A COMPARATIVE STUDY OF VARIOUS METHODS TO EVALUATE IMPEDANCE FUNCTION FOR SHALLOW FOUNDATIONS

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ABSTRACT

The dynamic response of rigid shallow foundations can be evaluated by impedance functions which is defined as a ratio between force or moment (input) versus displacement or rotation (steady state), (output) at the centroid of the base of massless foundation. The foundation vibration problems are described in six degrees of problems hence the impedance functions derived for translational modes. Stiffness and Radiation affects the dynamic soil-structure interaction as Impedance function is a function of stiffness as well as radiation damping. The present paper is comparative study of various methods like Barkan, Dominguez, Dobry and Gazetas for evaluation of impedance functions for various modes of vibration of shallow foundation. The pros and cons of the methods for estimation of impedance function was discussed. The stiffness in vertical direction shows similar trend for all four methods. The use of equivalent radius approach to compute dynamic stiffness and radiation damping causes large errors which leads wrong estimation of impedance functions.

KEYWORDS: *Impedance function, Foundation Vibration, dynamic soil-structure interaction.*

I. INTRODUCTION

A variable force or motion may be used as input and output variables in defining a system transfer function. For vibrating motion three particular choices are commonly used which is impedance functions, mobility functions and transmissibility functions. The dynamic impedance is a function of the soil-foundation system and nature and type of exciting loads and moments. For particular case of simple harmonic excitations the dynamic impedance function(s) can be defined as the ratio between force (or moment) R and the resulting steady state displacement (or rotations) U at the centroid of the base of massless foundation.

$$\mathbf{Sz} = \frac{Rz(t)}{Uz(t)} \tag{1}$$

in which $R_z(t)$= R_z exp (iωt) and is the harmonic vertical force; and $U_z(t)$= Uz exp (iωt) harmonic vertical displacement of the soil-foundation interface. The entity R_z is the total dynamic reaction against the foundation and includes normal traction against the base mat and shear traction along any vertical sidewalls. The similar impedance functions for the possible degrees of freedom is Sy- lateral sliding, S_x longitudinal swaying, S_{rx}- rocking impedance (moment-rotation ratio) along x axis, S_{ry}- rocking impedance with respect to y axis. S_t is a torsional impedance for rotation about z axis (vertical). In case of embedded foundation we can incorporate two more “cross-coupling” horizontal rocking impedance $S_{x\text{-}ry}$ and $S_{y\text{-}rx}$. This cross coupled impedance effect is very small in case of surface foundations but its effect increases as the depth of embedment increases.

II. REVIEW OF METHODS TO FIND IMPEDANCE

2.1 Barkan Method (1962):

Static or dynamic tests of model footings are useful for establishing relations between the applied loads and response of these footings for a particular subsoil conditions. A comprehensive program of carefully controlled model tests, exemplified by the vibration tests reported by Barkan (1962), Pauw (1952) and Fry (1963). The spring constant represents a linear relation between applied load and displacement of the foundation which implies a linear stress-strain relation for the soil. Therefore, it follows that theory of elasticity can provide some useful formulas for the spring constants obtained through the theory of elasticity for circular and rectangular footings resting on the surface of the elastic half-space. These expressions have been obtained for rigid footing except for the case of horizontal motion, for which the spring constant was obtained by assuming a uniform distribution of shearing stress on the contact area and computing the average horizontal displacement of this area. These formulas apply for situations corresponding to rigid block or mat foundations with shallow embedment.

Table 1: Spring Constant for Rigid Rectangular Footing Resting on Elastic Half-Space

Motion	**Spring Constant**
Vertical	$Kz = \frac{G}{(1-\upsilon)} * \beta_z * \sqrt{4cd}$
Horizontal	$Kx = 4*(1+\upsilon)*G*\beta_x*\sqrt{cd}$
Rocking	$Kr = \frac{G}{(1-\upsilon)} * \beta_\psi * 8 * cd^2$

Note: Values of $\beta_z, \beta_x, \beta_\psi$ are given in table below for various values of (d/c)

Table 2: Values of β_z, β_x, and β_ψ for various values of d/c

d/c	β_z	β_x	β_ψ
1	2.20	0.96	0.55
2	2.25	0.90	0.60
4	2.40	1.05	0.80
6	2.70	1.10	0.95
8	2.75	1.15	1.14
10	2.80	1.15	1.26

The effect of embedment is to increase the soil resistance to motion of the foundation; thus, the effective spring constant is increased. There are some limitations to this method which are as follows:

- Not applicable for Torsion Stiffness.
- Satisfactory solution for rocking and sliding motion of foundation were now known
- Not for embedded foundation
- Radiation Damping cannot be computed

2.2 Dominguez (1978):

It has been known for some time that the static stiffness of a typical rectangular foundation can be approximated with reasonable accuracy by the corresponding stiffness of "equivalent" circular foundations. For the translational modes in the three principal directions (x, y and z) the radius Ro of the 'equivalent' circular foundation is obtained by equating the areas of the contact surfaces; hence:

$$R_o = \sqrt[2]{\frac{2B*2L}{\pi}} \qquad (2)$$

For the rotational modes around the three principal axes, the 'equivalent' circular foundations have the same area moments of inertia around x, y arid "z, respectively, with those of the actual foundation. Thus, the equivalent radii are:

$$R_{ox} = \sqrt[4]{\frac{16*L*B^{3}}{3*\pi}} \tag{3}$$

For rocking around the x-axis;

$$R_{oy} = \sqrt[4]{\frac{16*B*L^{3}}{3*\pi}} \tag{4}$$

For rocking around the y-axis; and

$$R_{oz} = \sqrt[4]{\frac{16*B*L*(B^{2}+L^{2})}{6*\pi}} \tag{5}$$

For torsion around the z-axis.

Table 3: Spring Constant for Rectangular Rigid Foundation

Motion	**Spring Constant**
Vertical	$K_z = \frac{4*G*Ro}{(1-\upsilon)}*J_v*(L/B)$
Horizontal (x Direction)	$K_x = \frac{8*G*Ro}{(2-\upsilon)}*J_x*(L/B)$
Horizontal (y Direction)	$K_y = \frac{8*G*Ro}{(2-\upsilon)}*J_y*(L/B)$
Rocking (x Direction)	$K_{rx} = \frac{8*G*R_{ox}{}^{3}}{(3*(1-\upsilon))}*J_{rx}*(L/B)$
Rocking (y Direction)	$K_{ry} = \frac{8*G*R_{o}y^{3}}{(3*(1-\upsilon))}*J_{ry}*(L/B)$
Torsion	$K_{r\psi} = \frac{16}{3}*G*R_{0}z^{3}*J_t*(L/B)$

Note: Values of J's varies with various values of (L/B)

Table 4: Values of J's for various values of (L/B)

L/B	**J_v**	**J_x**	**J_y**	**J_{rx}**	**J_{ry}**	**J_t**
1	1.081	1.035	1.035	0.965	0.965	0.950
2	1.130	1.044	1.105	1.039	1.031	1.000
4	1.196	1.085	1.221	1.117	1.140	1.016
6	-	-	-	-	-	1.166
8	-	-	-	-	-	-
10	-	-	-	-	-	-

There are some limitations for this method also which are as follows:

- Cannot Compute Spring Constant for Embedded Foundations
- Equivalent Circle approximation is not good for long rectangular or a similar elongated foundation shape.
- Value of J's for higher L/B ratio is not available

2.3 Dobry (1986):

This method is to compute the effective dynamic stiffness (K) and radiation dashpots (C) of arbitrarily shaped, rigid surface machine foundations placed on reasonably homogeneous and deep soil deposits. The method is based on a comprehensive compilation of a number of analytical results, augmented by additional numerical studies and interpreted by means of simple physical models. This method is also based on theory of elasticity.

Table 5: Spring Constant for Foundation

Motion	Spring Constant
Vertical	$K_z = S_z*(\frac{2LG}{(1-\upsilon)})$
Horizontal (x Direction)	$K_x = K_y - (\frac{0.21*LG}{0.75-\upsilon} * (1-\frac{B}{L}))$
Horizontal (y Direction)	$K_y = S_y*(\frac{2LG}{(2-\upsilon)})$
Rocking (x Direction)	$K_{rx} = S_{rx}*(\frac{G}{1-\upsilon})*(Ix)^{0.75}$
Rocking (y Direction)	$K_{ry} = S_{ry}*(\frac{G}{1-\upsilon})*(I_y)^{0.75}$
Torsion	$K_t = S_t*(G)*(J)^{0.75}$

Table 6: Radiation Damping

Motion	Radiation Damping
Vertical	$C_z = (\frac{3.4}{\pi*(1-u)})*\rho*V_s*A$
Horizontal (x Direction)	$C_x = \rho*V_s*A$ For $L/B \geq 3$
Horizontal (y Direction)	$C_y = \rho*V_s*A$
Rocking (x Direction)	$C_{rx} = (\frac{3.4}{\pi*(1-u)})*\rho*V_s*I_x$
Rocking (y Direction)	$C_{ry} = (\frac{3.4}{\pi*(1-u)})*\rho*V_s*I_y$
Torsion	$C_t = \rho*V_s*J$

For spring constant there are some Variable which are S_z, S_y, S_{rx}, S_{ry}, and S_t which varies with area, length, width and there are as follows:

$S_z = 0.8$	for $A/4L^2 < 0.02$
$S_z = 0.73+1.54*(A/4L\text{^}2)\text{^}0.75$	for $A/4L^2 > 0.02$
$S_y = 2.24$	for $A/4L^2 < 0.16$
$S_y = 4.5*(A/4L\text{^}2)\text{^}0.38$	for $A/4L^2 > 0.16$
$S_{rx} = 2.54$	for $B/L < 0.4$
$S_{rx} = 3.2*(B/L)\text{^}0.25$	for $B/L > 0.4$
$S_{ry} = 3.2$	for $B/L \geq 0.2$
$S_t = 3.8+10.7*\{1-(B/L)\}\text{^}10$	for $B/L \geq 0.25$

Different plots were made which will be shown in results later.

2.4 Gazetus (1991):

A key step in current methods of dynamic analysis of soil-foundation structure systems under seismic or machine-type inertial loading is to estimate, using analytical or numerical methods, and the (dynamic) impedance functions associated with a rigid but massless foundation. In practical applications the selection of an appropriate method depends to a large extent on the size and economics of the project, as well as the availability of pertinent computer codes. Moreover, the method to be selected must adequately reflect the following key characteristics of the foundation- soil system and the excitation.

- The shape of the foundation-soil interface (circular, strip, rectangular, arbitrary).
- The amount of embedment (surface, partially or fully embedded foundation, piles).
- The nature of the soil profile (deep uniform or layered deposit, shallow stratum over bedrock).
- The mode of vibration and the frequency (ies) of excitation.

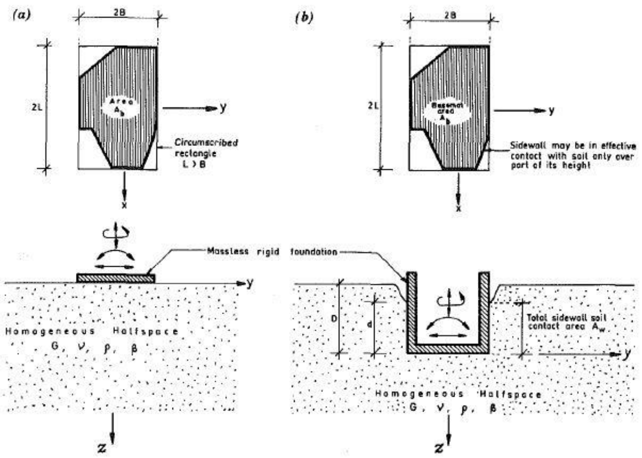

Fig.2: Surface Foundation of Arbitrary Shape (a) and Embedded Foundation of Arbitrary Basemat Shape (b) (Gazetus et al 1991)

Table 7: Dynamic Stiffness and Dashpot Coefficients for Arbitrarily Shaped Foundations on Surface of Homogeneous Half-Space

Motion (Surface)	**Spring Constant**
Vertical	$K_z = (\frac{2GL}{1-\upsilon})*(0.73+1.54*\chi^{0.75})$ where $\chi = \frac{A}{4*L^2}$
Horizontal (x Direction)	$K_x = K_y - (\frac{0.2GL}{0.75-\upsilon})*(1 - \frac{B}{L})$
Horizontal (y Direction)	$K_y = (\frac{2GL}{2-\upsilon})*(2+2.50*\chi^{0.85})$
Rocking (x Direction)	$K_{rx} =$ $(\frac{G}{1-\upsilon})*I_{bx}^{0.75}*(\frac{L}{B})^{0.25}*(2.4+0.5*(\frac{B}{L}))$
Rocking (y Direction)	$K_{ry} = (\frac{3G}{1-\upsilon})*I_{by}^{0.75}*(\frac{L}{B})^{0.15}$
Torsion	$K_t = 3.5*G*I_{bz}^{0.75}*(\frac{B}{L})^{0.4}*((\frac{I_{bz}}{B^4})^{0.2})$

Table 8: Dynamic Stiffness and Damping of Foundations Embedded in Half-Space with Arbitrary Basemat Shape

Motion (embedded)	**Spring Constant**
Vertical	$K_{z,emb} = K_z*(1+(1/21)*(\frac{D}{B})*(1+1.3*\chi))*(1+0.2*(\frac{A_w}{A_b})^{\frac{2}{3}})$
Horizontal (x Direction)	$K_{x,emb} = K_x*(\frac{K_{y,emb}}{K_y})$
Horizontal (y Direction)	$K_{y,emb} = K_y*(1+0.15*(\frac{D}{B})^{0.5})*(1+0.52*[(\frac{h}{B})*(\frac{A_w}{L^2})]\hat{}0.4)$
Rocking (x Direction)	$K_{rx,emb} = K_{rx}*\{1+1.26*\left(\frac{d}{B}\right)*[1+(\frac{d}{B})*((\frac{d}{D})^{-0.2})*((\frac{B}{L})^{0.5})]\}$
Rocking (y Direction)	$K_{ry,emb} = K_{ry}*\{1+1.26*((\frac{d}{L})^{0.6})*[1.5+((\frac{d}{L})^{1.9})*((\frac{d}{L})^{-0.6})]\}$
Torsion	$K_{t,emb} = K_t*\Gamma_w*\Gamma_{tre}$

There are also limitations for this method which is that formulas and charts are valid only for a constant depth of embedment and for a solid base mat shape.

III. METHODOLOGY

All the methods to find impedance functions for shallow foundation was compared with a MATLAB programme for all the modes of vibrations. The input values are width of foundation, dynamic frequency factor and angular velocity and the output values are dynamic stiffness of foundations for horizontal, vertical, rocking and torsional vibrations. Maximum 10 meter width of foundation is considered for calculation of dynamic stiffness.

IV. RESULTS

All the four methods are compared for the different stiffness which are vertical, horizontal, rocking and torsion vs width and dimensionless frequency factor. Below are the comparison of four methods:

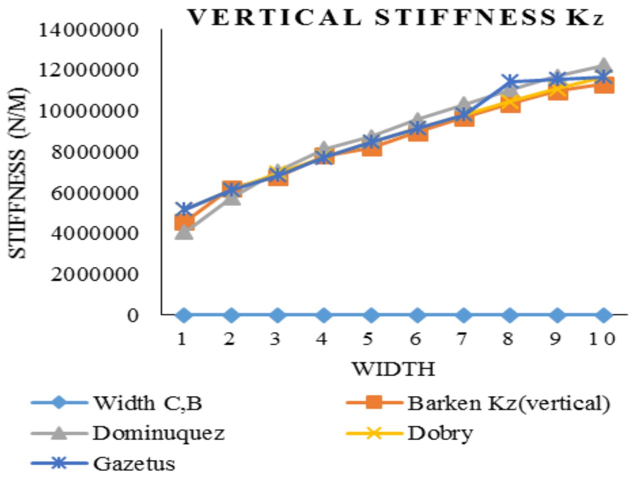

Fig.3: Vertical Stiffness vs Width

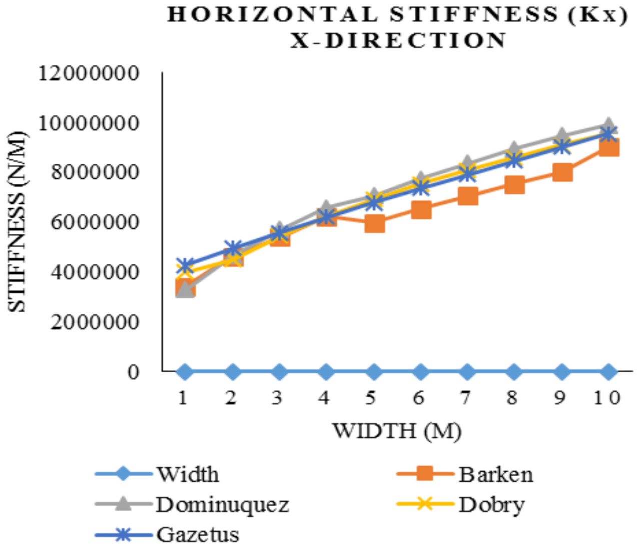

Fig.4: Horizontal stiffness vs Width

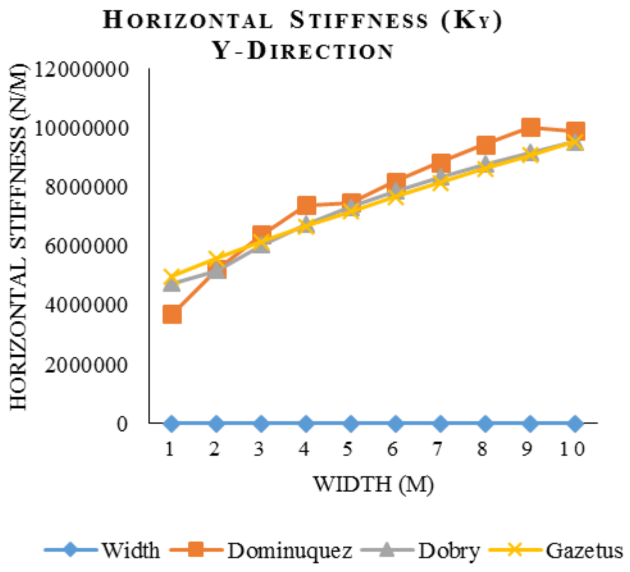

Fig.5: Horizontal Stiffness y-direction vs Width

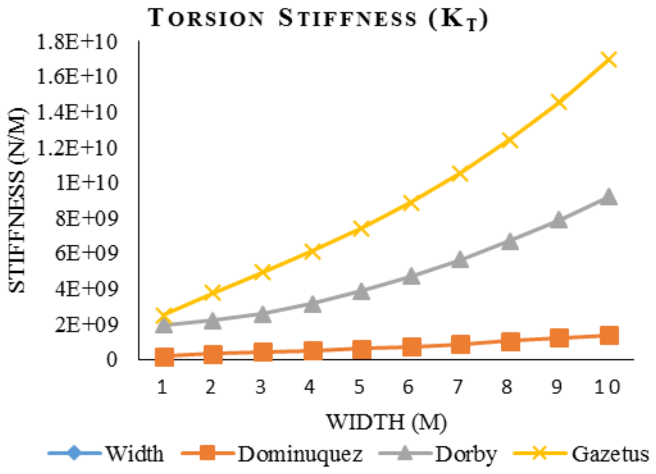

Fig.6: Torsion Stiffness vs Width

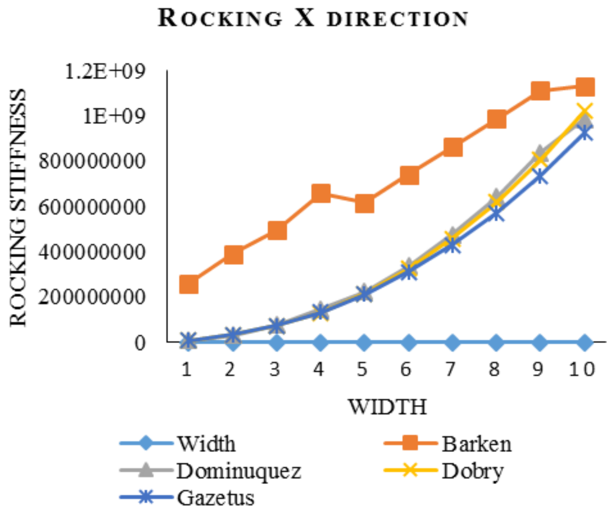

Fig.7: Rocking stiffness X-direction vs Width

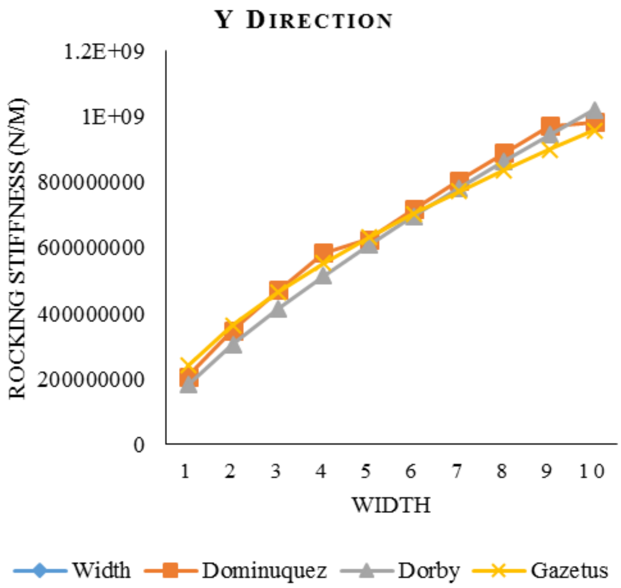

Fig.8: Rocking stiffness Y direction vs Width

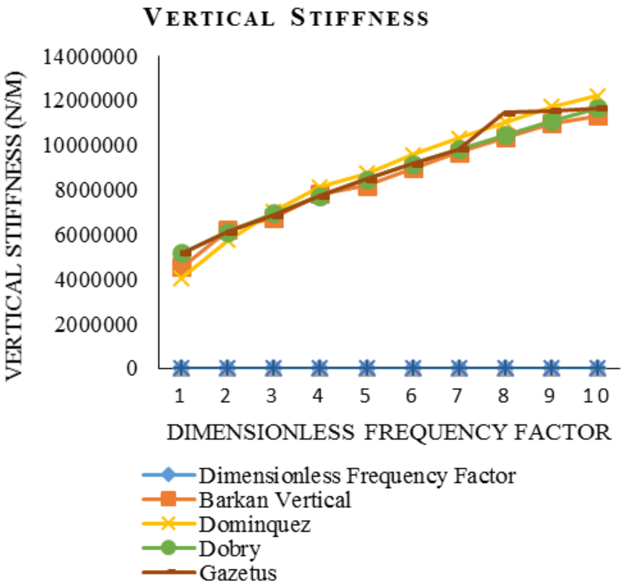

Fig.9: Vertical Stiffness vs Dimensionless Frequency Factor

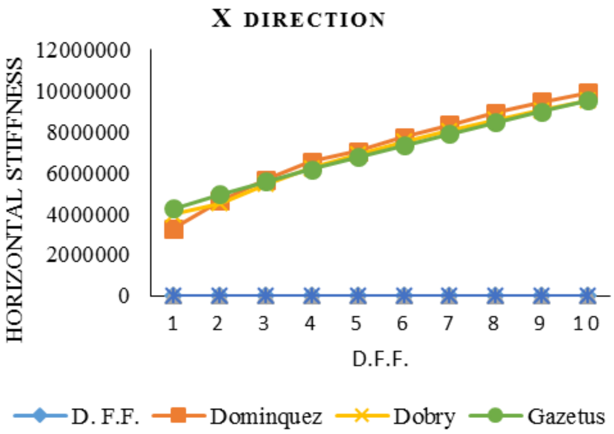

Fig.10: Horizontal vs D.F.F.

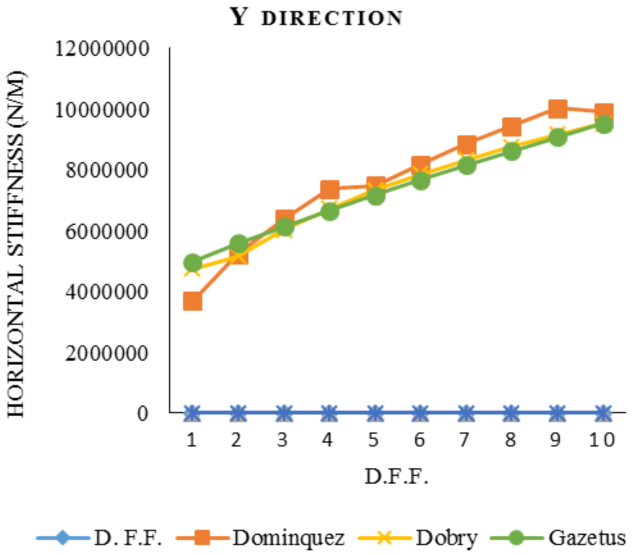

Fig.11: Horizontal Y vs D.F.F.

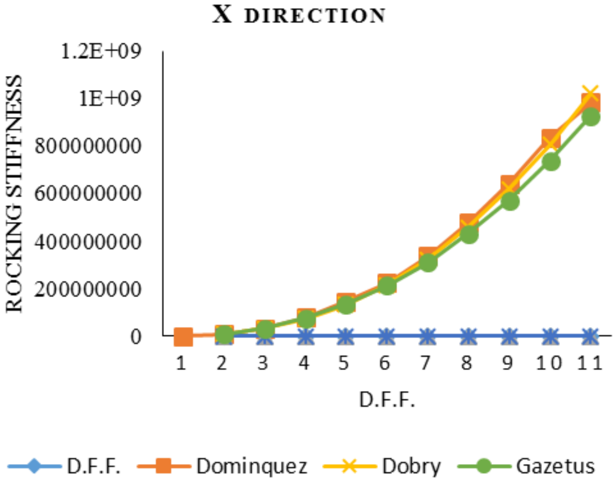

Fig.12: Rocking Vs D.F.F.

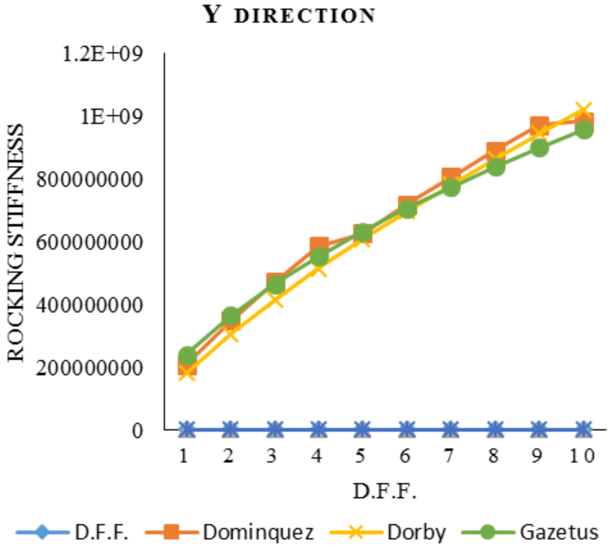

Fig.13: Rocking Y vs D.F.F

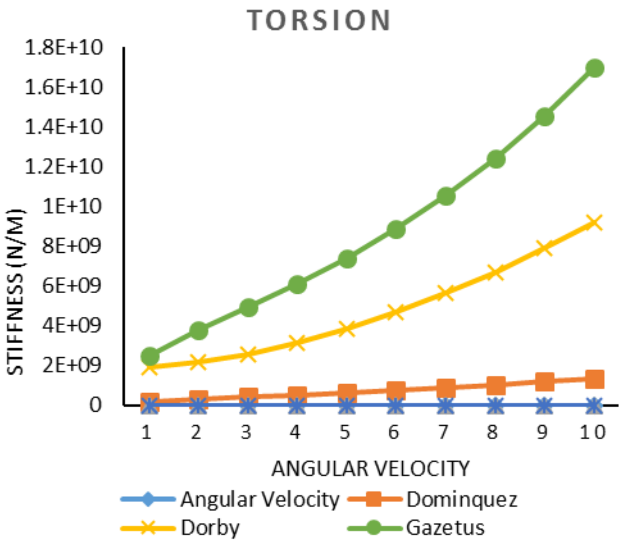

Fig.14: Torsion vs D.F.F

V. DISCUSSION

All the methods to find impedance for shallow foundation exhibits similar type of trends but for rocking and torsional stiffness, results of Dobry and Gazetas methods gives higher values of stiffness. In Barken method the results of stiffness is governed by value of J, which is irregularly varying with the ratio L/B which express irregularity in computation of stiffness in most of the modes of vibrations. Barken method is not applicable to find torsional stiffness and radiation damping. There is no satisfactory solution to find rocking and sliding motion of foundation. Barken method is giving solutions for just surface foundations and not able to compute stiffness for embedded foundations.

Dominguez method exhibits irregular approximations for ‘Equivalent Circle’ for elongated foundations. The value of J (Moment of Inertia about z-axis) for higher L/B ratio (L/B > 6) is not available hence not useful beyond certain dimensions of foundation. Gazetas approximations to find dynamic stiffness is applicable for both surface, partially embedded as well as embedded foundations. The side wall friction for various modes of vibrations incorporated in equations hence gives more realistic results in experimental studies. Moreover equivalent radius concept not creates any discrepancy in computation of dynamic stiffness. The comparative analysis indicates that the Gazetas method is covering all aspects of dynamic effects on shallow foundations for both surface as well as embedded foundations and exhibits reliable trend of results. It is most suitable for dynamic analysis of shallow foundation in layered soil.

VI. CONCLUSION

For Vertical Motion, the stiffness is similar for all the four proposed methods. As the width of foundation increases the stiffness is also increases. Barkan method is not suitable for Rocking and torsion Stiffness. Barkan and Dominquez is not applicable for embedded Foundation. Use of the equivalent circle approach to compute dynamic stiffness and radiation damping can cause large errors. As the L/B ratio increases the error in stiffness is also increases and v also increases. All the later three methods are quite similar as the values of different stiffness’s are similar but for the torsion stiffness the values are scattered for different values of width or frequency, etc. As the width or D.F.F increases value of stiffness is also increases whether it is a surface foundation or embedded foundation. Same implies for radiation damping whose values are also increases. It is suggested that the concept of equivalent radius should be reviewed analytically and the same should be verified by experimentations.

VII. FUTURE SCOPE

Stiffness values varies drastically for some methods which needs to be analyzed further. To verify the uncertainty in analytical outputs realistic scale model experimentations suggested.

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Notations:
K_z = Vertical Stiffness
K_x = Horizontal Stiffness in x direction
K_y = Horizontal Stiffness in y direction
K_{rx} = Rocking stiffness in x direction
K_{ry} = Rocking Stiffness in y direction
K_t = Torsion Stiffness
G = Shear Modulus
υ = Poison’s Ratio
I_x = Moment of inertia about x axis
I_y = Moment of inertia about y axis
J = Moment of inertia about z axis
ρ = Density
V_s = Shear Velocity
D = Depth of embedment
A = Area
B,c = Half width
L,d = Half Length
D.F.F. = ω*B/V_s
ω = angular Velocity
C_z = Vertical Radiation
C_x = Horizontal Radiation in X direction
C_y = Horizontal Radiation in Y direction
C_r = Rocking Radiation in X and Y direction
C_t = Torsion Radiation

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