A STUDY AND SIMULATION OF COMPUTER GENERATED HOLOGRAMS

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ABSTRACT

Holograms have been produced by optical techniques for the past few decades. Computer generated holography deals with the generation of holograms with the aid of computers. Holograms are constructed by recording the interference pattern obtained between the wavefields scattered from the object, called object wave, and a coherent reference wave. Computer generated holography utilises the wave theory of light to represent both the object and reference waves mathematically. In this work, Computer generated holograms are implemented using Matlab. Based on the Fraunhofer diffraction formula, the Fourier Transform operation is utilized to give the far field amplitude calculation required for calculating the interference pattern. The addition of the far field amplitudes of object wave and reference wave gives the hologram. The reconstructions of the object images from Computer Generated Holograms are also done. A method to avoid the overlapping between the object and the inverted image of object is discussed. The computer generated hologram was cropped at several positions and dimensions, and the reconstructed images are compared.

KEYWORDS: *Holography, wavefields, interference, Fourier Transform, Fraunhofer diffraction, Reconstruction*

I. INTRODUCTION

Holography is the technique of recording and reproducing a three dimensional image or scene very close to its reality. A hologram is a recording of the interference pattern formed when a point source of light (the reference beam) of fixed wavelength encounters light of the same fixed wavelength arriving from an object (the object beam). The wavefronts of light from the original object is recreated by the diffraction pattern when the hologram is illuminated by the reference beam only.[1]

Light has a phase and intensity but solely the intensity is recorded in traditional photography. However, a hologram stores the phase of light also due to the interference of the reference beam. The phase information is of most importance in holography because it provides the illusion of depth to the eyes and allows for an image to appear in three-dimensions. The procedure for creating holograms requires extremely specialised setups and equipments. By implementing the functions of the hologram creation setup employing a computer, many of the difficulties in creating a hologram can be eliminated.

The wave theory of light is utilised in computer generated holography to represent both the object and reference waves mathematically. With this knowledge, the superposition of these waves at any point in space can be calculated to obtain the interference pattern required for the hologram. Computer generated holograms do not require actual objects to generate the hologram as long as the light scattered or diffracted off the object could be represented mathematically. The light transmission and reflection properties of the object are no longer a problem since the ideal object wave can be computed mathematically, given its structure is properly described. Thus, a computer generated holographic image is computed by numerically simulating the physical phenomena of light diffraction and interference.

The fundamentals of computer generated holograms have been proposed by B. R. Brown and A.W.Lohmann in 1967 in [2].The simple implementations of computer generated Fourier holograms are explained by A.Vijayakumar et al in [3]. The techniques of digital holography and diffractive optics in scalar diffraction domain are discussed by Giuseppe A. Cirino et al in [5]. Color holograms and correction of chromatic distortion are discussed by Li-chein Lin in [6]. The mathematical explanations of computer generated holograms are explained by J.W.Goodman in [7]. The application of holography in security aspects are discussed in the papers [8], [9], [10] and [11].

The paper is arranged in the following sections. The first section gives a basic idea of computer generated holograms. The second part explains the theory and the mathematical aspects. The third section contains the details of the simulation and algorithm. The next section consists of the results obtained.

II. COMPUTER GENERATED HOLOGRAMS

Computer Generated Holography is an optical numerical technique, which avoid the traditional light interference recording process by computer numerical calculation and record the hologram directly. It does not require the actual light and the actual existence of the recording medium and reduces the difficulty of the realization of holography. Computer Generated Holograms has many applications in digital storage and display systems. Numerical generation and encryption of hologram are two important topics that have instigated abundant research and analysis works within the past twenty years.

An important need for synthesizing computer generated holograms is to create optical wavefronts from objects that do not physically exist. The process of synthesizing a hologram usually consists of the following steps[2] . The first step is to compute the propagation of the complex amplitude from the object to the hologram plane. The second step is to encode the complex amplitude as a real, non-negative function from which the hologram can be generated on a graphic output device. The simulation of the interference fringes caused by interaction between the reference beam and object beam in conventional holography can be considered as an example for this. The next steps are to make the hologram and to reduce it to a reasonable size for diffracting light. The final photo reduction step could be eliminated by using special output devices such that hologram can be written directly in the desired size.

An important property of hologram is that information about the object is spread over the entire image plane; therefore each and every portion of hologram contains the complete information of the object. This property can be utilised so as to make holograms effective for communication.

III. MATHEMATICAL EXPLANATION

The theory of computer generated holograms is well explained by J.W.Goodman in "Introduction to Fourier Optics" [7].Consider a planar object in an opaque (*m, n*) plane called as the object plane. Let (*x,y*) be the hologram plane which is at a *z* from object plane. The amplitude transmittance functions for the object wave and the reference waves are *To*(*m,n*) and *Tr*(m,n) respectively. The Fresnel diffraction formula is given by

$$U\left(x,y\right)=\frac{e^{jkz}e^{j\frac{k}{2z}\left(x^2+y^2\right)}}{j\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{T\left(m,n\right)exp\left[j\frac{k}{2z}\left(m^2+n^2\right)\right]\right\}exp\left[-j\frac{2\pi}{\lambda z}(xm+yn)\right]dmdn \tag{1}$$

The phase factors outside the integral can be neglected, it can be seen that the kernel that transforms *T*(*m*,n) to *U*(*x,y*) is the kernel of two dimensional Fourier transform. Fraunhofer diffraction formula is obtained from Fresnel Integral of diffraction by the following condition

$$\frac{k}{2z}\left(m^2+n^2\right)_{\max}<<1 \tag{2}$$

The Fraunhofer diffraction (m,n) formula is used to relate the far field amplitude U(x,y) (at the hologram plane) to the object plane (m,n) by the equation

$$U\left(x,y\right)=\frac{e^{jkz}e^{j\frac{k}{2z}\left(x^2+y^2\right)}}{j\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{T\left(m,n\right)\right\}exp\left[-j\frac{2\pi}{\lambda z}(xm+yn)\right]dmdn \quad (3)$$

For the calculation of the diffracted field amplitude profile for object and reference , in the equation(3), T(m,n) is to be replaced by the sum of To(m,n) and Tr(m,n).

$$U\left(x,y\right)=\frac{e^{jkf}e^{j\frac{k}{2f}\left(x^2+y^2\right)}}{j\lambda f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[T_o\left(m,n\right)+T_r\left(m,n\right)\right]exp\left[-j\frac{2\pi}{\lambda z}(xm+yn)\right]dmdn \quad (4)$$

Consider the reference beam to be a plane wave in the image plane. Therefore, a point source is chosen as the reference, which, when Fourier transformed, yields uniform illumination in the image plane. Dirac delta function, when Fourier transformed, produces a constant i.e., independent of the position in the image plane in spatial domain [3].

The intensity distribution is calculated in the image plane from equation given below :

$$I=UU^* \quad (5)$$

The reconstruction is done by evaluating the function T(x,y) using the Fraunhofer diffraction formula. This is given in equation (6)

$$U\left(u,v\right)=\frac{e^{jkf}e^{j\frac{k}{2f}\left(u^2+v^2\right)}}{j\lambda f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{T\left(x,y\right)\right\}exp\left[-j\frac{2\pi}{\lambda f}(xu+yv)\right]dxdy \quad (6)$$

The exponential term before the integral vanishes when the intensity is calculated. The integration results in four terms [4]. The first term corresponds to the Fourier transform of a constant. The second term is the autocorrelation function of T_o (m,n) which produces a peak at the geometric center of the observation plane which will be removed by filtering. The third and fourth terms are of interest in holography and correspond to the interference between the object wave and reference wave from the point source. The evaluation of the third and fourth integrals yields two separate real images of the object at the (u,v) plane. This means that two transmittance functions of the object are reproduced at the back focal plane of the lens such that one is the conjugate of the other.

IV. SIMULATION SETUP

4.1. Simulation algorithm for Generation and Reconstruction of computer generated holograms

The computer generated holograms were generated and reconstructed using Matlab (version R2012a). The main steps to follow in the development of a computer generated hologram and the reconstruction of the object from the hologram is shown in the figure 1.

The main steps in the simulation algorithm are discussed below:

Step 1: Construction of reference wave:

The reference wave is taken as a plane wave which can be constructed using a Dirac delta function. The transfer function of the matrix can be generated by making only one element of the square matrix as constant and the rest of the elements are set to zero. There is no restriction in choosing the position. Nonetheless, if the center i.e. (((N/2) +1),((N/2)+1)) is selected, it simplifies future hologram reconstruction.

Step 2: Construction of object wave:

The object can be either generated or loaded. For simpler objects like a square object, it is easier to generate and this method directly gives the matrix. In the case of other objects, for example, a picture, loading the image is necessary. The loaded image is converted into matrix for further manipulation.

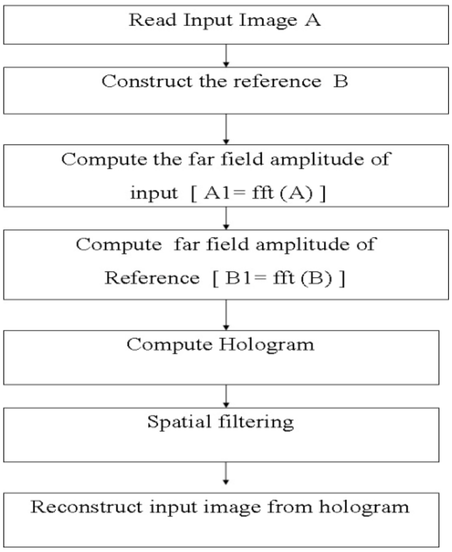

Figure 1: Flow chart for the simulation algorithm

4.1.1. Method to rearrange the input image to avoid overlapping in the reconstructed image:
As discussed in section 3, the reconstruction of hologram produces two separate real images of the object at the (u, v) plane. If these two images are overlapping, it will be difficult to identify the object image from the reconstructed hologram. So the input image has been modified so as to identify both the object and its inverted image in the reconstructed hologram without any confusion.
A 256 x 256 image has been generated and the input image was resized to fit the top left quarter of the image. This has resulted in non-overlapping images in the reconstructed hologram. The input image and its modified part are shown in figure 2 and figure 3 respectively. The reference image is shown in figure 4.
Step 3: Construction of far field patterns of object and reference.
The Fourier Transform operation gives the far field amplitude calculation based on the Fraunhofer diffraction formula. This is solved using the Discrete Fourier Transform algorithm (DFT). But, DFT calculation increases time proportional to N^2. The speed of computation is reduced to N log N using the Fast Fourier Transform algorithm. This is used to calculate the far field amplitude U(x, y).

Figure 2. Input image

Figure 3. Modified input

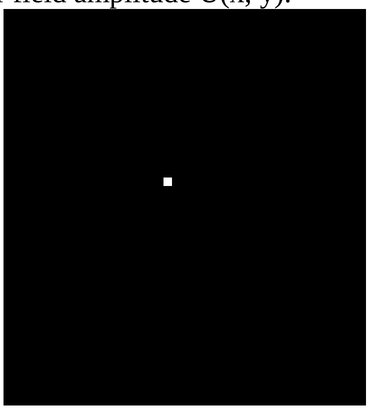

Figure 4. Reference image

Step 4: Hologram Computation and spatial filtering
The addition of the far field matrices of the object and reference is performed by matrix addition. The resulting matrix is the hologram matrix. The square of the matrix gives the intensity values across the hologram plane. Filtering is done, using matrix subtraction.

Step 5: Reconstruction of the hologram

The matrix corresponding to the filtered hologram is Fourier transformed. The intensity of the resulting matrix is calculated by squaring it. The matrix is imaged. The reconstructed image shows two images of the object out of which one is inverted according to the theory

4.2. Cropped hologram reconstruction:

If we tear a photograph into pieces, from one of the pieces, retrieval of full information is not possible. But, in a hologram, when broken into pieces, every single piece has the full information. This hologram reconstruction variation is demonstrated in computer generated Fourier holograms by cropping the original 256 x 256 hologram at different positions and reconstructing them. The convergence of the reconstructed image towards the original image with the increase in size of the hologram is visible from results.

V. RESULTS

Computer generated holograms were generated and reconstructed using the simulation algorithm discussed in this paper .The simulations were done using Matlab v R2012a. The holograms were cropped at different positions and were reconstructed. The results obtained are shown below.
The effect of overlapping of object and its inverted image during reconstruction can be inferred from figure 7. The far field pattern of the input image in figure 2 is shown in figure 5, its hologram in figure 6, corresponding reconstructed image in figure 7 respectively.

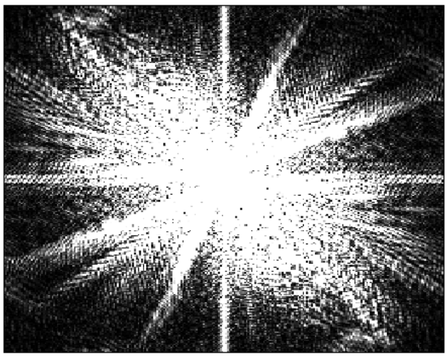

Figure5. Far Field Pattern of Figure 2

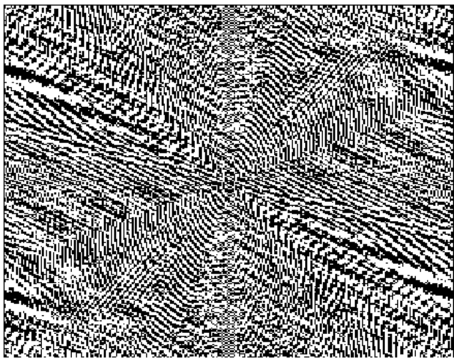

Figure6. Computer Generated Hologram of Figure 2

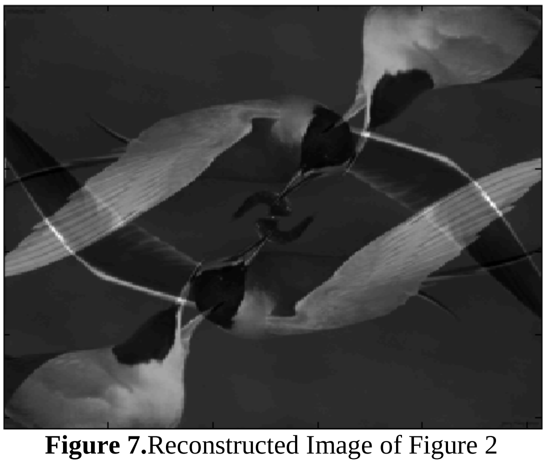

Figure 7. Reconstructed Image of Figure 2

The holograms are reconstructed by rearranging the input image to avoid overlapping as explained in section 4.1.1 in figure 10.The far field pattern of the modified input image in figure 3 is shown in figure 8, its hologram in figure 9, and corresponding reconstructed image in figure 10 respectively.

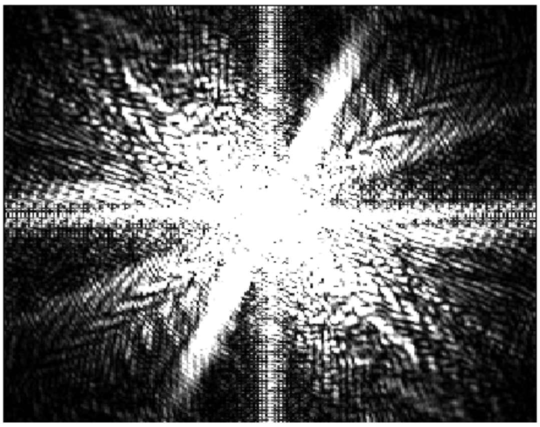

Figure8. Far Field Pattern of Figure 3

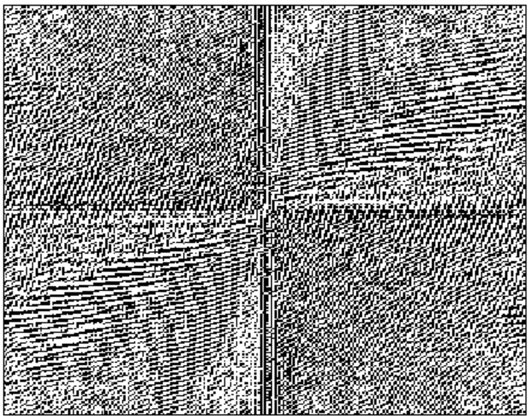

Figure9. Computer Generated Hologram of Figure 3

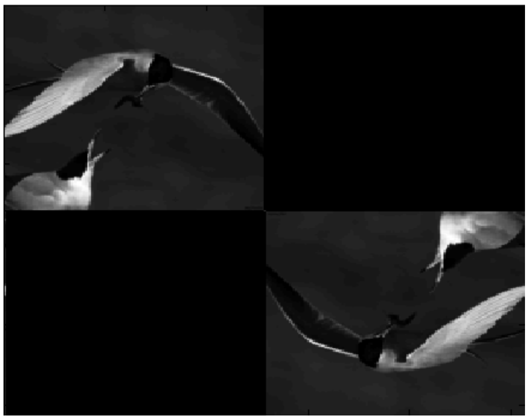

Figure10. Reconstructed Image of Figure 3

The hologram given in figure9 was cropped at different positions and dimensions and these cropped portions are reconstructed. The property of hologram is that every portion of the hologram contains information about the object image. The cropped holograms and corresponding reconstructed images from them are shown in sequential order from figure 11 to 16.It can be inferred from these figures that the quality and size of the reconstructed image depends on the position and size of cropped hologram.

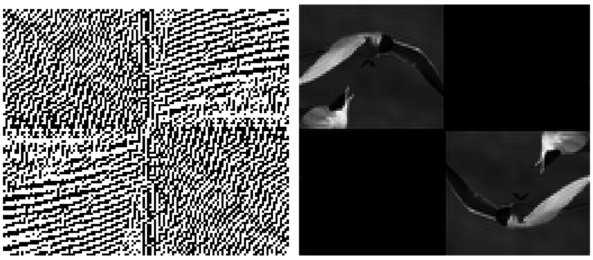

Figure 11,12

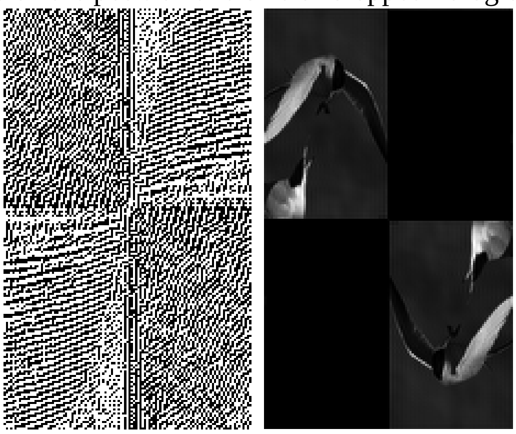

Figure 13,14

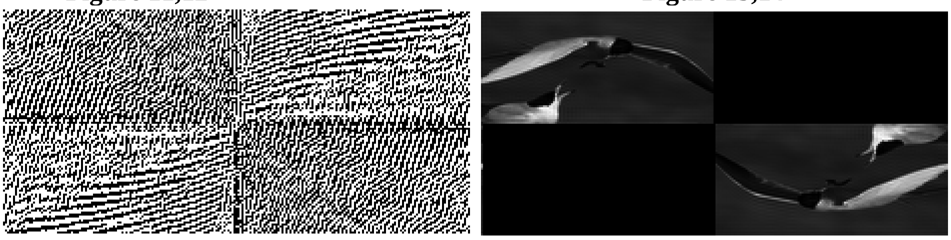

Figure 15,16

Figure 11-16 shows cropped portions of holograms and corresponding reconstructed images

VI. CONCLUSION

This paper discusses the basic principles and implementation of computer generated holograms. A simple algorithm for simulation of computer generated holograms was presented. Fourier transform operation was utilized to obtain the hologram of the object. The reconstruction of object image from the computer generated hologram was done. The image reconstructed from hologram contains the reproduction of the object image and its conjugate image. A method to rearrange the input image so as to identify both the object and conjugate in the reconstructed image was discussed in the paper. An interesting property of hologram is that every portion of the hologram contains information about the object. The hologram was cropped at different positions and dimensions and these cropped portions were reconstructed to get the object image. It was found that the reconstructed image depend on the position and size of cropping of the hologram.

VII. FUTURE SCOPE

Computer Generated Holography has been a field of research for the past few decades. Holographic storage is emerging as a promising field of high capacity data storage. Airplane pilots use Holographic optical elements for navigation. Holograms are used on credit cards and debit cards to provide increased security in order to minimize counterfeiting. Holographic displays are becoming popular nowadays. Computer generated holography helps to produce holograms without the use of highly accurate optical setups. Also, the holograms of objects which do not physically exist can also be produced.

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