WAVE PROPAGATION IN A HOMOGENEOUS ISOTROPIC MAGNETO-THERMO-ELASTIC CYLINDRICAL PANEL

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ABSTRACT

This work investigates the three dimensional wave propagation of a homogeneous isotropic magneto thermo elastic cylindrical panel in the context of the linear theory of thermo elasticity. Three displacement potential functions are introduced to uncouple the equations of motion. A Bessel function solution with complex arguments is directly used to analyze the frequency equations with traction-free boundary conditions. The special cases have also been deduced for magneto elastic, thermo elastic and elasto-kinetic at various levels from the present analysis. The numerical example which demonstrates the present method is studied for the material magneto-strictive cobalt iron oxide (CoFe2O4). The computed non-dimensional phase velocity and attenuation coefficient are plotted in the form of dispersion curves. The coupling effect among thermal, magnetic and elastic in magneto thermo elastic material provides a mechanism for sensing thermo mechanical disturbances in the design of sensors and surface acoustic damping filters.

KEYWORDS: *Wave propagation, isotropic cylindrical panel, Bessel function, attenuation coefficient.*

I. INTRODUCTION

The interaction between the magnetic and thermal fields plays a vital role in geophysics for understanding the effect of Earth's magnetic field on seismic waves. With the development of active material systems, there is a significant interest in the coupling effects between the elastic, magnetic and temperature for their application in sensing and actuation. The analysis of thermally induced vibration of magneto elastic cylindrical panel is usually encountered in the design of structures, atomic reactors, steam turbines, supersonic aircraft, space shuttle and other devices operating at elevated temperature. In the field of non-destructive evaluation, laser-generated waves have attracted great attention owing to their potential application to non-contact and non-destructive evaluation of sheet materials. Thermoelectric currents in the presence of magnetic fields can cause pumping and stirring of liquid metal coolants in nuclear reactors and molten metal in industrial metallurgy. In the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Moreover, it is well recognized that the investigation of the magneto thermal effects on elastic wave propagation has bearing on many seismological applications. This study may be used in applications involving non-destructive testing (NDT), qualitative nondestructive evaluation(QNDE) of large diameter pipes and health monitoring of other ailing infrastructure in addition to checking and verifying the validity of FEM and BEM for such problems.

The static analysis cannot predict the behavior of the material due to the rapid thermal stress changes. Green and Lindsay [1] and Lord and Shulman [2] modified the Fourier law and constitutive relations. It is used to get hyperbolic equation for heat conduction by taking into account of the time needed for acceleration of heat flow and relaxation of stresses. The theory of thermo elasticity is initially studied by Nowacki[3]. A special feature of the Green-Lindsay model is that it does not violate the classical

Fourier's heat conduction law. The obtained results were compared by a special application of the frozen stress technique of photo elasticity. Paul and Muthiayalu[4] studied magneto thermo elastic free vibrations in an infinite plate by verifying the numerical result for aluminum alloy. Ponnusamy[5] has obtained the frequency equation of free vibration of a generalized thermo elastic solid cylinder of arbitrary cross section by using Fourier expansion collocation method. Sharma and Sidhu [6] studied the propagation of plane harmonic thermo elastic wave in homogeneous isotropic, cubic crystals and anisotropic materials in the context of generalized thermo elasticity. The three dimensional vibration analysis of a transversely isotropic thermo elastic cylindrical panel has been investigated by Sharma [7].The application of powerful numerical tools like finite element or boundary element methods to these problems is also becoming important. The theory of magneto thermo-elasticity has aroused much applications in many industrial appliances particularly in nuclear devices, where a primary magnetic field exists.

Sherief and Ezzat[8] used the Laplace transform technique to find the distribution of thermal stresses and temperature in a generally thermo elastic electrically conducting half-space under sudden thermal shock and permeated by a primary uniform magnetic field. Wang and Dai [9] presented magneto-thermo-dynamic stresses and perturbation of magnetic field vector in an orthotropic thermo elastic cylinder.

II. FORMULATION OF THE PROBLEM

In cylindrical co-ordinates, the three dimensional stress equations of motions and strain displacement relations and heat conduction equations, Maxwell equation for magnetic field, in the absence of body force for linearly elastic medium are as follows:

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho\frac{\partial^2 u}{\partial t^2}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}(\sigma_{r\theta}) = \rho\frac{\partial^2 v}{\partial t^2}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho\frac{\partial^2 w}{\partial t^2} \qquad (1)$$

$$K\left[T_{,rr} + r^{-1}T_{,}r + r^{-2}T_{,\theta\theta} + T_{,zz}\right]$$

$$= \rho C_v T_{,t} + T_o\left(\frac{\partial}{\partial t}\left[\beta(e_{rr} + e_{\theta\theta}) + \beta e_{zz} - P_3\psi_{,z}\right] + \tau_o\frac{\partial^2}{\partial t^2}\right) + \rho C_v \tau_o T_{,tt}$$

$$r^{-1}\frac{\partial}{\partial r}(rB_r) + r^{-1}\frac{\partial}{\partial\theta}(B_\theta) + \frac{\partial}{\partial z}(B_z) = 0$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.

The stress-strain relations are

$$\sigma_{rr} = (\lambda + 2\mu)e_{rr} + \lambda e_{\theta\theta} + \lambda e_{zz} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{\theta\theta} = \lambda e_{rr} + (\lambda + 2\mu)e_{\theta\theta} + \lambda e_{zz} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{zz} = \lambda e_{rr} + \lambda e_{\theta\theta} + (\lambda + 2\mu)e_{zz} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{\theta z} = \mu\gamma_{\theta z} + r^{-1}d_{15}\frac{\partial\psi}{\partial\theta}$$

$$\sigma_{rz} = \mu\gamma_{rz} + d_{15}\frac{\partial\psi}{\partial r}$$

$$\sigma_{r\theta} = \mu\gamma_{r\theta}$$

Magnetic induction displacements are

$$B_r = d_{15}\gamma_{rz} - \mu_{11}\frac{\partial\psi}{\partial r}$$

$$B_\theta = d_{15}\gamma_{z\theta} - r^{-1}\mu_{11}\frac{\partial\psi}{\partial\theta}$$

$$B_z = d_{31}(e_{rr} + e_{\theta\theta}) + d_{31}e_{zz} - \mu_{33}\frac{\partial\psi}{\partial z} + P_3T$$

The strain displacements are

$$e_{rr} = \frac{\partial u}{\partial r}$$

$$e_{\theta\theta} = r^{-1}u + r^{-1}\frac{\partial v}{\partial\theta} \qquad (2)$$

$$e_{zz} = \frac{\partial w}{\partial z}$$

Shear strain displacements are

$$\gamma_{r\theta} = \frac{\partial v}{\partial r} - r^{-1}v + r^{-1}\frac{\partial u}{\partial\theta}$$

$$\gamma_{z\theta} = \frac{\partial v}{\partial z} + r^{-1}\frac{\partial w}{\partial\theta}$$

$$\gamma_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \qquad (3)$$

Substituting the equation (3) in equation(2), yields

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\left(r^{-1}u + r^{-1}\frac{\partial v}{\partial\theta}\right) + \lambda\frac{\partial w}{\partial z} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{\theta\theta} = \lambda\frac{\partial u}{\partial r} + (\lambda + 2\mu)\left(r^{-1}u + r^{-1}\frac{\partial v}{\partial\theta}\right) + \lambda\frac{\partial w}{\partial z} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{zz} = \lambda\frac{\partial u}{\partial r} + \lambda\left(r^{-1}u + r^{-1}\frac{\partial v}{\partial\theta}\right) + (\lambda + 2\mu)\frac{\partial w}{\partial z} - \beta T + d_{31}\frac{\partial\phi}{\partial z}$$

$$\sigma_{\theta z} = \mu\left(\frac{\partial v}{\partial z} + r^{-1}\frac{\partial w}{\partial\theta}\right) + r^{-1}d_{15}\frac{\partial\psi}{\partial\theta}$$

$$\sigma_{rz} = \mu\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right) + d_{15}\frac{\partial\psi}{\partial r}$$

$$\sigma_{r\theta} = \mu\left(\frac{\partial v}{\partial r} - r^{-1}v + r^{-1}\frac{\partial u}{\partial\theta}\right)$$

$$B_r = d_{15}\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)-\mu_{11}\frac{\partial \psi}{\partial r} \qquad (4)$$

$$B_\theta = d_{15}\left(\frac{\partial r}{\partial z}+r^{-1}\frac{\partial w}{\partial \theta}\right)-r^{-1}\mu_{11}\frac{\partial \psi}{\partial \theta}$$

$$B_z = d_{31}\left(\frac{\partial u}{\partial r}+r^{-1}u+r^{-1}\frac{\partial v}{\partial \theta}\right)+d_{31}\frac{\partial w}{\partial z}-\mu_{33}\frac{\partial \psi}{\partial z}+P_3T$$

Substituting the equation (4) in equation (1), yields,

$$(\lambda+2\mu)\left(\frac{\partial^2 u}{\partial r^2}+r^{-1}\frac{\partial u}{\partial r}-r^{-2}u\right)+r^{-2}\mu\frac{\partial^2 u}{\partial \theta^2}+\mu\frac{\partial^2 u}{\partial z^2}+r^{-1}(\lambda+\mu)\frac{\partial^2 v}{\partial r\,\partial \theta}$$
$$+(\lambda+\mu)\frac{\partial^2 w}{\partial r\partial z}-r^{-2}(\lambda+3\mu)\frac{\partial v}{\partial \theta}-\beta\frac{\partial T}{\partial r}+(d_{31}+d_{15})\frac{\partial^2 \phi}{\partial r\partial z}=\rho\frac{\partial^2 u}{\partial t^2} \qquad (5a)$$

$$\mu\left(\frac{\partial^2 v}{\partial r^2}+r^{-1}\frac{\partial v}{\partial r}-r^{-2}v\right)+r^{-1}(\lambda+\mu)\frac{\partial^2 u}{\partial r\partial \theta}+r^{-2}(\lambda+\mu)\frac{\partial u}{\partial \theta}+r^{-2}(\lambda+2\mu)\frac{\partial^2 v}{\partial \theta^2}$$
$$+\mu\frac{\partial^2 v}{\partial z^2}+r^{-1}(\lambda+\mu)\frac{\partial^2 w}{\partial \theta\partial z}-r^{-1}\beta\frac{\partial T}{\partial \theta}+r^{-1}(d_{31}+d_{15})\frac{\partial^2 \phi}{\partial \theta\partial z}=\rho\frac{\partial^2 v}{\partial t^2} \qquad (5b)$$

$$\mu\left(\frac{\partial^2 w}{\partial r^2}+r-1\frac{\partial w}{\partial r}+r-2\frac{\partial^2 w}{\partial \theta^2}\right)+(\lambda+\mu)\left[\frac{\partial^2 u}{\partial r\partial z}+r-1\left(\frac{\partial^2 v}{\partial \theta\,\partial z}+\frac{\partial u}{\partial z}\right)\right]+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}$$
$$-\beta\frac{\partial T}{\partial z}+d_{31}\frac{\partial^2 \phi}{\partial z^2}+d_{15}\left(\frac{\partial^2 \phi}{\partial r^2}+r-1\frac{\partial \phi}{\partial r}+r-2\frac{\partial^2 \phi}{\partial \theta^2}\right)=\rho\frac{\partial^2 w}{\partial t^2} \qquad (5c)$$

$$k\left(\frac{\partial^2 T}{\partial r^2}+r^{-1}\frac{\partial T}{\partial r}+r^{-2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)-\rho\left(C_v\frac{\partial T}{\partial t}-\tau_o\rho C_v\frac{\partial^2 T}{\partial t^2}-T_o\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\right)$$
$$\left[\beta\left(\frac{\partial u}{\partial r}+r^{-1}u+r^{-1}\frac{\partial v}{\partial \theta}\right)+\beta\frac{\partial w}{\partial z}+P_3\frac{\partial \psi}{\partial z}\right]=0 \qquad (5d)$$

$$d_{15}\left[\frac{\partial^2 w}{\partial r^2}+r^{-2}\frac{\partial^2 w}{\partial \theta^2}+r^{-1}\frac{\partial w}{\partial r}\right]+(d_{13}+d_{15})\left(r^{-1}\frac{\partial^2 v}{\partial \theta\,\partial z}+r^{-1}\frac{\partial u}{\partial z}+\frac{\partial^2 u}{\partial \theta\,\partial z}\right)$$
$$+d_{31}\frac{\partial^2 w}{\partial z^2}-\mu_{33}\frac{\partial^2 \phi}{\partial z^2}-\mu_{11}\left(\frac{\partial^2 \phi}{\partial r^2}+r^{-2}\frac{\partial^2 \phi}{\partial \theta^2}+r^{-1}\frac{\partial \phi}{\partial r}\right)+P_3T=0 \qquad (5e)$$

To uncouple the equations (5a) to (5d), the mechanical displacement u, v and w along the radial, circumferential and axial directions are assumed following:

$$u = r^{-1}\psi_{,\theta}-G,r$$

$$v = r^{-1}G_{,\theta}-\psi_{,r}$$

$$w = w_{,z}$$

$$\phi = \chi_{,z}$$

Hence

$$\left((\lambda+2\mu)\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)G-(\lambda+\mu)\frac{\partial^2 w}{\partial z^2}-(d_{31}+d_{15})\frac{\partial^2\psi}{\partial z^2}+\beta T=0$$

and $\left(\mu\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)\psi=0$ (6a)

where $\nabla_1^2=\frac{\partial^2}{\partial r^2}+r^{-1}\frac{\partial}{\partial r}+r^{-2}\frac{\partial^2}{\partial\theta^2}$

$$\left((\lambda+2\mu)\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)G-(\lambda+\mu)\frac{\partial^2 w}{\partial z^2}+\beta T=0$$

and (6b)

$$\left(\mu\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)\psi=0$$

$$\left(\mu\nabla_1^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)w-(\lambda+\mu)\nabla_1^2 G-\left(d_{15}\nabla_1^2+d_{31}\frac{\partial^2}{\partial z^2}\right)\chi-\beta T=0 \quad (6c)$$

$$\left(k\nabla_1^2+k\frac{\partial^2}{\partial z^2}-\rho C_\gamma\frac{\partial}{\partial t}\right)T-T_o\frac{\partial}{\partial t}\left(-\beta\nabla_1^2 G-\beta\frac{\partial^2 w}{\partial z^2}+P_3\frac{\partial^2\chi}{\partial z^2}\right)=0 \quad (6d)$$

$$(d_{13}+d_{15})\nabla_1^2 G-\left(d_{15}\nabla_1^2+d_{31}\frac{\partial^2}{\partial z^2}\right)w+\left(\mu_{11}\nabla_1^2+\mu_{33}\frac{\partial^2}{\partial z^2}\right)\chi-P_3 T=0 \quad (6e)$$

III. Solution to the Problem

The equations (6a) to (6e) are coupled partial differential equations with three displacements, heat conduction components and magnetic potential. To uncouple these equations, assume the transverse wave along the axial direction z to be zero.Ψ gives a purely transverse wave. Hence the solutions of the equations (6a) to (6e) can be presented in the following form:

$$G(r,\theta,z,t)=\bar{G}(r)\sin(m\pi)z\cos\left(\frac{n\pi}{\alpha}\right)\theta\, e^{iwt}$$

$$w(r,\theta,z,t)=\bar{w}(r)\sin(m\pi)z\sin\left(\frac{n\pi}{\alpha}\right)\theta\, e^{iwt}$$

$$\chi(r,\theta,z,t)=\bar{\chi}(r)\sin(m\pi)z\cos\left(\frac{n\pi}{\alpha}\right)\theta\, e^{iwt}$$

$$T(r,\theta,z,t)=\bar{T}(r)\sin(m\pi)z\cos\left(\frac{n\pi}{\alpha}\right)\theta\, e^{iwt}$$

(7)

$$\psi(r,\theta,z,t)=\bar{\psi}(r)\sin(m\pi)z\cos\left(\frac{n\pi}{\alpha}\right)\theta\, e^{iwt}$$

Introducing the non-dimensional quantities

$$r^1 = \frac{r}{R};\ z^1 = \frac{z}{L};\ \bar{T} = \frac{T}{T_o};\ \delta = \frac{n\pi}{\alpha};\ t_L = \frac{m\pi R}{L};\ \Omega^2 = \frac{\rho w^2 R^2}{\mu};$$

$$R = \frac{a+b}{2};\ \bar{\lambda} = \frac{\lambda}{\mu}\quad \bar{\beta} = 1;\ \bar{K} = 1;\ \chi' = \frac{\chi}{\phi_o};\ P = \frac{P_3\ (2+\bar{\lambda})}{\beta\ d_{31}\ \mu}$$

$$d_1 = \frac{d_{15} + d_{31}}{d_{31}\ \mu}\quad d_2 = \frac{d_{15}}{\mu d_{31}} \qquad (8)$$

$$\eta_1 = \frac{\mu_{11}(2+\bar{\lambda})}{\mu\ d_{31}{}^2};\ \mu_p = \frac{\mu d_{31}\phi_o}{2+\bar{\lambda}};\ \beta^* = \frac{\mu\beta T_o R^2}{2+\bar{\lambda}};\ \in_1 = \frac{\beta^2 T_o \mu}{\rho C_\gamma (2+\bar{\lambda})}$$

Rewriting the equations (6a) to (6e) and (7) in a convenient form of equations as follows:

$$(\nabla_2^2 + g_1)\bar{G} + (1+\bar{\lambda})g_2\ \bar{w} - \mu_p\ g_5\ \bar{\chi} + \beta^*\bar{T} = 0$$

where $\nabla_2^2 = \frac{\partial^2}{\partial r'^2} + r^{-1}\frac{\partial}{\partial r'} - \frac{\delta^2}{r'^2}$

where

$$g_1 = C_2(\Omega^2 - t_L^2);\ g_2 = c_3 t_L^2$$

$$g_3 = \Omega^2 - \frac{C_1}{C_2}\ t_L^2$$

$$g_5 = d_1 t_L^2$$

$$C_2(\nabla_2^2 + g_3)\bar{w} + (1+\bar{\lambda})C_3\nabla_2^2\bar{G} + d_2\mu_p(\nabla_2^2 - g_6)\bar{\chi} - \beta^*\bar{\beta}\bar{T} = 0$$

$$\frac{\in_1 c_2\,\Omega^2}{i\chi^*}\left(\nabla_2^2\ \bar{G} + \bar{\beta}\,t_L^2\ \bar{w} - P\mu_p t_1^2\,\bar{\chi}\right) + \beta^*(\nabla_2^2 + g_4)\bar{T} = 0$$

$$d_1\nabla_2^2\bar{G} - d_2(\nabla_2^2 - g_6)\bar{w} + \eta_1\mu_p\left(\nabla_2^2 + g_7\right)\bar{\chi} - \beta^* P\bar{T} = 0$$

(9)

$$\eta_1 = \frac{\mu_{11}\ (2+\bar{\lambda})}{\mu\ d_{31}^2};\qquad g_7 = -\in t_L^2$$

The determinant form of the system of equations in equation (9) as follows:

$$\begin{vmatrix} \nabla_2^2 + g_1 & (1+\bar{\lambda})g_2 & -\mu_p g_5 & \beta^* \\ -C_3(1+\bar{\lambda})\nabla_2^2 & C_2(\nabla_2^2 + g_3) & d_2\mu_p(\nabla_2^2 - g_6) & -\beta^*\bar{\beta} \\ d_1\nabla_2^2 & -d_2(\nabla_2^2 - g_6) & \eta_1\mu_p(\nabla_2^2 + g_7) & -\beta^* P \\ \frac{\in_1 C_2\,\Omega^2}{i\chi^*}\nabla_2^2 & \bar{\beta}t_L^2 & -P\mu_p t_L^2 & \beta^*(\nabla_2^2 + g_4) \end{vmatrix} (\bar{G}, \bar{w}, \bar{\chi}, \bar{T}) = 0 \quad (10)$$

Equation (10) on simplification reduces to the following differential equation

$$[\nabla_2^{\,8} + A\nabla_2^{\,6} + B\nabla_2^{\,4} + C\nabla_2^{\,2} + D]\,\overline{G} = 0$$

where

$$A = \frac{1}{\Delta}[C_2\eta_1 g_1 + C_2\eta_1 g_4 - C_2\eta_1 g_3 + d_2^{\,2} g_4 - 2d_2^{\,2} g_6 + C_2\eta_1 g_1 g_7$$

$$+ C_2\eta_1 g_1 g_3 + d_2^{\,2} g_4 g_1 - d_2^{\,2} g_1 g_6 - C_2^{\,2} \in_1 \Omega^2 \eta_1 / i\chi^*]$$

$$B = \frac{1}{\Delta}[C_2\eta_1 g_3 g_7 + C_2\eta_1 g_3 g_4 - C_2\eta_1 p^2 t_L^{\,2} - 2d_2^{\,2} g_4 g_6 - d_2\overline{\beta} p t_L^{\,2} + d_2^{\,2} g_6 g_4$$

$$+ d_2^{\,2} g_6^{\,2} - d_2 p\overline{\beta} + \overline{\beta}\eta_1 t_L^{\,2} - C_2 p^2 t_L^{\,2} g_1 + C_2\eta_1 g_1 g_3 g_7 + C_2\eta_1 g_1 g_3 g_4$$

$$- d_2^{\,2} g_1 g_4 g_6 - d_2\overline{\beta}^2 t_L^{\,2} p g_1 - d_2^{\,2} g_1 g_6 - d_2 C_3(1+\overline{\lambda})\, g_4 g_5 + d_2 C_3(1+\overline{\lambda}) g_4 g_6$$

$$+ d_2 C_3 g_4 g_5 g_6 + C_3\overline{\beta}(1+\overline{\lambda}) g_5 t_L^{\,2} - d_2 C_3\overline{\lambda} g_4 g_5 + d_2 C_3\overline{\lambda} g_4 g_6 + d_2 C_3\overline{\lambda} g_4 g_5 g_6$$

$$+ C_2 d_1 g_4 g_5^{\,2} + C_2^{\,2} p \in_1 \Omega^2 g_5 / iX^* + C_2 d_1 g_3 g_5 + C_2 d_2 \in_1 \Omega^2 g_5 \overline{\beta}$$

$$+ d_2 C_3(1+\overline{\lambda}) p t_L^{\,2}\eta_1 - C_3(1+\overline{\lambda}) t_L^{\,2}\eta_1 p - d_1 C_2 p t_L^{\,2} - C_2^{\,2} \in_1 \Omega^2 \eta_1 g_1 / i\chi^*$$

$$- C_2^{\,2} \in_1 g_3 \Omega^2 \eta_1 / i\chi^* - d_1 d_2\overline{\beta} t_L^{\,2} + d_2^{\,2} C_2 \Omega^2 t_1 g_6 / i\chi^*]$$

$$C = \frac{1}{\Delta}[C_2\eta_1 g_4 g_7 + C_1\eta_1 g_3 g_4 g_7 - C_2 p^2 t_L^{\,2} - d_2^{\,2} g_4 g_6^{\,2} + 2d_2\overline{\beta} p t_L^{\,2} g_6 + \overline{\beta}^2 \eta_1 t_L^{\,2} g_7$$

$$+ d_2 g_1 g_4 g_7 + C_2\eta_1 g_3 g_4 g_1 g_7 - C_2 p^2 t_L^{\,2} g_1 - d_2^{\,2} g_1 g_4 g_6 + d_2^{\,2} g_6^{\,2} g_1 - d_2\overline{\beta} p g_1 + \overline{\beta} g_1 \eta_1 t_L^{\,2}$$

$$- d_2 g_4 g_5 C_3(1+\overline{\lambda}) + C_2 g_5 p \in_1 \Omega^2 g_3 / i\chi^* + d_1 g_5^{\,2}\overline{\beta} t_L^{\,2} - d_2 g_5\overline{\beta} \in_1 \Omega^2 C_2 g_6 / i\chi^*$$

$$- C_3(1+\overline{\lambda}) d_2 t_L^{\,2} g_6 - C_3(1+\overline{\lambda})\overline{\beta}\,\eta_1 t_L^{\,2} g_7 + d_1 C_2 p t_L^{\,2} - C_2 d_2 \in_1 \Omega^2 \overline{\beta} g_5$$

$$- C_2^{\,2} \in_1 \Omega^2 g_3\, g_7 \eta_1 / i\chi^* + d_2^{\,2} t_1 \in_1 \Omega^2 g_6 / i\chi^* + d_1 d_2\overline{\beta}\, t_L^{\,2} g_6]$$

$$D = \frac{1}{\Delta}[d_2^{\,2} g_1 g_4 g_6^{\,2} + 2d_2\overline{\beta} p g_1 g_6 t_L^{\,2} + \overline{\beta}\eta_1 t_L^{\,2} g_1 g_7 - d_2^{\,2} \in_1 C_2 \Omega^2 g_6^{\,2} / i\chi^*] \qquad (11)$$

and $\Delta = C_2^{\,2}\Omega_1 + d_2^{\,2} + C_2\eta_1 g_1 + d_2^{\,2} g_1$

The solution of the equation (11) are obtained as,

$$\overline{G}(r) \quad = \quad \sum_{i=1}^{4}[A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r)]$$

$$\overline{W}(r) = \sum_{i=1}^{4} L_i [A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r)]$$

$$\overline{\chi}(r) = \sum_{i=1}^{4} M_i [A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r)]$$

$$\overline{T}(r) = \sum_{i=1}^{4} N_i [A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r)]$$

$$\overline{\psi}(r) = A_5 J_\delta(Kr) + B_5 Y_\sigma(Kr) \qquad (12)$$

where $K^2 = \Omega^2 - t_L^2$

The arbitrary constants L_i, M_i and N_i are obtained from

$$\begin{bmatrix} g_2(1+\overline{\lambda}) & g_5\mu_p & -\beta^* \\ C_2(\nabla_2{}^2 + g_3) & d_2\mu_p(\nabla_2{}^2 - g_6) & -\beta^*\overline{\beta} \\ -d_2(\nabla_2{}^2 - g_6) & \eta_1\mu_p(\nabla_2{}^2 + g_7) & \beta^* p \end{bmatrix} \begin{bmatrix} L_i \\ M_i \\ N_i \end{bmatrix} = \begin{bmatrix} \nabla_2{}^2 + g_1 \\ C_3\nabla_2{}^2(1+\overline{\lambda}) \\ -d_1\nabla_2{}^2 \end{bmatrix}$$

$$L_i = \frac{1}{\Delta}\Big\{(\alpha_i{}^2 + g_1)[-pd_2(\alpha_i - g_6) + \overline{\beta}\eta_1(\alpha_i{}^2 + g_7)]$$

$$-g_5[-pC_3(1+\overline{\lambda})\alpha_i{}^2 + \overline{\beta}\eta_1(\alpha_i{}^2 + g_7)]$$

$$-1[\eta_1 C_3 \alpha_i{}^2(\alpha_i{}^2 + g_7) + d_1 d_2 \alpha_i{}^2(\alpha_i{}^2 - g_6)]\Big\}$$

$$M_i = \frac{1}{\Delta\mu_p}\Big\{g_2(1+\overline{\lambda})[-pC_3(1+\overline{\lambda})\alpha_i{}^2 - \overline{\beta}d_1\alpha_i{}^2]$$

$$-(\alpha_i{}^2 + g_1)[-pC_2(\alpha_i{}^2 + g_3) - \overline{\beta}d_2(\alpha_i{}^2 - g_6)]$$

$$-[-d_1 C_2 \alpha_i{}^2(\alpha_i{}^2 + g_3) + C_3(1+\overline{\lambda})\alpha_i{}^2 d_2(\alpha_i{}^2 - g_6)]\Big\}$$

$$N_i = \frac{1}{\Delta\beta^*}\Big\{g_2(1+\overline{\lambda})[-d_2{}^2(\alpha_i{}^2 - g_6)^2 - \eta_1 C_3(1+\overline{\lambda})(\alpha_i{}^2 + g_7)\alpha_i{}^2]$$

$$-g_5[-d_2(\alpha_i{}^2 + g_3)(\alpha_i{}^2 - g_6)C_2 + C_3(1+\overline{\lambda})d_2\alpha_i{}^2(\alpha_i{}^2 - g_6)]$$

$$+(\alpha_i{}^2 + g_1)[\eta_1 C_2(\alpha_i{}^2 + g_3)(\alpha_i{}^2 + g_7) + d_2{}^2(\alpha_i{}^2 - g_6)^2]\Big\}$$

IV. BOUNDARY CONDITIONS

The secular equations for the three dimensional vibration of cylindrical panel subjected to the traction free boundary conditions at the upper and lower surfaces at r = a, b listed below

(i) The traction free non-dimensional mechanical boundary conditions for a stress free edge are given by

$$\sigma_{rr} = 0; \qquad \sigma_{r\theta} = 0; \qquad \sigma_{rz} = 0$$

(ii) Thermal Condition

$$T_{,r} + hT = 0$$

Where h is the surface heat transfer co-efficient, $h \to 0$ thermally insulated surface and $h \to \infty$ refers to an isothermal one.

(iii) Magnetic Condition

$B_{,r} = 0$

If $\sigma_{rr} = 0$,

$$\sum_{i=1}^{4}(2+\bar{\lambda})A_i\left\{\left[\frac{\delta}{r'^2}J_\delta(\alpha_i r') - \frac{\alpha_i}{r'}J_{\delta+1}(\alpha_i r') + \frac{((\alpha_i r')^2R^2-\delta^2)}{r'^2}J_\delta(\alpha_i r')\right]\right.$$

$$\left. +\bar{\lambda}\left[\frac{\delta(\delta-1)}{r'^2}J_\delta(\alpha_i r') - \frac{\alpha_i}{r'}J_{\delta+1}(\alpha_i r')\right] + \left[t_L{}^2L_i\overline{W} - \beta_1\bar{T}R^2N_i - d_{31}M_i\bar{\chi}\right]\right\}$$

$$+A_5\left[(\bar{\lambda}+2)\frac{K\delta}{r'}J_{\delta+1}(Kr') - \frac{\delta(\delta-1)}{r'^2}J_\delta(Kr')\right]$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r'^2}J_\delta(Kr') - \frac{K}{r'}\delta J_{\delta+1}(Kr')\right] = 0$$

If $\sigma_{r\theta} = 0$,

$$\sum_{i=1}^{4}A_i\left[\frac{2\delta(\alpha_i)}{r'}J_{\delta+1}(\alpha_i r') - \frac{2\delta}{r'^2}(\delta-1)J_\delta(\alpha_i r')\right]$$

$$+A_5\left[\frac{K_1R^2r'^2}{r'^2}J_\delta(Kr') - \frac{2\delta(\delta-1)}{r'^2}J_\delta(Kr')\right]$$

$$+\left[\frac{K}{r'}J_{\delta+1}(Kr')\right] = 0$$

If $\sigma_{rz} = 0$,

$$\sum_{i=1}^{4}A_i\left[t_n(1-L_i)\left[\frac{\delta}{r'}J_\delta(\alpha_i r') - \alpha_i J_{\delta+1}(\alpha_i r')\right] + A_5\left(\frac{t_L}{r'}\delta J_\delta(Kr')\right)\right] = 0$$

If $T_{,r} = 0$,

$$\sum_{i=1}^{4}N_i\left[\frac{\delta}{r'}J_\delta(\alpha_i r') - \alpha_i J_{\delta+1}(\alpha_i r')\right] = 0$$

If $B_{,r} = 0$,

$$\sum_{i=1}^{4}\left\{d_{15}t_L\left[(1+\mu_{11}M_i - L_i)\left(\frac{\delta}{r'}J_\delta(\alpha_i r') - \alpha_i J_{\delta+1}(\alpha_i r')\right)\right]\right\} + A_5\left(\frac{t_L}{r'}J_\delta(Kr')\right) = 0$$

V. FREQUENCY EQUATIONS

$|a_{ij}| = 0$ for $i, j = 1, 2, 3, \dots 10$

where

$$a_{11} = (2+\bar{\lambda})[\delta J_{\delta}(\alpha_1 r_1')/r'^2 - \alpha_1 J_{\delta+1}(\alpha_1 r_1')/r'$$

$$+((\alpha_1 r_1')^2 R^2 - \delta^2) J_{\delta}(\alpha r_1')/r_1'^2]$$

$$+\bar{\lambda}[\delta(\delta-1) J_{\sigma}(\alpha_1 r_1')/r'^2 - \alpha_1 J_{\delta+1}(\alpha_1 r_1')/r']$$

$$+(t_L{}^2 L_1 - \beta_1 R^2 N_1 - d_{31} t_L{}^2 M_1) J_{\delta}(\alpha_1 r_1')$$

$$a_{13} = (2+\bar{\lambda})[\delta J_{\delta}(\alpha_2 r_1') - \frac{\alpha_2 J_{\delta+1}}{r_1'}(\alpha_2 r_1') + \frac{((\alpha_2 r_1')^2 R^2 - \delta^2)}{r_1'^2} J_{\delta}(\alpha_2 r_1')$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r_1'^2} J_{\delta}(\alpha_2 r_1') - \frac{\alpha_2 J_{\delta+1}}{r_1'}(\alpha_2 r_1')\right]$$

$$+(t_L{}^2 L_2 - \beta R^2 N_2 - d_{31} t_L{}^2 M_2) J_{\delta}(\alpha_2 r_1')$$

$$a_{15} = (2+\bar{\lambda})[\frac{\delta J_{\delta}}{r_1'^2}(\alpha_3 r_1') - \frac{\alpha_3 J_{\delta+1}}{r_1'}(\alpha_3 r_1') + \left(\frac{(\alpha_3 r_1')^2 R^2 - \delta^2)}{r_1'^2}\right) J_{\delta}(\alpha_3 r_1')$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r_1'^2} J_{\delta}(\alpha_3 r_1') - \frac{\alpha_3 J_{\delta+1}}{r_1'}(\alpha_3 r_1')\right]$$

$$+(t_L{}^2 L_3 - \beta_1 R^2 N_3 - d_{31} t_L{}^2 M_3) J_{\delta}(\alpha_3 r_1')$$

$$a_{17} = (\bar{\lambda}+2)\left[\frac{\delta J_{\delta}(\alpha_4 r_1')}{r_1'^2} - \frac{\alpha_4 J_{\delta+1}(\alpha_4 r_1')}{r_1'} + \frac{((\alpha_4 r_1')^2 R^2 - \delta^2)}{r_1'^2} J_{\delta}(\alpha_4 r_1')\right]$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r_1'^2} J_{\delta}(\alpha_4 r_1') - \alpha_4 J_{\delta+1}(\alpha_4 r_1')\right]$$

$$+(t_L{}^2 L_4 - \beta_1 R^2 N_4 - d_{31} t_L{}^2 M_4) J_{\delta}(\alpha_4 r_1')$$

$$a_{19} = (\bar{\lambda}+2)\left[\frac{K J_{\delta}}{r_1'} J_{\delta+1}(K r_1') - \frac{\delta(\delta-1)}{r_1'^2} J_{\delta}(K r_1')\right]$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r_1'^2} J_{\delta}(K r_1') - \frac{K\delta J_{\delta+1}(K r_1')}{r_1'}\right]$$

$$a_{19} = (\bar{\lambda}+2)\left[\frac{KJ_\delta}{r_1'}J_{\delta+1}(Kr_1') - \frac{\delta(\delta-1)}{r_1'^2}J_\delta(Kr_1')\right]$$

$$+\bar{\lambda}\left[\frac{\delta(\delta-1)}{r_1'^2}J_\delta(Kr_1') - \frac{K\delta J_{\delta+1}(Kr_1')}{r_1'}\right]$$

$$a_{21} = \frac{2\delta}{r_1'}\alpha_1 J_{\delta+1}(\alpha_2 r_1') - \frac{2\delta(\delta-1)}{r_1'^2}J_\delta(\alpha_1 r_1')$$

$$a_{23} = \frac{2\delta\alpha_2}{r_1'}J_{\delta+1}(\alpha_2 r_1') - \frac{2\delta(\delta-1)}{r_1'^2}J_\delta(\alpha_2 r_1')$$

$$a_{25} = \frac{2\delta\alpha_3}{r_1'}J_{\delta+1}(\alpha_3 r_1') - \frac{2\delta(\delta-1)}{r_1'^2}J_\delta(\alpha_3 r_1')$$

$$a_{27} = \frac{2\delta\alpha_4}{r_1'}J_{\delta+1}(\alpha_4 r_1') - \frac{2\delta(\delta-1)}{r_1'^2}J_\delta(\alpha_4 r_1')$$

$$a_{29} = \frac{KR^2 r_1'^2}{r_1'^2}J_\delta(Kr_1') - \frac{2\delta(\delta-1)}{r_1'^2}J_\delta(Kr_1') + \frac{K}{r_1'}J_{\delta+1}(Kr_1')$$

$$a_{31} = t_L(1-L_1)\left[\frac{\delta}{r_1'}J_\delta(\alpha_1 r_1') - \alpha_1 J_{\delta+1}(\alpha_1 r_1')\right]$$

$$a_{33} = t_L(1-L_2)\left[\frac{\delta}{r_1'}J_\delta(\alpha_2 r_1') - \alpha_2 J_{\delta+1}(\alpha_2 r_1')\right]$$

$$a_{35} = t_L(1-L_3)\left[\frac{\delta}{r_1'}J_\delta(\alpha_3 r_1') - \alpha_3 J_{\delta+1}(\alpha_3 r_1')\right]$$

$$a_{37} = t_L(1-L_4)\left[\frac{\delta}{r_1'}J_\delta(\alpha_4 r_1') - \alpha_4 J_{\delta+1}(\alpha_4 r_1')\right]$$

$$a_{39} = t_L\,\frac{\delta J_\delta}{r_1'}(Kr_1')$$

$$a_{41} = N_1\left[\frac{\delta J_\delta}{r_1'}(\alpha_1 r_1') - \alpha_1 J_{\delta+1}(\alpha_1 r_1')\right]$$

$$a_{43} = N_2\left[\frac{\delta J_\delta}{r_1'}(\alpha_2 r_1') - \alpha_2 J_{\delta+1}(\alpha_2 r_1')\right]$$

$$a_{45} = N_3\left[\frac{\delta J_\delta}{r_1'}(\alpha_3 r_1') - \alpha_3 J_{\delta+1}(\alpha_3 r_1')\right]$$

$$a_{47} = N_4\left[\frac{\delta J_\delta}{r_1'}(\alpha_4 r_1') - \alpha_4 J_{\delta+1}(\alpha_4 r_1')\right]$$

$$a_{49} = 0$$

$$a_{51} = t_L\left[(1+\mu_{11}M_1 - L_1)\left(\frac{\delta J_\delta}{r_1'}(\alpha_1 r_1') - \alpha_1 J_{\delta+1}(\alpha_1 r_1')\right)\right]$$

$$a_{53} = t_L\left[(1+\mu_{11}M_2 - L_2)\left(\frac{\delta J_\delta}{r_1'}(\alpha_2 r_1') - \alpha_2 J_{\delta+1}(\alpha_2 r_1')\right)\right]$$

$$a_{55} = t_L\left[(1+\mu_{11}M_3 - L_3)\left(\frac{\delta J_\delta}{r_1'}(\alpha_3 r_1') - \alpha_3 J_{\delta+1}(\alpha_3 r_1')\right)\right]$$

$$a_{57} = t_L\left[(1+\mu_{11}M_4 - L_4)\left(\frac{\delta J_\delta}{r_1'}(\alpha_4 r_1') - \alpha_4 J_{\delta+1}(\alpha_4 r_1')\right)\right]$$

$$a_{59} = \frac{t_L}{r_1'} J_\delta(K r_1')$$

Here a_{ij} (j = 2, 4, 6, 8, 10) can be obtained by replacing a modified Bessel function of first kind in a_{ij} (j = 1, 3, 5, 7, 9) with those of second kind and a_{ij} (j = 6, 7, 8, 9, 10) can be obtained by replacing r_1' in a_{ij} (i = 1, 2, 3, 4, 5) with r_2'.

VI. NUMERICAL RESULTS AND DISCUSSION

The frequency equation(11) is numerically solved for magneto-strictive cobalt iron oxide ($CoFe_2O_4$) material. For the purpose of numerical computation we consider the closed circular cylindrical panel with the center angle α= 2π and the integer n must be even since the panel vibrates in circumferential full wave. In fact the frequency equation for a closed cylindrical panel can be obtained by setting δ =l (l = 1, 2, 3..) where l is the circumferential wave number in equation(11). An isotropic material properties of $CoFe_2O_4$are as follows:

λ = 3.75 x 10^9 Nm^{-2}
μ = 2.5 x 10^9 Nm^{-2}
T_0 = 298 K
C_γ = 420 J kg^{-1} k^{-1}
ρ = 5.3 x 10^3 kg m^{-3}
β_1 = 1.52 x 10^6 $NK^{-1}m^{-2}$
p_3 = - 452 x 10^{-6} C K^{-1} m^{-2}
μ_{11}= -344.66 x 10^{-6} Ns^2C^{-2}
μ_{33} = 91.72 x 10^{-6} Ns^2
d_{31} = 8.2 x 10^{-1} $NA^{-1}m^{-1}$
d_{33} = 1.0 $NA^{-1}m^{-1}$
d_{51} = 7.810^{-1} $NA^{-1}m^{-1}$
β_3= 1.53 x 10^6 $NK^{-1}m^{-2}$
K_1 = K_3 = 1.5 W m^{-1} K^{-1}

Zeros of the frequency equations are evaluated using a computer program in MATLAB software which has been developed for this purpose. The roots of the algebraic equation (11) are complex for all considerable values of wave number, therefore the waves are attenuated in space.
The attenuation equation is given by:

$$c^{-1} = v^{-1} + i\omega^{-1}q$$

In tables (1) and (2),a comparison is made for the non-dimensional frequencies of symmetric and anti-symmetric modes of vibration among the longitudinal, flexural, torsional modes of thermally insulated and isothermal boundary conditions for an isotropic cylindrical-panel with a/b =0.25. From these tables it is clear that as the first five vibration modes increase, the non-dimensional frequencies also increase in both symmetric and anti-symmetric modes of vibration and also it is clear that the non-dimensional frequency profiles exhibit high amplitude for anti-symmetric mode compared with symmetric mode of vibration.

Table 1: Non-dimensional frequencies for first five symmetric modes of longitudinal, flexural and torsional vibration of isotropic cylindrical panel with a/b = 0.25.

Mode	Thermally insulated			Isothermal		
	Longitudinal mode	Flexural mode	Torsional mode	Longitudinal Mode	Flexural mode	Torsional mode
S1	1.3937	1.3927	1.5565	1.2289	1.2278	1.4295
S2	1.6542	1.6533	1.8391	1.4614	1.4604	1.7886
S3	1.9176	1.9156	2.1227	1.7009	1.7019	2.0529
S4	2.1832	2.1802	2.4048	1.9486	1.9475	2.2504
S5	2.5840	2.5810	2.8318	2.3381	2.3375	3.5245

Table 2: Non-dimensional frequencies for first five anti-symmetric modes of longitudinal, flexural and torsional vibration of isotropic cylindrical panel with a/ b = 0.25.

Mode	Thermally insulated			Isothermal		
	Longitudinal mode	Flexural mode	Torsional mode	Longitudinal mode	Flexural mode	Torsional mode
S1	1.4069	1.3405	1.2414	1.4049	1.3237	1.3588
S2	1.6702	1.6039	1.4868	1.5611	1.6502	1.6123
S3	1.9360	1.8722	1.7452	1.9182	1.9076	1.8682
S4	2.2408	2.1458	2.0174	1.0768	1.1753	2.1264
S5	2.6137	2.5641	2.4496	2.5680	2.6670	2.5183

Table 3: Non-dimensional frequencies for first five symmetric modes of longitudinal, flexural, and torsional vibration of isotropic cylindrical panel with a/b = 0.5.

Mode	Thermally insulated			Isothermal		
	Longitudinal mode	Flexural mode	Torsional mode	Longitudinal mode	Flexural mode	Torsional mode
S1	1.4558	1.4543	1.5153	1.4049	1.4037	1.5588
S2	1.7260	1.7251	1.8827	1.6611	1.5602	1.7123
S3	1.9967	1.9057	2.0511	1.9182	1.9076	1.9682
S4	2.1778	2.2648	3.0213	2.1768	2.1753	2.5264
S5	2.6754	2.6732	3.8303	2.5680	2.5670	2.6183

From tables (3) and (4), it is observed that as the modes increase the non-dimensional frequencies also increase, whereas the dispersion of longitudinal and flexural modes are almost same and the torsional mode gets dominant in symmetric and anti-symmetric cases. The amplitude of all modes of vibrations is increased in magnitude with respect to aspect ratios also. The temperature change of insulated boundary condition is more pertinent in these vibrational modes.

The dispersion of displacement, temperature change and perturbed magnetic field in case of fundamental modes for the symmetric and anti symmetric cases of the cylindrical panel plays a vital role in smart material applications. This type of model analysis is very important in bio-sensing applications in nuclear magnetic resonance(NMR), magnetic resonance imaging (MRI) and echo planar imaging (EPI).

6.1. Dispersion curves

In figures (1)and (2), the variations of the non-dimensional frequency(Ω) of an elastic cylindrical panel with respect to the parameter $t_L = m\pi R/L$ have been shown for different values of the thickness to mean radius of the panel ($t^* = 0.05$, 0.1, 0.25 and 0.5) for isothermal and thermally insulated boundary conditions respectively. From Figure 1 it is observed that the non-dimensional frequency of the panel shows almost linear variation with respect to t_L for the increasing thickness parameter t^*.But in Figure 2 some dispersion is observed from the linear behavior of frequency with respect to t_L for different values of t^*. In both the cases at small values of the parameter t_L in the range $0 \le t_L \le 1$ the values of frequency are almost steady for different values of the thickness parameter $t^* = 0.05$, 0.1, 0.25 and 0.5, where for higher values of t_L the frequency is large and starting dispersive curves are increasing curve according to the increasing values of t^*.

Table 4: Non-dimensional frequencies of symmetric and anti-symmetric modes of longitudinal, flexural, torsional vibrations of thermally insulated and isothermal boundary conditions for a isotropic cylindrical panel with a/b = 0.5

Mode	Thermally insulated			Isothermal		
	Longitudinal mode	Flexural mode	Torsional mode	Longitudinal mode	Flexural mode	Torsional mode
S1	1.4237	1.5028	1.6295	1.4125	1.4337	1.4768
S2	1.6842	1.6842	1.7986	1.5611	1.6602	1.6323
S3	1.9576	1.9239	1.9529	1.8180	1.9176	1.9682
S4	2.3232	2.1832	2.7204	2.2768	2.1753	2.6264
S5	2.6840	2.5840	3.9245	2.6443	2.5870	3.5183

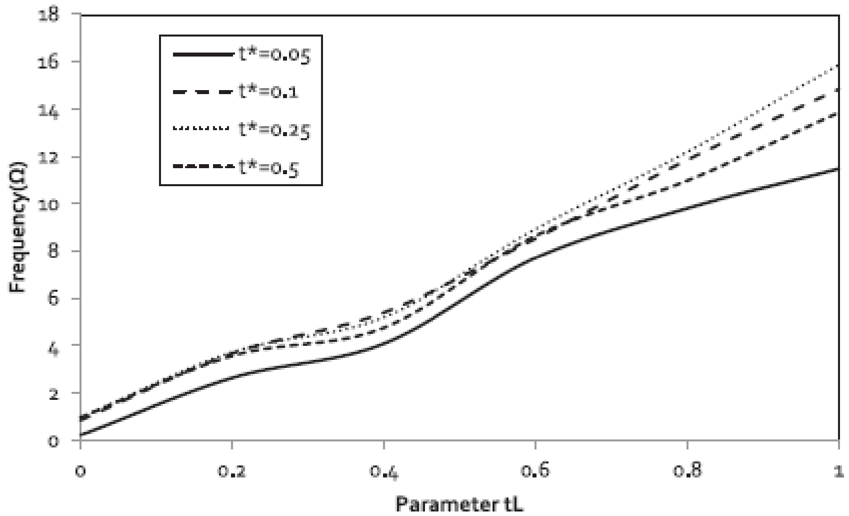

Figure 1. Dispersion curves for frequency versus t_L with different t* for isothermal cylindrical panel.

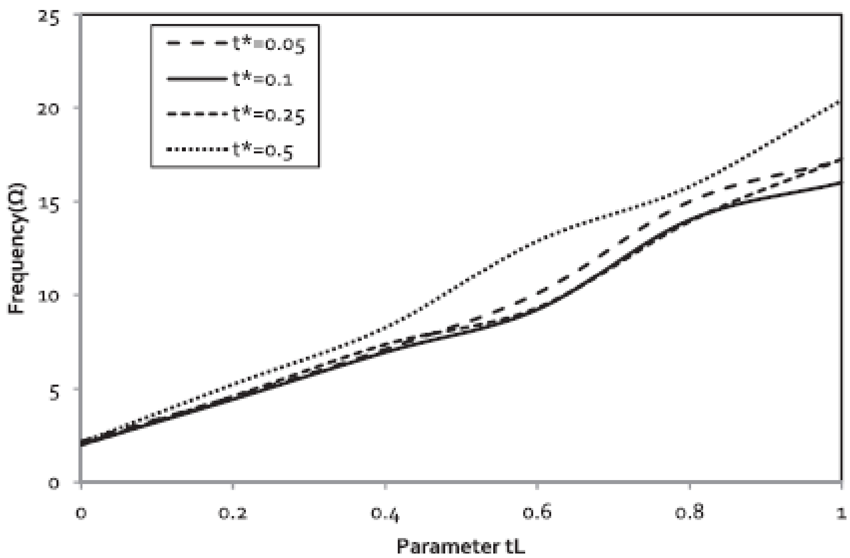

Figure 2 Dispersion curves for frequency versus t_L with different t* for thermally insulated cylindrical panel.

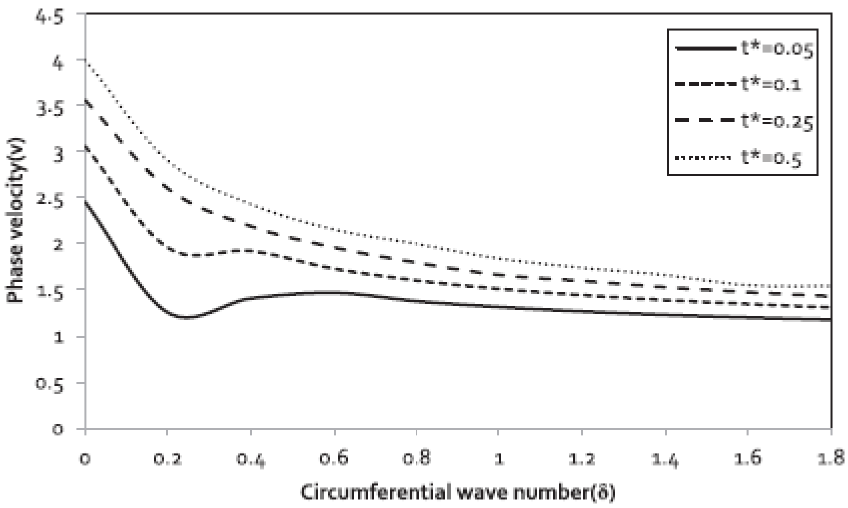

Figure 3 Dispersion curves for phase velocity versus circumferential wave number with different t* for isothermal cylindrical panel.

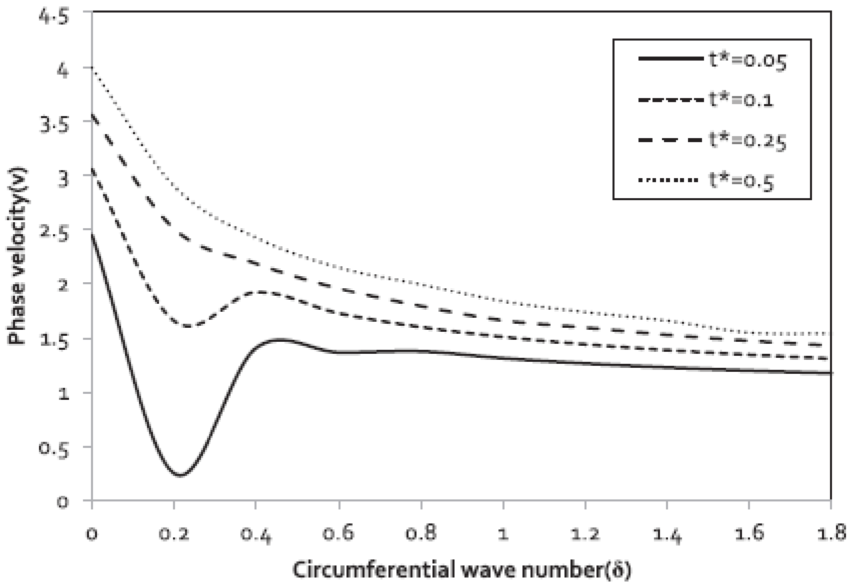

Figure 4 Dispersion curves for phase velocity versus circumferential wave number with different t* for thermally insulated cylindrical panel.

The variation of the non-dimensional phase velocity with respect to the wave number for isothermal and thermally insulated cylindrical panel is shown in figures (3) and (4)respectively. From these curves it is clear that the phase velocity curves are dispersive only for small values of wave number in the range $0 \leq \delta \leq 0.4$.But for higher values of wave number, these become non-dispersive for both isothermal and insulated boundary conditions. But there is a small deviation in the magnitude of frequency in insulated boundary which might happen because of the dissipation of energy and random behavior of molecules due to thermal and magnetic waves and strong alignment of molecules. The phase velocity of higher modes of propagation attains quite large values at vanishing wave number which sharply slashes down to become steady and asymptotic with increasing wave number due to the coupling effect of magnetic and thermal fields.

VII. CONCLUSION

The three dimensional vibration analysis of a homogeneous isotropic magneto thermo elastic cylindrical panel subjected to traction free boundary conditions has been considered for this paper. For this problem, the governing equations of three dimensional linear elasticity have been employed and solved by a Bessel function solution with complex arguments. The effect of various mechanical parameters of a closed magneto-strictive cobalt iron oxide cylindrical panel is investigated for isothermal and thermally insulated boundary. The results are depicted as dispersion curves. A significant effect of the coupling of magnetic and thermal is observed on phase velocity and attenuation coefficient. These results provide useful reference solution for modeling and design to the engineers and designers.

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