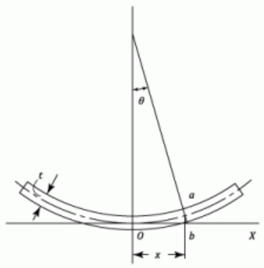


Figure.3: Circular Plate with the Symmetrically Loaded.

For this study we have: [11, 12]

$$13.\ \hat{f} = \left(\frac{6}{t^2}\right)M_{xy} = \left(\frac{3P}{4\pi t^2}\right)\left[\left(1+\frac{1}{m}\right)2Ln\frac{R}{r} + \left(1-\frac{1}{m}\right)\left(\frac{R^2-r^2}{R^2}\right)\right]$$

In the same manner:

$$14.\ M_{yz} = \left(\frac{P}{8\pi}\right)\left\{\left(1+\frac{1}{m}\right)2Ln\frac{R}{x} + \left(1-\frac{1}{m}\right)\left[\frac{2R^2-r^2}{R^2} - \frac{r^2}{x^2}\right]\right\}$$

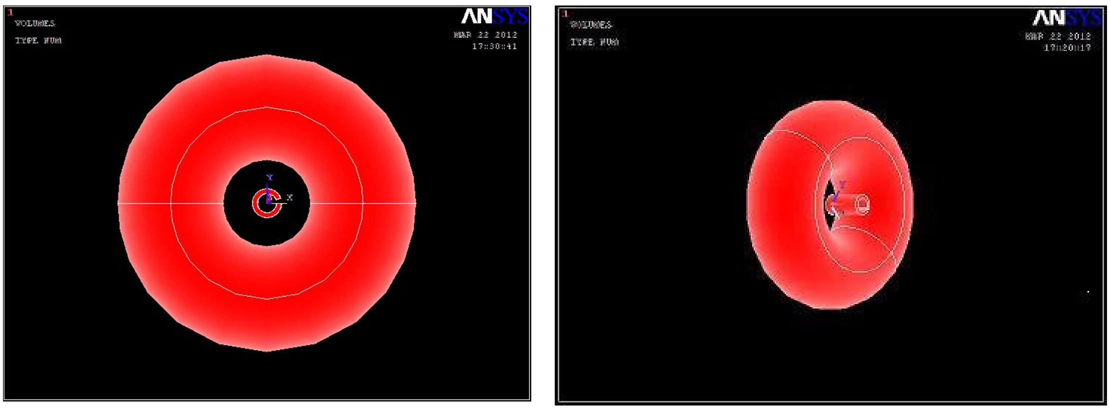
Finally:

$$15.\ \hat{f}_z = \left(\frac{3P}{4\pi t^2}\right)\left[\left(1+\frac{1}{m}\right)2Ln\frac{R}{r} + \left(1-\frac{1}{m}\right)\left(\frac{R^2-r^2}{R^2}\right)\right] = f_x$$

Mentioned equations are formulised to the prime of the study, and they clearly demonstrated as simple equations, for the deformation of the cantilever. [13, 14]

II.1 Simulation With The Software (ANSYS)

It is highly recommended to using the ANSYS Software in the mechanical engineering to modelling the behaviour of the materials and then finding the reasonable functions from the mathematics science to modelling the processes. [15] All the methods, which are used to model in this study, are based on the relational operators between the mechanical engineering and the applied mathematics, for the study on the deformation of the elastic foundation. [16]



(a) (b)

Figure.4: a) 2D and b) 3D picture of the cylinder and the plate that is attached to it

In the next sections, all parts of the cylindrical structure are going to be considered and the mathematical modelling of the structure is going to be discussed and set. [17]

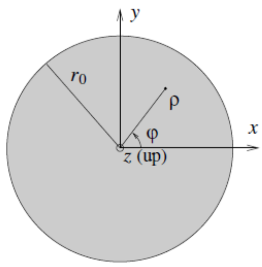


Figure.5: coordinate system of a problem

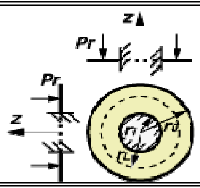
The problem of circular plates subjected to uniform or harmonic loads is one of the classical problems in elasticity theory, which also is encountered frequently in practice. Timoshenko and Goodier (1970) presented a three-dimensional solution for uniformly loaded isotropic circular plates with simply supported edges. [18, 19] Analytical solutions of isotropic circular plates with either remained or simply supported edges subjected to a uniform load, which were derived based on thin plate theory with Kirchhoff hypothesis, can be found in (Timoshenko and Woinowsky-Krieger, 1959). [20] Lekhnitskii (1968) investigated problems of anisotropic circular plates with clamped and simply supported boundaries. In the next section behaviour of the loads and forces are discussed and the equations of them to be reflected the deformation of a foundation are stablished. [21, 22]

III. EQUATIONS OF THE SECTIONS AND PARTS OF THE STRUCTURE

It should be beginning with the equations of a plate that is loaded by a concentrated force p has the form of:

$$1.\quad \varpi = P_r \gamma^2 f_0(\varepsilon) / (4D) \rightarrow D = r_0 - r_i$$

Table.2: Situation of loaded forces on a circular plate

shape	*Load*	*Object*	*Units*
Outer circular plate	p_o	**1st order tensor**	**m**
Inner circular plate	p_i	**1st order tensor**	**m**
	**I**	**2nd order tensor**	**1**

By integrating EQ.1, which is the basic influenced function for the condition of $p = 1$, it can be obtained the basic solutions for some special problems, which are generated and contents addressable in table.2. [23] Let assumed specific kind of the loading for the sampled force that is stated on the Table.3. It's been already mentioned in Table.3 a sample structure and it properties. It can be obtained the Young's modulus of the elasticity of 200Gpa, which is originally related to the property of the elastic foundation with the loading forces in the axially and non-axially orientation of the situated structure. All the solutions for this special problem are satisfied with the equations and formulas as follow. [24]

Table.3: Special problem about the loading with the specific properties

Inputs		
Loading:	***Ring Loading P_r =***	*15N/M*
	Loading Radius r_L =	*0.25m*
Geometry:	***Outer Radius r_o =***	*0.5m*
	Inner Radius r_i =	*0.1*
	Thickness h =	*2mm*
Material:	***Young's modulus E=***	*200GPa*
	Poisson's ratio ν =	*0.3*
Output:	***Unit of displacement w =***	*3mm*

Also a plate which is loaded by the uniform forces $q\cos n\theta$ that are distributed in the circumferences which are reduced in radius and they can be denoted by α. [25] This kind of consideration already used in the principles of the actions, in reply to above study that is represented the deflection at a specific point with the coordinates γ, ϕ in the form of:

$$\textbf{2.}\quad w = \frac{q\alpha l^3}{4D}\int_0^{2\pi} f_0(\sqrt{\alpha^2+\gamma^2-2\alpha\gamma(\theta-\phi)})\cos n\theta d\theta$$

In order to compute the mentioned integral (EQ.2), it is very aposematic to use the additional formulas for the cylindrical structure that it can be followed by:

$$\textbf{3.}\quad z_0(\sqrt{\alpha^2+\gamma^2-2\alpha\gamma(\theta-\phi)}) = 2\sum_{n=0}^{\infty}{}' J_n(\alpha)Z_n(\gamma)\cos n(\theta-\phi)$$

EQ.3 is clearly exemplified a new generation of integral formula for the cylindrical structure and the most important remark in this kind of study is to observe the sign ' into the 3.formula, which is introduced as a coefficient 1/2 for each n=0 case. This formula holds for $\alpha \le \gamma$. If $\alpha \ge \gamma$, then should be interchanged α and γ in the right hand side of the EQ.3, and then we have:

$$\textbf{4.}\quad z_0 = H_0^{(1)}(\sqrt{i}\sqrt{\alpha^2+\gamma^2-2\alpha\gamma\cos(\theta-\phi)})$$

The first major problem is to access the best results, and then it is much recommended to use imaginary regions in the EQ.4, which they can be, took into the real parts, and then it can be converted to the following as an imaginary function: [26]

For $\gamma \le \alpha$:

$$\textbf{5.}\quad w = w_1 = \pi\alpha q l^3[u_n(\gamma)f_n(\alpha) - v_n(\gamma)g_n(\alpha)]\cos n\phi/(2D)$$

For $\gamma \ge \alpha$:

$$\textbf{6.}\quad w = w_{II} = \pi\alpha q l^3[u_n(\alpha)f_n(\gamma) - v_n(\alpha)g_n(\gamma)]\cos n\phi/(2D)$$

According to the recent studies on the mathematical applications in the imaginary regions, we are exemplified all the results, with the imaginary coordinates. Equations 5 and 6 are replaced too, from the real ones, as imaginary functions for simplicity. When n=0 set on those (EQ, 5-6), then, they can be obtained in the changed form, and then they can be used in the axially symmetric problems.

The main issue that is stated in this article is to find the relationship between the nonlinear elliptic equations on a circular plate and the applications of oriented Differential equations, when an elastic foundation is going to crammed on the structure. It is well known that, the solutions are not always minimals of the functional. In general, they are stationary solutions. Consider for example to one of the parameters that is very important to remark, which is the elastic foundation (this condition is one of the conditions to cause of using the differential equations for modelling the structures as the same case). Presented study is complicated the conditions because of it should be focused on the forces and then presumably they can effect on a deformation of a whole shapes of the result, then it is very important to use an adaptable elastic cantilever to cause the minimum deformation. These conditions are generally related to the nature of the elastic materials. After try to find the nonlinear equations in a circular plate, now we should scrutiny to the adaptable and affecting parameters that are sets in the Table.4. In Table.4 three terms are stated. Stress (T), Strain rate (D), Unity Tensor (I). Term T is one of the orders has a different meaning, and is very important to contradistinguish between one another's. This term is essentially broached to show the stress imposed, when the forces are going to act on the structure. Anyhow this vaudeville is very common way to expose the differences between the symbols. It can be obtained from the Table.4 all the variables of the tensors, but they have variable reflected in the different locations, which they are actually based on the model of the structure.

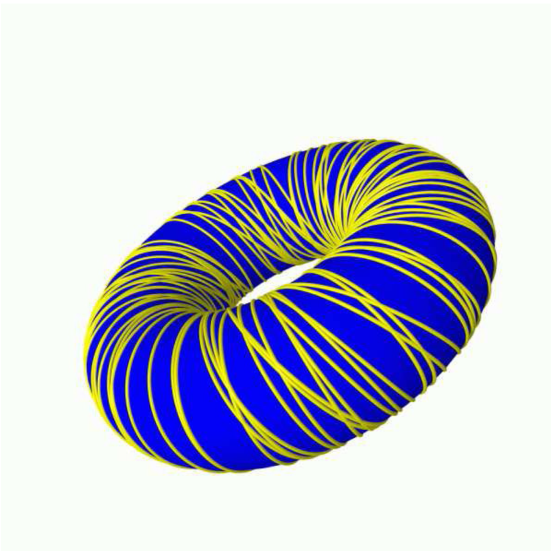


Figure.6: The rotationally symmetric torus, embedded in R3

Unity tensor (I) is very important Term to the simulation of the accurate equations. Last statement can steerage the engineering models with the nonlinear equations in the mathematics science.

Table.4: Properties of the forces that are caused the deformation of an elastic foundation

Quantity	Symbol	Object	Units
stress	**T**	2nd order tensor	N/m^2
strain rate	**D**	2nd order tensor	1/s
unity tensor	**I**	2nd order tensor	1

There are regular inspections of the equations, while should be explained some notations and preliminaries. And they must be companionship with the new methods (LEMMAS'). The thorough analysis of the solution in the next part is the objective of this article.

III.1 PRELAMINARIES AND NOTATION

Let $v \in A_p, K \geq 1$, then Let (γ_n) be a family in $A_p(K)$ and Let Ω be an open bounded set.

Let (a_h) be a family of functions satisfying: [4]

(1) $f_\Omega \left|Da_h\right|^p \gamma_h d_y \leq C_1 \prec \infty$ For every $h \in N$.

(2) There exists a function $u \in W^{1,p}(\Omega, \upsilon)$ such that $u_h \to u$ in $l^1(\Omega)$. Moreover Let (a_h) be a family of vector functions in R^n such that.

(3) $f_\Omega \left|a_h\right|^q \gamma_h^{-1/(p-1)} d_y \leq C_2 < \infty$ For every $h \in N$.

(4) $div\ (a_h) = f \in L^\infty(\Omega)$ On $C_0^1(\Omega)$ for every $h \in N$.

(5) There exists $a \in \left[L^q(\Omega, \upsilon^{-1/(p-1)})\right]^n$ such that $a_h \to a$ weakly in $[l^1(\Omega)]^n$. Then:

$$\textbf{6.}\quad \int_\Omega \left(a_h, D_{u_h}\right)\zeta\, dy \;\to\; \int_\Omega \left(a, Du\right)\zeta\, dy$$

For every $\zeta \in C_0^\infty(\Omega)$. (First LEM)

When the domain of the integral is divergence to ∞ , then it can be converted to the sigma as an indefinite series of the numbers. [27] This kind of changing of the equations should be plain sailing from now on. All the proposed processes can be checked into the different problems.

$$\textbf{7.}\quad q = \sum_{n=0}^{\infty} \Box_n(\zeta)[a_n \cos n\theta + b_n \sin n\theta]$$

EQ.7 with the helpmeet of the last formulas 5&6 made the easy way to construct the basic solutions for the loading that it can be represented by:

$$\textbf{8.}\quad q = \sum_{n=1}^{\infty} \xi^n [a_n \cos n\theta + b_n \sin n\theta]$$

$$\textbf{9.}\quad q = \sum_{n=1}^{\infty} \xi^{-n} [a_n \cos n\theta + b_n \sin n\theta]$$

When n is a positive integer ($n > 0$), then the integrating process is going to be simple from the last ones. [28] If we remind all the previous formulas, which they are basically equated from the Bessel functions, and then we have:

$$\textbf{10.}\quad \frac{d}{dz}[z^n J_n(z)] = z^n J_{n-1}(z)$$

,

$$\textbf{11.}\quad \frac{d}{dz}[z^n H_n^{(1)}(z)] = z^n H_{n-1}^{(1)}(z)$$

Because the EQs.10-11 have an imaginary parts, then we can set $z = \xi\sqrt{i}$ as a relationship between the real and imaginary parts of the equation, and then should be integrating and dividing them into the real and imaginary ones. Then it can be observed one of the simple ways that is dividing the complicated equations of functions into the simple ones.

The current model is clearly converged to the simple functions, that they are just demonstrated the whole shape of the cylindrical structure of this study, but this is true, just when it works in the imaginary region. Then it should be followed by:

$$\textbf{12.}\quad \int \xi^n u_{n-1}(\xi) d\xi = \xi^n [u_n(\zeta) + \upsilon_n(\xi)] / \sqrt{2},$$

$$\textbf{13.}\quad \int \xi^n u_{n-1}(\xi) d\xi = -\xi^n [u_n(\zeta) - \upsilon_n(\xi)] / \sqrt{2},$$

$$\textbf{14.}\quad \int \xi^n f_{n-1}(\xi) d\xi = \xi^n [f_n(\zeta) + g_n(\xi)] / \sqrt{2},$$

15. $\int \xi^n g_{n-1}(\xi) d\xi = -\xi^n [f_n(\zeta) - g_n(\xi)] / \sqrt{2}$,

We can easily consider the solutions of the loading which they varied from one another and those are related to the properties of the elastic foundation, when we used them across the cylindrical structure. [29] Presented study exposed a circular plate that is lying on an elastic foundation and modelled it, with the different equations and then derived the best and suitable functions for the results. [30]

IV. CONCLUSIONS AND RECOMMENDATIONS

Procedures that are used in this paper, essentially related to the application of the nonlinear equations into the special problem.The latter could not accommodate dissipative processes, but the enlarged one does. Recurring to the classical dissipative formulation then requires this Projection or "return to reality". In the mean time, a variational formulation has indeed been proposed. This article utilized all the nonlinear equations to solving in a special problem in the mechanical engineering, which was the deformation of the elastic foundation with a cylindrical plate attached to it, when it comes to be situated on a loading position. There is scope for further work. This conventional approach of study can be follow and continue to rise an advanced statements, with an equating the Nonlinear equations to the special cases with the determinate material. The presented approach can be extended to the other rotational symmetric shapes. The transmission coefficients of incoming waves can also be described by this approach**.** The presented paper takes on a deterministic approach to analyze the nonlinear equations on a circular plate with the special kind of the material. [31] And we may conclude that the stability analysis of the same structures calls for further research.

REFERENCES

[1] . Timoshenko, S.P., Woinowsky-Krieger, S., 1959. Theory of Plates and Shells (2nd Ed.). McGraw Hill, New York.

[2] . Thompson, J.M.T. and Hunt, G.W. (1984). Elastic Instability Phenomena. Wiley, New York.

[3] . Nirenberg, L., An extended interpolation inequality, Ann. Sc. *Norm. Sup.* Pisa, cl. *Sc.* IV. ser. 20(1966), 733-737.

[4] . *Khashayar Teimoori, Mahdi Pirhayati* (2012) *"Scrutinize The Abrasion Phenomenon With The Brunt Forces In The Joints Of The Robots And Proposed A New Way To Decrease That"*, *International Journal of Advances in Engineering & Technology, IJAET Vol. 3: Issue. 1. 14-20 March*

[5]. Braides, A. & Defranceschi, A., Homogenization of multiple integrals, Oxford University Press, New York, 2008.

[6]. Chiado Piat, V. & Defranceschi, A., Homogenization of monotone operators, *Nonlinear Anal.,* 14:9(1998), 717-732.

[7]. Defranceschi, **A.,** An introduction to homogenization and G-convergence, Lecture notes, School on homogenization, ICTP, Trieste, 1993.

[8]. Lions, J. -L., Lukkassen, D., Persson, L. -E. & Wall, P., Reiterated homogenization of nonlinear monotone operators, *Chin. Ann. Math.,* 22B:1(2011), 1-12.

[9]. Lukkassen, D. & Wall, P., On weak convergence of locally periodic functions, *J.Nonlinear. Math. Phys.,* **9:** 1(2002), 42-57 2010.

[10]. Tonti E., A mathematical model for physical theories, ((Rend. Lincei)) (1970) (in press).

[11]. N. M. Rivière, *Singular integrals and multiplier operators,* Ark. Mat. 9 (1971), no. 2, 243-278.

[12]. L. P. Rothschild and E. M. Stein, *Hypoelliptic differential operators and nilpotent groups* (preprint).

[13]. B. F. Jones, Jr., *A class of singular integrals,* Amer. J. Math. **86** (1964), 441-462. MR **28** #4308.

[14]. W. Connett and A. Schwartz, *A multiplier theorem for ultraspherical series,* Studia Math. 51 (1974), 51-70. MR 50 # 10674.

[15]. Luo, J.Z., Liu, T.G., Zhang, T., 2004. Three-dimensional linear analysis for composite axially symmetrical circular plate. *International Journal of Solid and Structures*, **41**: 3689-3706.

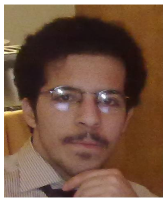
[16]. Ding, H.J., Xu, R.Q., Guo, F.L., 1999. Exact axisymmetric solution of laminated transversely isotropic piezoelectric circular plates (II)−Exact solution for elastic circular plates and numerical results. *Science in China* (*Series E*), **42**:470-478.

[17]. ABRAMOWITZ M., STEGUN I.A. (1964). Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables. U.S. Government Printing Of_ce, Washington, 1046p.

[18]. *on the existence of certain singular integrals,* Acta Math. 88 (1952), 85-139. MR 14,

[19]. Bensoussan, A. & Frehse, J., Nonlinear partial differential equations and applications,Springer-Verlag, New York, 2001.Alexey I. Andrianov & Aad J. Hermans; HYDROELASTICITY OF ELASTIC CIRCULAR PLATE,
[20]. Widman, K. O., Holder continuity of solutions of elliptic equations ,Manuscr. Math., 5(1971), 299-308.
[21]. Lame, G. (1852). *Lecons sur la theorie mathematique d'elasticite des corps solides*, Bachelier, Paris, France., Reprint: 2006, ISBN: 2-87647-261-9.
[22]. Koiter,W.T. (1960). A consistent first approximation in general theory of thin elastic shells. In Theory of Thin Elastic Shells, First IUTAM Symp. (Edited by W.T. Koiter), pp 12–33. North-Holland, Amsterdam.
[23]. Voyiadjis, G.Z. and Woelke, P. (2006). General non-linear finite element analysis of thick plates and shells. Int. J. Solids Struct., 43, 2209–2242.
[24]. Woelke, P., Abboud, N., Daddazio, R. and Voyiadjis, G. (2008). Localization, damage and fracture modeling in shell structures. Proceedings of World Congress of Computational Mechanics, Venice, Italy.
[25]. Bensoussan, A., Lions, J. L. & Papanicolaou, G., Asymptotic analysis for periodic structures, North Holland, Amsterdam, 1978.
[26]. Stephani, H. (1989). Differential equations: Their solutions using symmetries, Cambridge University Press, Cambridge.
[27]. G. Karpilovsky, Commutative Group Algebras, Monographs and Textbooks in Pure and Applied Mathematics, Vol. 78, Marcel Dekker, New York, 1983.
[28]. F. Verhulst, Nonlinear Differential Equations and Dynamical Systems, Springer, Berlin, 1990.
[29]. S. Wolfram, The Mathematica Book, 4th ed., Wolfram Media/Cambridge University Press, Champaign/Cambridge, 1999.
[30]. C. Truesdell and W. Noll, *The Nonlinear Field Theories of Mechanics*, Handbuch der Physik, Vol. III/3, Springer-Verlag, Berlin New York, 1965.
[31]. V. I. Smirnov, A Course of Higher Mathematics, Pergamon Press, Vol. V, 1964.

Authors



Khashayar Teimoori was born in Tehran, Iran in 1992. Currently he is pursuing B.Tech-final year degree in the field of mechanical engineering from Islamic Azad University, Science and Research Branch of Tehran-Iran and he is study in the field of applied mathematics in the Payame Noor University at the same time. Now he is a member in the technical societies as ASME, AMS (American Mathematical Society), ISME (Iranian Society of Mechanical Engineering), and IMS (Iranian Mathematical Society).His special interests are Analytical and computational methods in chaos, solving the problems in nonlinear equations and designing the complicated structures.