TRANSIENT ANALYSIS OF PIEZOLAMINATED COMPOSITE PLATES USING HSDT

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ABSTRACT
Piezoelectric materials have excellent sensing and actuating capabilities have made them the most practical smart materials to integrate with laminated structures. Integrated structure system can be called a smart structure because of its ability to perform self-diagnosis and quick adaption to environment changes. An analytical procedure has been developed in the work based on higher order shear deformation theory subjected to electromechanical loading for investigating transient characteristics of smart material plates. For analysis two displacement models are to be considered i.e., model-1 accounts for strain in thickness direction is zero whereas in model-2 in-plane displacements are expanded as cubic functions of the thickness coordinate. Navier's technique has been adopted for obtaining solutions of anti-symmetric cross–ply and angle-ply laminates of both model-1 and model-2 with simply supported boundary conditions. For obtaining transient response of a laminated composite plate attached with piezoelectric layer Newmark's method has been used. Effect of thickness coordinate of composite laminated plates attached with piezoelectric layer subjected to electromechanical loadings is studied.

KEYWORDS: *Anti-symmetric, electromechanical, piezoelectric layer, Navier's technique, Newmark's method, smart materials.*

NOMENCLATURE

A	- Extension Stiffness Matrix.
B	- Bending-Extension Coupling Matrix.
D_S	- Elasticity matrix relating shear force and shear strains.
D_b	- Elasticity matrix relating moments and bending strains.
E_i	- Young's modulus of elasticity in the ith direction.
I	- Moment of inertia.
F_i	- In-plane force resultants.
M_r	- Moment resultants.
F_t	- Transverse force resultants.
σ	- Stress vector.
Q	- Elastic constant matrix.
ε	- Strain vector.
e	- Piezoelectric constant matrix.
E	- Electric field intensity vector.
Δt	-Time increment.
δU	- Virtual strain energy.
δV	- Virtual work done by applied forces.
δK	- Virtual kinetic energy.
z	- Distance of a point along the z-axis.
x, y, z	- Cartesian co-ordinates.
u, v, w	- Components of deformation in x, y, z axes.
$\varepsilon_o, \varepsilon_o^*$	- Strain components.
L, L^*	- Bending curvatures.
φ , φ *	- Transverse shear strains.

I. INTRODUCTION

Laminated composite plates have found widespread applications in construction of engineering structures due to its light weight, high specific strength, high specific stiffness as well as excellent fatigue and corrosion resistance properties. As structures are exposed to various conditions in their service life, the material properties have been degraded. S. J. Lee et al [1] carried out transient analysis of composite laminated plates with embedded smart material layers using classical, first order and third order plate theories. Numerical simulations are adopted by them using finite element method and presented the results for studying the effects of boundary conditions, lamination scheme and loading. Shaochong Yang and Qingsheng Yang [2] investigated dynamic behavior of laminated plates with nonlinear elastic restraints by a varied constraint force model using a systematic numerical procedure. They also considered the restraining moments of flexible pads, with pads are modeled by translational and rotational springs. Through transient analysis they obtained the time histories of transverse displacements at various points of the laminated plate. Suleyman Başturk et al [3] investigated the nonlinear dynamic response of basalt/nickel FGM composite plates under blast load. They adopted Power Law Model (PLM) and Homogenous Laminated Model (HLM) to model the basalt/nickel FGM composite plates. Equations of motion for the plate are derived by them using principle of virtual work. They also investigated the effects of different approximations to model the basalt/nickel FGM composite plates. Chien-Ching Ma et al [4] studied the transient analysis of piezoelectric bi-materials subjected to dynamic concentrated force and electric charge. They solved the problem using Laplace transform and inverse Laplace transform method by means of Cagniard's method. They went for numerical calculations for examining transient behavior of field quantities in detail. B. L. Wang and N. Noda [5] came to know the importance of piezoelectric materials also called as smart structures used in composite laminated plates to ensure structural rigidity and understanding the fracture of the structure. In order to reduce the problem to the solution of singular integral equations Fourier transformation technique has been used. They also investigated the influences of crack position on stress intensity factors and layer thickness. Ruifeng Wang et al [6] developed 3 D finite element formulation for multiferroic composite and implemented into ABACUS software for its transient analysis. They showed that transient response can be influenced by input signal which could be tuned for strongest electric output. Jafar Rahiminasab and Jalil Rezaeepazhand [7] analyzed transient vibration of 3 layer sandwich plate based on classical plate theory with electrorheological fluid (ER). They employed Hamilton's principle and used constant average acceleration scheme for deriving and integration of finite element equations of motion. From results they found that change in electric field will have influence on system natural frequencies. Hassan A. Khayyat [8], studied to increase the physical understanding of the different phenomena which was taking place during the offset impact of an automotive bumper beam. He also studied to validate a modeling procedure for the system's crash performance. For the development and validation of modeling procedures and for the crash performance of the bumper beam he used the experimental database. J. N. Reddy et al [9] developed computational tools for nonlinear analysis of smart composite systems with embedded piezoelectric layers. They mainly focused on linear static analysis using FEM and nonlinear finite element formulation based on third order shear deformation theory. Effects of transverse normal stress/strains, transverse shear deformation has been incorporated by the theoretical model presented by Kant et al [10]. They adopted Navier's technique for obtaining solutions in closed form with solving boundary problem. Shiyekar et al [11] presented analytical solutions for cross-ply composite laminates integrated with piezoelectric fiber-reinforced composite (PFRC) actuators which are under bidirectional bending. For analyzing smart material plates subjected to electromechanical loading, higher order shear and normal deformation theory (HOSNT12) has been used by them. Based on Hamilton's principle and finite element methods, linear response of piezothermoelastic plate has outlined by Fariborz Heidary et al [12]. They presented numerical results for a piezolaminated plate subjected to thermomechanical loadings. With use of electric potential difference across piezo layers, they suppressed the vibrations on piezolaminated composite plate. In the above literature this work was not done. The transient analysis of anti-symmetric cross–ply and angle-ply laminates attached with piezo layer of both model-1 and model-2 with simply supported boundary conditions is carried out.

II. FORMULATION OF HSDT

In formulating the higher-order shear deformation theory, a composite plate of $0 \le x \le a$; $0 \le y \le b$ attached with an actuator and is simply supported along four sides of the plate is considered.

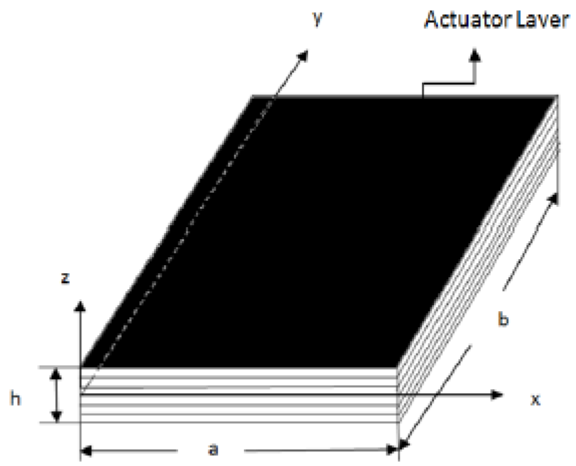

Fig. 1. Composite laminated plate attached with piezoelectric layer [11]

In order to approximate 3D-elasticity plate problem to a 2D one, the displacement components $u(x,y,z,t), v(x,y,z,t)$ and $w(x,y,z,t)$ at any point in the plate are expanded in terms of the thickness coordinate. The elasticity solution indicates that the transverse shear stress varies parabolically through the plate thickness. This requires the use of a displacement field, in which the in-plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field for model -1which assumes w (x, y, z) constant through the plate thickness and thus setting $\varepsilon_z = 0$ is expressed as [10]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad \text{……. (1)}$$

The displacement field for model-2 in which the in-plane displacements are expanded as cubic functions of the thickness coordinate in addition the transverse normal strain may vary nonlinearly through the plate thickness is expressed as [11]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned}\right\} \quad \text{…… (2)}$$

Where the parameters u_0, v_0 are the in plane displacements and w_o is the transverse displacement of a point (x, y) on the mid plane. The functions θ_x, θ_y are rotations of the normal to the mid plane about y and x–axes, respectively. The parameters u_0^*, v_0^*, w_0^*, θ_x^*, θ_y^*, and θ_z^* are the corresponding higher–order deformation terms and they represent higher–order transverse cross sectional deformation modes.

The linear constitutive relations for elastic layer coupled with piezoelectric layer are [12]:

$$\{\sigma\} = [Q]\ \{\varepsilon\} - [e]\ \{E\} \quad \text{……(3)}$$

The governing equations of displacement field are [12]:

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \qquad \text{…… (4)}$$

Substituting virtual strain energy (δU), virtual workdone (δV) & virtual kinetic energy (δK) in Eq.4 and integrating through the thickness of the laminate, and rewriting in-plane force, moment resultants, transverse force resultants and inertias in matrix, it is obtained as:

$$\begin{Bmatrix} F_i \\ F_i^* \\ -- \\ M_r \\ M_r^* \\ -- \\ F_t \\ F_t^* \end{Bmatrix} = \begin{bmatrix} \overline{A} & | & \overline{B} & | & 0 \\ \overline{B^t} & | & \overline{D_b} & | & \overline{0} \\ \overline{0} & | & \overline{0} & | & \overline{D_s} \end{bmatrix} \begin{Bmatrix} \varepsilon_o \\ \varepsilon_o^* \\ -- \\ L \\ L^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \qquad \text{…… (5)}$$

Following are the mechanical and electrical in-plane boundary conditions used for both cross-ply (SS1) and angle-ply (SS2) laminates of both model-1 and model-2.

The SS-1 boundary conditions for the higher order displacement model-1 are:

At edges x = 0 and x = a

$v_0 = 0$, $w_0 = 0$, $\theta_y = 0$, $M_x = 0$, $v_0^* = 0$, $\theta_y^* = 0$, $M_x^* = 0$, $N_x = 0$,
$N_x^* = 0$, $\xi = 0$ ….. Eq. (6a)

At edges y = 0 and y = b

$u_0 = 0$, $w_0 = 0$, $\theta_x = 0$, $M_y = 0$, $u_0^* = 0$, $\theta_x^* = 0$, $M_y^* = 0$, $N_y = 0$,
$N_y^* = 0$, $\xi = 0$ ….. Eq. (6b)

The SS-2 boundary conditions for the higher order displacement model-1 are:

At edges x = 0 and x = a

$u_0 = 0$, $w_0 = 0$, $\theta_y = 0$, $N_{xy} = 0$, $M_x = 0$, $u_0^* = 0$, $\theta_y^* = 0$, $M_x^* = 0$,
$N_{xy}^* = 0$, $\xi = 0$ ….. Eq. (7a)

At edges y = 0 and y = b

$v_0 = 0$, $w_0 = 0$, $\theta_x = 0$, $N_{xy} = 0$, $M_y = 0$, $v_0^* = 0$, $\theta_x^* = 0$, $M_y^* = 0$,
$N_{xy}^* = 0$, $\xi = 0$ ….. Eq. (7b)

The SS-1 boundary conditions for the higher order displacement model – 2 are:

At edges x = 0 and x = a

$v_0 = 0$, $w_0 = 0$, $\theta_y = 0$, $\theta_z = 0$, $M_x = 0$, $v_0^* = 0$, $w_0^* = 0$,
$\theta_y^* = 0$, $\theta_z^* = 0$, $M_x^* = 0$, $N_x = 0$, $N_x^* = 0$, $\xi = 0$ …..Eq. (8a)

At edges y = 0 and y = b

$u_0 = 0$, $w_0 = 0$, $\theta_x = 0$, $\theta_z = 0$, $M_y = 0$, $u_0^* = 0$, $w_0^* = 0$,
$\theta_x^* = 0$, $\theta_z^* = 0$, $M_y^* = 0$, $N_y = 0$, $N_y^* = 0$, $\xi = 0$ …..Eq. (8b)

The SS-2 boundary conditions for the higher order displacement model-2 are:

At edges x = 0 and x = a

$u_0 = 0$, $w_0 = 0$, $\theta_y = 0$, $\theta_z = 0$, $N_{xy} = 0$, $M_x = 0$, $w_0^* = 0$, $u_0^* = 0$, $\theta_y^* = 0$,
$\theta_z^* = 0$, $M_x^* = 0$, $N_{xy}^* = 0$, $\xi = 0$ …..Eq. (9a)

At edges y = 0 and y = b

$v_0 = 0$, $w_0 = 0$, $\theta_x = 0$, $\theta_z = 0$, $N_{xy} = 0$, $M_y = 0$, $v_0^* = 0$, $w_0^* = 0$, $\theta_x^* = 0$,
$\theta_z^* = 0$, $M_y^* = 0$, $N_{xy}^* = 0$, $\xi = 0$ …..Eq. (9b)

In Newmark's integration method, the time derivatives are approximated, using difference approximations, and therefore solution is obtained only for discrete times and not as a continuous function of time.

$$\{\hat{K}\}\{u_{t+\Delta t}\} = \{\hat{F}_{es}\} + \{\hat{F}_{pz}\} = \{F\},$$

Where, $\{\hat{K}\} = [K] + a_3[M]$

$$\{F\} = [F]_{t+\Delta t} + [M](a_3\{u_t\} + a_4\{\dot{u}_t\} + a_5\{\ddot{u}\}) \quad \text{…… (10)}$$

The Eq. (10) represents a system of algebraic equations among the discrete values of $\{u_t\}$ at time t to t+Δt in terms of known values at time. At the first time step t = 0, the values u_0, $\dot{u}_0, \ddot{u}_0$ are the initial known quantities. The transient response for both symmetric and anti-symmetric cross-ply and angle-ply laminates are estimated.

III. NUMERICAL RESULTS

A simply supported piezolaminated composite plate is considered for analysis. Composite plate is made of graphite/epoxy material. Piezoelectric material of PFRC attached at top of composite laminated plate.

The material properties of graphite/epoxy are [11]:

$$\frac{E_1}{E_2} = 25, \ \frac{G_{12}}{E_2} = 0.5, \ \frac{G_{23}}{E_2} = 0.2, \ E_2 = E_3 = 10^6 \ N/cm^2$$
$$G_{12} = G_{13} \ and \ \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

Material properties for PFRC layer are [12]:

C_{11} = 32.6 GPa, $C_{12} = C_{21}$ = 4.3 GPa; $C_{13} = C_{31}$ = 4.76 GPa; $C_{22} = C_{33}$ = 7.2 GPa; C_{23} = 3.85 GPa; C_{44} = 1.05 GPa; $C_{55} = C_{66}$ = 1.29 GPa; e_{31} = - 6.76 C/m^2; $g_{11} = g_{22}$ = 0.037E - 9 C/V m; g_{33} = 10.64E - 9 C/V m.

Fig. 2 and 3 shows non-dimensionalized shear stresses (τ_{xz}) as a function of thickness coordinate for cross-ply laminated composite plate with and without piezo layer subjected to uniform distributed load to side to thickness ratio (a/h) of 25. The induced stresses are more in top layers of laminated composite plates this is because of piezoelectric effect will be more on top layers. From these figures it is observed that the maximum percentage variation of stresses between with and without piezo layer is 14.56 in model-1 and 14.89 in model-2 of angle-ply laminated composite plates. Fig. 4 and 5 contains non-dimensionalized transverse shear stresses for cross-ply and angle-ply laminated composite plates with and without piezo layer of model-1 and model-2 under applied transverse loads as function of thickness coordinates. As the thickness coordinate is heading towards mid plane the transverse shear stress is maximum in case of angle-ply piezolaminated plates than that of cross-ply piezolaminated plates. The maximum variation in transverse shear stresses between with and without piezo layer is 10.59% in model-1 and 11.34% in model-2 of angle-ply laminated composite plates. Fig. 6 and 7 shows the effect of non-dimensionalized transverse shear stress (τ_{yz}) against thickness coordinate (z/h) for simply supported laminated composite plates with and without piezo layer subjected to uniform distributed load. The maximum variation in transverse shear stresses between with and without piezo layer is 10.59% in model-1 and 11.34% in model-2 of angle-ply laminated composite plates.

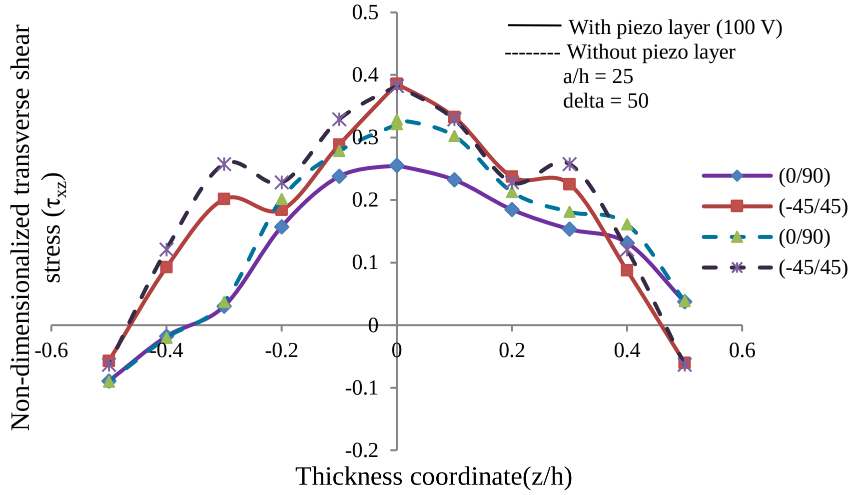

Fig. 2. Non-dimensionalized transverse shear stress (τ_{xz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-1 subjected to uniform distributed load

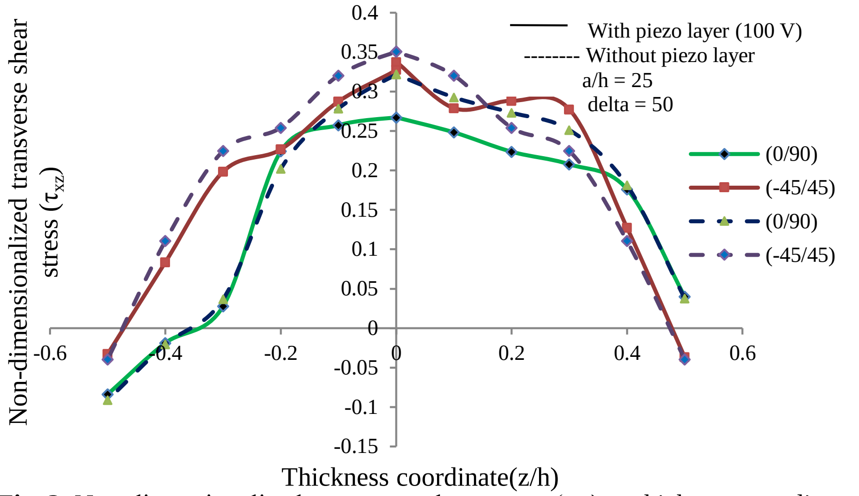

Fig. 3. Non-dimensionalized transverse shear stress (τ_{xz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-2 subjected to uniform distributed load

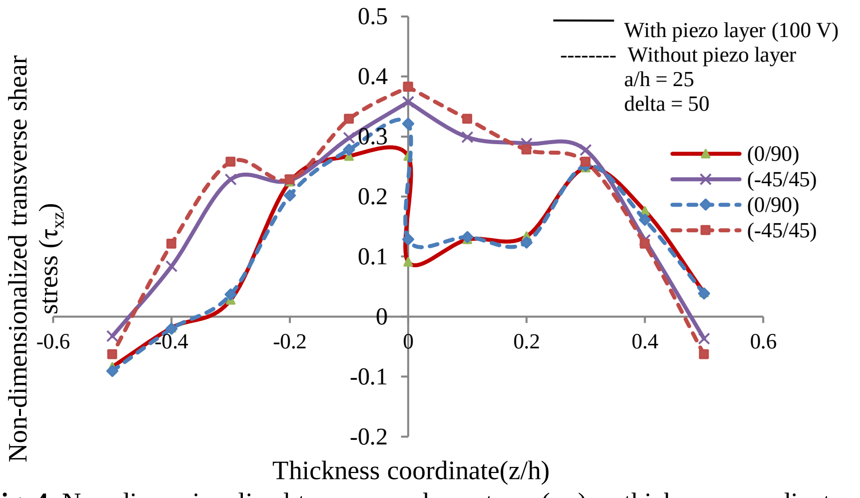

Fig. 4. Non-dimensionalized transverse shear stress (τ_{xz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-1 subjected to uniform distributed load

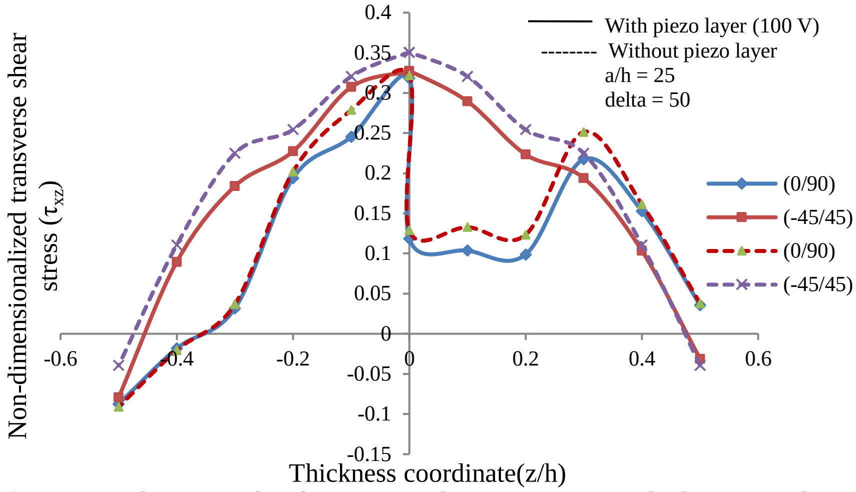

Fig. 5: Non-dimensionalized transverse shear stress (τ_{xz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-2 subjected to uniform distributed load

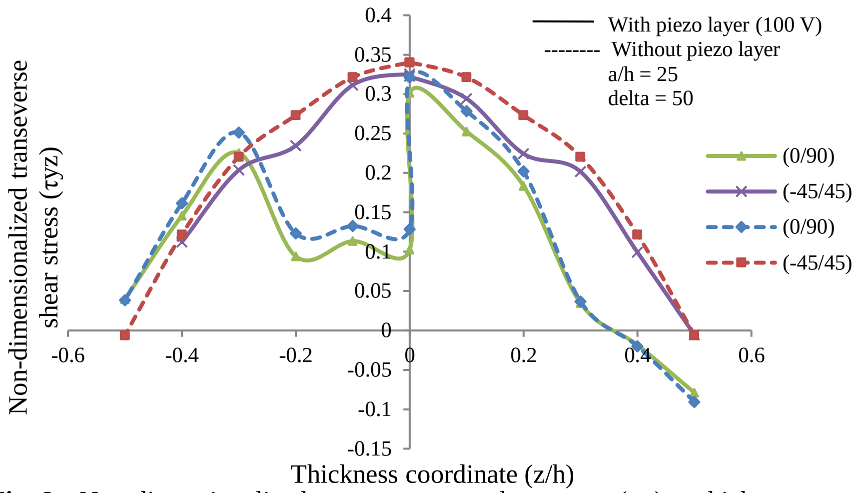

Fig. 6. Non-dimensionalized max. transverse shear stress (τ_{yz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-1 subjected to uniform distributed load

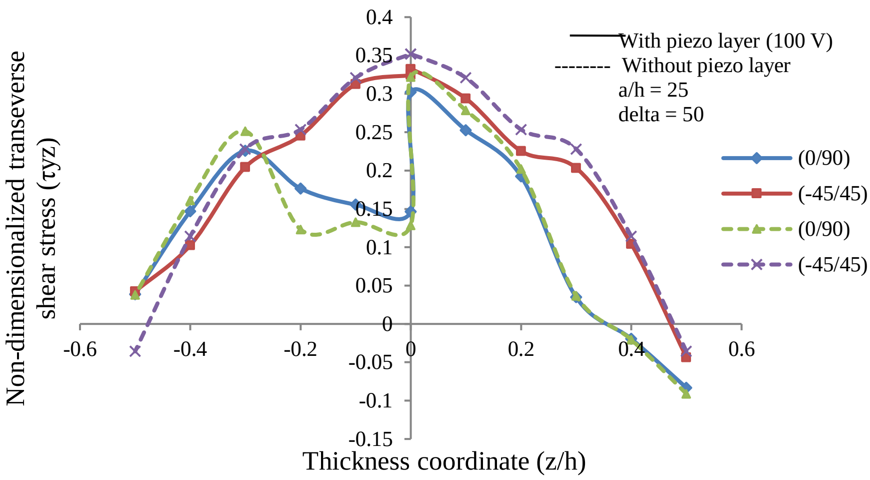

Fig. 7. Non-dimensionalized max. transverse shear stress (τ_{yz}) vs thickness coordinate (z/h) of simply supported laminated composite plate for model-2 subjected to uniform distributed load

IV. CONCLUSIONS

In this paper an analytical procedure is developed using higher order shear deformation theory for composite plates attached with piezo layer which are subjected to electromechanical loading. Transient response of antisymmetric cross-ply and angle-ply composite laminates attached with piezo layer has been analyzed. Effect of shear stress with respect to variation of thickness coordinates for both cross-ply and angle-ply composite laminates attached with piezo layer for both model-1 and model-2 are studied.

V. FUTURE SCOPE OF WORK

The work can be further extended as

- The transient response of symmetric cross-ply and angle-ply composite laminates attached with piezo layer can be analyzed.
- The bending and buckling analysis also can be carried out for both symmetric and anti-symmetric piezolaminated composite plates.

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