STABILITY ANALYSIS AND CONTROLLER DESIGN FOR THE ROLL ANGLE CONTROL OF AN AIRCRAFT

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ABSTRACT
The Flight stability and automatic control model of a general Navion aircraft showed a serious inversion in the step response analysis and thus a compensator is required to bring the system to a stable state. Hence, this paper investigates the stability of general Navion aircraft and using PID toolkit of Matlab 2014 software, a compensator was developed that corrected the inversion problem earlier encountered. The results obtained showed that the percentage required for the system to go over its steady state value was 8.1672%. The time taken for the system response to reach the target value from an initial state of zero was 0.2427secs. The time taken for the system to reach steady state after the initial rise was 3.0384secs.

KEYWORDS: *Aircraft, Controller Design, Matlab, Roll angle Control, Stability analysis.*

I. INTRODUCTION

Stability investigation of aircrafts began at the start of the 20th century. Prior to this period, there was the lack of understanding on the relationship between stability and control in relation to aircrafts. Early ideas regarding stability and control came from the works of pioneers such as Albert Zahm of the United States, Alphonse Penaud of France and Frederick Lanchester in England [1]. However, the development of automatic pilots (autopilots) followed the success of the Wright brothers in developing a powered man-carrying airplane. The Wright brothers work was a modification of the work of Samuel Pierpont Langley who designed an unmanned powered aircraft model. Langley was later engaged by the war department to develop a man-carrying airplane and his dual attempts failed until the Wright brothers successfully carried out their first flight.
The first automatic flight controller in the world was designed by the Sperry brothers, the fast advancement of high performance military, commercial and general aviation aircraft design has required the development of many technologies viz; aerodynamics, structures, materials, population and flight control. Aircraft designs of this dispensation rely so much on automatic control systems to monitor and control various subsystems of the aircraft. This is advancement in aircraft technology [2], [3].
An aircraft in flight moves along three principle axes namely:

1. Vertical Axis
2. Lateral Axis
3. Longitudinal Axis

- **Vertical Axis:** In movement along the vertical axis, the aircrafts rudder plays a major role here. Movement along this axis is known as yawing.
- **Lateral Axis:** In movement along the lateral axis, the aircrafts elevator plays the crucial role here. .movement along this axis is known as pitching.
- **Longitudinal Axis:** In movement along the longitudinal axis, the aircrafts ailerons play the crucial role. Ailerons are hinged control surfaces attached to the trailing edge of the wings [8]. Figure 1 shows the axes of an aircraft.

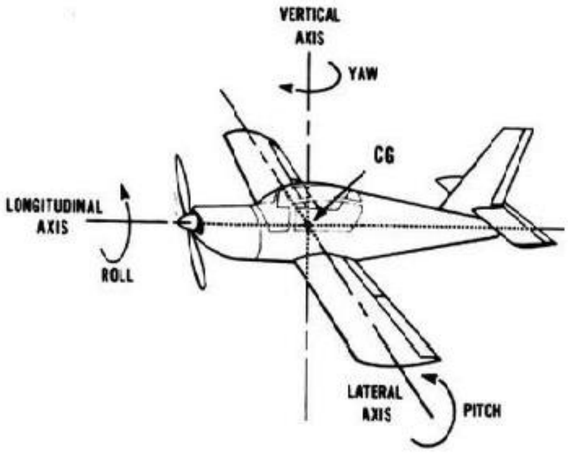

Figure 1. Axes of the Airplane

Roll control is achieved by deflecting the ailerons in a differential manner. The two ailerons are interconnected and move in opposite direction to each other. [4]
Roll angle control of an aircraft is a lateral stability problem, which when solved will increase the lateral stability of the aircraft thus making the aircraft stable around the longitudinal axis. [10].

This work is organised in the following order. Section 2 talks about the Methodology employed in achieving the controller design. State space analysis is used to model the general Navion aircraft and its state equations are obtained. Section 3 talks about the stability analysis of the aircraft model obtained in 2.0. A Proportional-Integral-Derivative(PID) controller is developed and the gains used to derive a model to correct the inversion problem encountered. The next section is devoted to future work that can be carried out in this area of research and a concluding section is given finally.

II. METHODOLOGY

2.1 Modelling of a Roll Angle Control System

Roll angle control is a lateral dynamics problem. In modelling such a system, the moments and dynamics around its axis must be considered.
Newton's second law of motion is important in airplane dynamics because a fixed axis of the earth is used as the references frame.

For lateral dynamics of an aircraft, it is assumed that the aircraft is in steady, cruise with altitude and velocity (v) as constants [5], [9]. Also, the speed of the aircraft and flight references conditions are symmetric with the propulsive forces constant. Thus,

$$v = p = q = r = \emptyset = \varphi = 0 \tag{1}$$

Where p,q and r are the angular rates of the roll, pitch and yaw axes respectively, $\emptyset \; and \; \varphi$ are the roll and yaw attitudes
The lateral dynamics of an aircraft is described by the following kinematics and dynamics

$$y + mg\ \mathrm{C}_{\theta}\mathrm{S}_{\theta} = m\left(\frac{dv}{dt} + ru - pw\right) \tag{2}$$

$$L = I \times \frac{dp}{dt} - Ix_z\frac{dr}{dt} + qr\left(I_z - I_y\right) - Ix_zPq \tag{3}$$

$$N = -Ix_z\frac{dp}{dt} + I_z\frac{dr}{dt} + pq\left(I_y - I_x\right) - Ix_zqr \tag{4}$$

It is assumed that an airplane in motion will have small deviations about a steady flight condition [1], [5]. This applying the small disturbance theory, all the variables in equation (2), (3), (4) are replaced by a reference value plus a perturbation or disturbance. Thus, with assumptions made, the linearized small disturbance lateral rigid body equations of motion are given by; [1]

$$\left(\frac{d}{dt} - y_v\right)\Delta u - y_p\,\Delta p + \left(u_o - y_r\right)\Delta r - \left(g\ \cos\theta\right)\Delta\phi = y_{0r}\,\Delta_{\sigma r} \tag{5}$$

$$-\mathcal{L}_v\,\Delta_v + \left(\frac{d}{dt} - \mathcal{L}_p\right)\Delta p - \left(\frac{\mathrm{I}xz}{\mathrm{I}v}\frac{d}{dt} + \mathcal{L}_r\right)\Delta r = \mathcal{L}_{oa}\Delta\,\sigma_a + \mathcal{L}_{or}\,\Delta\sigma r \qquad (6)$$

$$-N_v\,\Delta_v - \left(\frac{\mathrm{I}xx}{\mathrm{I}z}\frac{d}{dt} + N_p\right)\Delta p + \left(\frac{d}{dt} - N_r\right)\Delta r = N\sigma_a\Delta_{\sigma a} + N_{\sigma r}\,\Delta\sigma r \qquad (7)$$

Rearranging and collecting terms, equations (5), (6), (7) can be written in the state variable form:

$$\dot{x} = A\,x + Bn \qquad (8)$$

Where the matrices A and B are defined as

$$A = \begin{bmatrix} y_r & yp & -(N_o - y_r) & g\cos\theta \\ \mathcal{L}_v^x + \frac{\mathrm{I}x2\,N_p^x}{\mathrm{I}z} & \mathcal{L}_p^x + \frac{2x2\,N_r^x}{\mathrm{I}x} & \mathcal{L}_p^x + \frac{1x2\,N_r^x}{\mathrm{I}x} & 0 \\ N_v^x + \frac{\mathrm{I}x2}{\mathrm{I}_z}\mathcal{L}_r^x & N_p^x + \frac{\mathrm{I}x2}{\mathrm{I}_z}\mathcal{L}_p^x & N_r^x + \frac{\mathrm{I}x2}{\mathrm{I}_z}\mathcal{L}_r^x & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad (9)$$

$$B = \begin{bmatrix} 0 & y_{\sigma r} \\ \mathcal{L}_a^x + \frac{\mathrm{I}x2\,N_{\sigma a}^x}{\mathrm{I}x} & \mathcal{L}_{\sigma r}^x + \frac{\mathrm{I}x2\,N_{\sigma r}^x}{\mathrm{I}x} \\ N_{\sigma a}^x + \frac{\mathrm{I}x2}{\mathrm{I}_z}\mathcal{L}_{\sigma a}^x & N_{\sigma a}^x + \frac{\mathrm{I}x2}{\mathrm{I}_z}\mathcal{L}_{\sigma a}^x \\ 0 & 0 \end{bmatrix} \qquad (10)$$

$$x = \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} \quad \text{and} \quad n = \begin{bmatrix} \Delta\sigma a \\ \Delta\sigma r \end{bmatrix} \qquad (11)$$

If the inertia product $I_{xz}= 0$, equation (8) can be expressed in the form:

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} yv & yp & -(N_0 - y_r) & g\cos\theta \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} D_v \\ D_p \\ D_r \\ D\phi \end{bmatrix} + \begin{bmatrix} 0 & y_{\sigma r} \\ L\sigma_a & L\sigma_r \\ N_{\sigma a} & N_{\sigma r} \\ 0 & 0 \end{bmatrix} \qquad (12)$$

The lateral directional equations of motion consist of the side force, rolling moment and yawning moment equations of motion. Sometimes, it is convenient to use the side slip angle $D\beta$ instead of the side velocity [1], [3].

The relationship between both quantities is given by the equation

$$\Delta\beta \approx \tan^{-1}\frac{\Delta V}{\mu_0} = \frac{\Delta V}{\mu_0} \qquad (13)$$

Therefore, equation (12) can be expressed in terms of $\Delta\beta$ as

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) & \frac{g\cos\theta_0}{u_0} \\ L\beta & Lp & Lr & 0 \\ N\beta & Np & Nr & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta r}}{u_0} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta_{\delta a} \\ \Delta_{\delta r} \end{bmatrix} \qquad (14)$$

The solution to equation (14) is obtained by expanding the following determinant

$$|\lambda_a I - A| = 0 \qquad (15)$$

Where 1 and A are the identity and lateral stability matrices respectively.

It is worthy to note that whole the transfer function gives the relationship between the output and input of a system, in the case of dynamics, it specifies the relationship between the motion variables and the control input [1]

Since and our interest lies in the roll angle control and not the yaw angle control the matrix of equation (15) can be represented as

$$\begin{bmatrix} \Delta\dot{B} \\ \Delta\dot{P} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) & \frac{g\cos\theta_0}{u_0} \\ L\beta & Lp & Lr & 0 \\ N\beta & Np & Nr & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 \\ L_{\delta a} \\ N_{\delta a} \\ 0 \end{bmatrix} [D_{\delta a}] \qquad (17)$$

Using the data from a General NAVION aircraft, [1] the derivatives for the matrix elements in equation 17 are given as

Table 1: Lateral Dynamic Derivatives for NAVION aircraft.

Pitching Velocities	**Y-Force Derivative**	**Yawing Moment Derivative**	**Rolling Moment Derivative**
Pitching Velocity	$Y_v = 0.254$	$N_v = 0.025$	$L_v = -0.091$
Side Slip Angle	$Y_B = -44.665$	$N_B = 4.549$	$L_B = -15.969$
Rolling Rate	$Y_p = 0$	$N_p = -0.349$	$L_p = -8.395$
Yawing Rate	$Y_r = 0$	$N_r = -0.76$	$L_r = 2.19$
Rudder Deflection	$Y_{dr} = 12.433$	$N_{dr} = -4.613$	$L_{dr} = 23.09$
Aileron Deflection	$Y_{da} = 0$	$N_{da} = -0.224$	$L_{da} = -28.916$

Substituting the values in Table 1 above into (16), the state space model in (17) is obtained.

$$\begin{bmatrix} \Delta\dot{B} \\ \Delta\dot{P} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.254 & 0 & -1.0 & -0.182 \\ -10.02 & -8.40 & 2.19 & 0 \\ 4.488 & -0.350 & -0.350 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta B \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 \\ -28.916 \\ 0.224 \\ 0 \end{bmatrix} [D_{\sigma a}] \qquad (17)$$

Considering a Single Input Single Output (SISO) Open-loop system, matrices C and D are defined as:

$$C = [\ 0\ 0\ 0\ 1] \qquad (18)$$

$$D = [0] \qquad (19)$$

III. Results and Discussion

3.1 Stability Analysis

Stability is the most critical problem that is involved in linear control systems. The questions of stability are:

1. Under what condition can systems can became unstable.
2. If the system is unstable, how can the system be stabilized.

To determine the stability of the following system, the Matlab software is used and the transfer function is gotten as follows:

```
A = [-0.254     0      -1      0.182;
      -16.02    -8.40     2.19       0;
       4.488    -0.350  -0.760       0;
          0        1       0        0];

B = [0; -28.916; -0.244; 0];

C = [0 0 0 1];

D = [0];
```

Using

```
System = ss (A,B,C,D);
```

We get the continuous-time state space model. The system transfer function is gotten by putting the code.

```
G(s) = tf (System);
```

The continuous time transfer function that represents the aileron deflection roll angel is given by

$$\frac{\Delta\,\phi(s)}{\Delta\sigma_{\alpha\,(s)}} = G_{(s)} \tag{20}$$

$$G_{(s)} = \frac{-28 = 925 - 29.86 - 139.4}{s^4 + 9.414\,s^3 + 13.975s^2 + 48.04s + 0.4271}$$

The poles and zero of the open- loop transfer code are given by the code;

Zeros = zero (Gs); Poles = pole (Gs);

Zeros = $-0.5162 + 2.134i, -0.5162 - 2.134i$

Poles = $-8.4328 + 0.0000i, -0.4862 + 2.3336i,\ -0.4862 - 2.3336i,\ -0.0089 + 0.0000i$.

Since all the roots of the characteristic equation have negative real parts, thus, the system is said to be dynamically stable. Also the Routh Hurwitz stability criterion helps us to determine if all the roots of the characteristic equation given by (21) lie in the left half of the s-plane.

$$s^4 + 9.414\,s^3 + 13.97s^2 + 48.04s + 0.42 \tag{21}$$

The characteristic equation in (21) satisfies the Routh Hurwitz criterion by inspection because

i. The equation has no missing terms.
ii. The coefficients are all of the same sign.

The step response is obtained using matlab by typing the code step (System); in the command window. The step response is shown in figure 2.

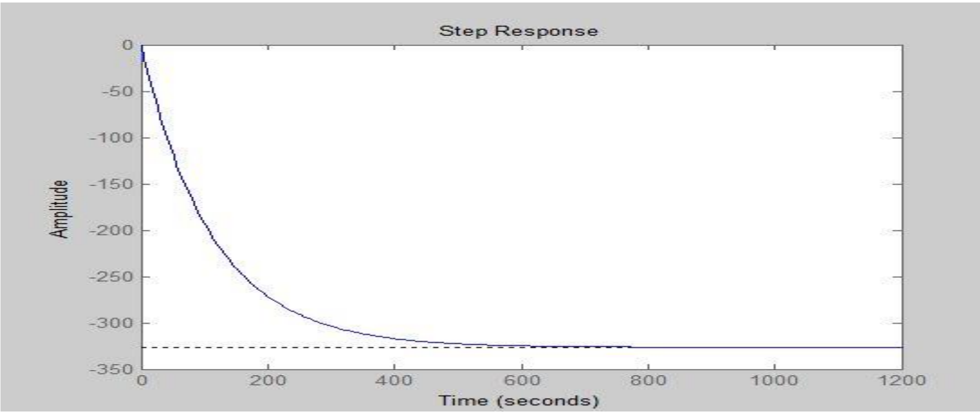

Figure 2: Step Response of Roll Angle Model of Navion Airplane.

Examining figure 2, it is seen that the dynamical characteristics of the NAVION aircraft not acceptable as there is an inversion in the loop. Also the overshoot, rise and settling time must be modified using feedback control.

3.2 CONTROLLER DESIGN

To design a controller for the plant, we have to first determine if the system is controllable.

Using an m-file from matlab, the command below test for controllability

```
% this Program tests for the controllability of the system
A= [-0.254    0     -1    0.182;
   -16.02   -8.40   2.19    0;
    4.488   -0.350  -0.760    0;
    0        1        0    0];
B = [0; -28.916; -0.244; 0];
u = ctrb (A, B);
```

```
n = length (A);
rang = rank(u);
if rang==n
display ('Object is Controllable');
else
display ('Object is not controllable');
end
```

In arriving at the desired design specification the step information of the system was looked at from the step information obtained from the system, a controller with the following characteristics is intended to be designed and simulated.

$Overshoot \leq 10\%$

$Rise\ time \leq 2s$

$Settling \leq 5s$

The choice of controller to be used for the design is the classical PID controller. The reason for the choice of this controller is because the PID controller gave a better performance during system tuning to achieve desired. Design requirement compared to using a proportional or integral or PI or PD controller.

Using the PID tuner application in Matlab, the plant model in equation (20) is imported and tuned till appropriate and close to values of over shoot, rise time and settling time are achieved. The diagram below shows the configuration of the simple feedback system used in this design.

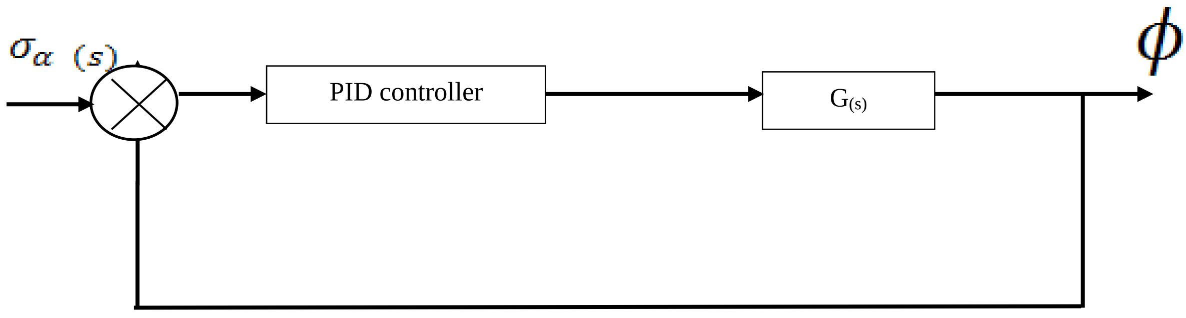

Figure 3: closed loop control system

Using the PID tuner application on Matlab, our values for PID controller to be used in this design are

$K_p = -2.1413, K_i = -1.2626, K_j = -0.123$

Inputing and assigning the following values on matlab to the equation

$$G_{(c)} = K_p + K_i x \frac{1}{s} + K_d * s \qquad (22)$$

This is the general form of a PID controller.

From figure above, the closed loop transfer function can be expressed as:

$$\frac{G_{(c)} * G_{(s)}}{1 + G_{(s)} G_{(c)}} \qquad (23)$$

Using the code below on matlab, the closed loop transfer function (G_{closed}) of our system.

$$G_{closed} = Feedback\left(G_{(s)} * G_{(c)}, 1\right) \quad (24)$$

This gives the step response shown in figure below.

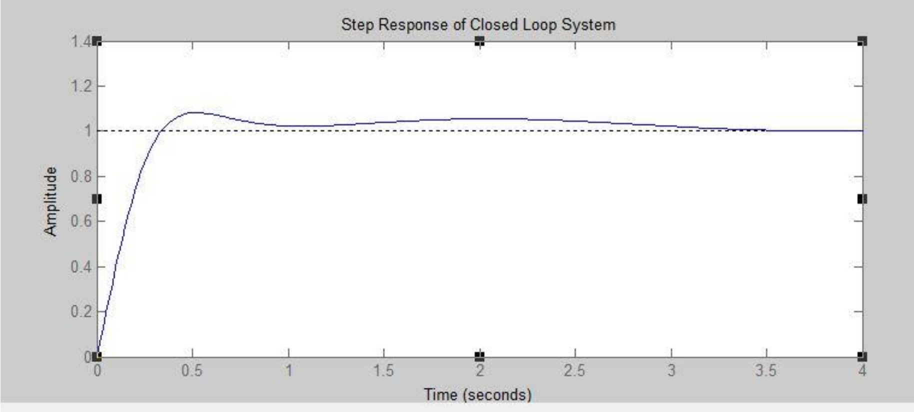

Figure 4: Step Response of the closed Loop Feedback System.

The step characteristics of our closed loop system are as shown in figure 4 above. From the step plot, we can see that our designed controller compensated for the Inversion earlier encountered in the initial system design. The Overshoot using equation (26) was roughly 8% and zero steady state error was achieved. The closed loop continuous time transfer function for our design is given as

$$Gclosed = \frac{3.557s^4\ 4 + 65.59\, s^3 + 117.6s^2 + 336.2s + 176}{s^5 + 12.97\, s^4 + 79.5s^3 + 165.6s^2 + 336.6s + 176} \quad (26)$$

The root locus of the closed loop feedback system is as shown in figure 5. It specifies a stable system as it satisfies the requirement of stability as the entire zeros lie in the left hand part of the S-plane.

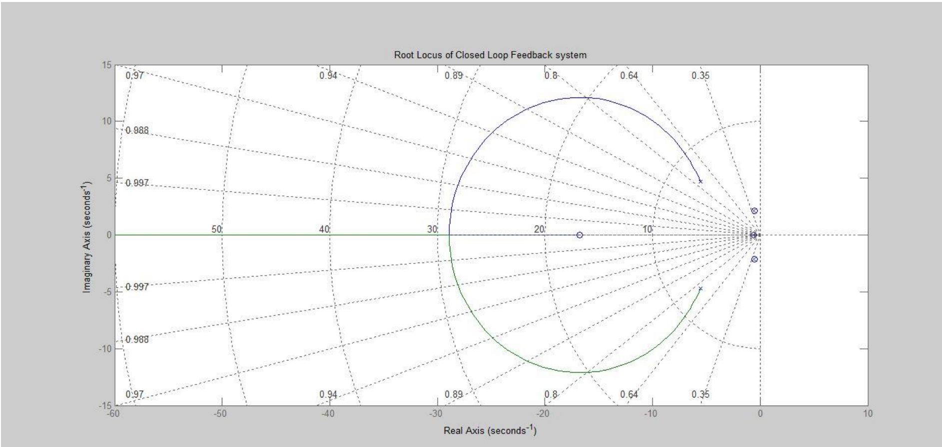

Figure 5: Root locus of Closed loop system.

IV. Conclusions

The lateral dynamical equations of an aircraft gives the roll and yaw axis model. The focus of this paper is the roll angle autopilot control. From our design, we achieved an overshoot of *8.1672%*, a rise time of *0.2427seconds* and a settling time of *3.0384seconds*. Further research could could be carried out using a different set of set parameters and controller. The controller choosen would have to be optimized to achieve system stability.

V. Future Work

Further research work could be carried out in this area of research. with the rise of intelligent control techniques, such techniques could be employed in the development of a more robust and adaptive controller. Also, the PID controller parameters obtained during auto tuning could be employed with new set of parameters for the controller. Also, as this is a hard real time critical system in which a delay could be catastrophic, the time it takes the system to respond to the controller input should be optimised for a more perfect response.

REFERENCES

[1] R. Nelson. *Flight stability and automatic control*. New York: McGraw-Hill. 1998.
[2] Lucio R. Riberio and Neusa Maria F. Oliveira. "UAV Autopilot Controllers Test Platform Using Matlab/Simulink and X-Plane", 40th ASEE/IEEE Frontiers in Education Conference, October 27-30, 2010, Washington, DC. 2010
[3] Omur Akyazi, A.Sefa Akpinar and M. Ali Usta, "A self-tuning fuzzy Logic Controller for aircraft roll Control System", International Journal of Control Science and Engineering. 2012
[4] E. Gouthami and M. A. Rani, "Modelling of Closed Loop PID Controller for an Auto-Pilot Aircraft Roll Control", International Conference on recent Advances in Communication, VLSI and Embedded Systems. 2014
[5] Vishnu. G.N, Dileep. M.V and Prahalad. K.R, "Design of Fuzzy Logic Controller for Lateral Dynamics Control of Aircraft by Considering the Cross-Coupling Effect of Yaw and Roll on Each Other," *International Journal of Computer Applications.* vol. 47, no. 13. 2012
[6] Branimir. S, Ljubisa .V and Caslav .M, "The Application of the root Locus Method for the design of Pitch Controller of an F-104A Aircraft", Journal of Mechanical Engineering. 2009
[7] Katsuhiko Ogata. Modern Control Engineering, 4th Edition, New Jersey, Pearson Publishers. 2002
[8] Phillips. C. L and Parr J.M, Feedback Control Systems, 5th Edition, India, Pearson Publishers.2011
[9] Nurbaiti Wahid, Nurhaffizah Hassan," Self-tuning Fuzzy PID Controller Design for Aircraft Pitch Control",978-0-7695-4668-1/. IEEE. 2012
[10] Vishnu. G.N, Dileep. M.V and Prahalad. K.R, "Aircraft Yaw Control System using LQR and Fuzzy Logic Controller", *International Journal of Computer Applications.* vol. 47, no. 9.

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