TUNING OF A FIRST-ORDER LAG-LEAD COMPENSATOR USED WITH A VERY SLOW SECOND-ORDER PROCESS

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ABSTRACT

Lag-lead compensators are well known in automatic control engineering. They have 3 or 4 parameters to be adjusted (tuned) for proper operation depending on the compensator order. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure. A selected very slow second-order process has a time response to a unit step input of 150 seconds settling time is controlled using a first-order lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of square of errors of the control system using MATLAB. Four functional constrains are used to control the performance of the lag-lead compensated control system. The result was reducing the settling time to only 0.666 seconds. The steady-state characteristics of closed-loop control system using the first-order lag-lead compensator were excellent. It was possible reduce the steady-state error to less than 0.01 using the tuned first-order lag-lead compensator.

KEYWORDS: *First-order Lag-lead Compensator, Very Slow Second-order Process, Compensator Tuning, Control System Performance.*

I. INTRODUCTION

Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications. In the present work we follow the research school using the time response specifications. Still the subject is interested to automatic control researchers.

James, Frederick and Taylor (1987) discussed the application of expert system technique to the design of lead-lag compensators for linear SISO systems [1]. Loh, Cai and Tan (2004) studied the auto-tuning of phase lead-lag compensators using the frequency response of the plant using relays with hysteresis [2]. Chang (2004) used phase-lag and phase-lead compensators to control servo control systems [3]. Wang (2006) developed a non-trial-and-error procedure to design lag-lead compensators based on the idea of Yeung-Wang-Chen's graphical-based non-trial-and –error method for 3-parameters lag-lead compensators and Wang's result on the exact and unique solution for single lag and lead compensator design [4]. Zhang, Liu, Dang, Zhang and Ou (2006) used a lag-lead compensator to control asynchronous linear motors for better performance and application to active mass driver control system for vibration control of civil engineering structures [5].

Panda and Padhy (2007) used the genetic algorithm optimization technique to design thyristor controlled series compensator-based controller to enhance the power system stability [6]. Nassirharand (2008) developed an educational software utility for designing linear compensators based on the Youla parameterization technique and an exact model matching criterion [7]. Wang (2009) provided an approach for phase-lead/lag compensators to achieve the desired specifications of gain and phase margins for all-pole stable plant with time-delay [8]. Cao, Watkins and O'Brien (2009) discussed a compensator graphical user interface implemented by MATLAB to enable the user to design a continuous time compensator using the root-locus and Bode plot [9]. Li, Sheng and Chen

(2010) derived the impulse response of the distributed order lead-lag compensator using MATLAB and compared with the numerical inverse Laplace Transformation method [10]. Zanasi and Cuoghi (2011) presented three different methods for the synthesis of lead-lag compensator meeting the phase margin and the gain crossover frequency [11]. Nandar (2012) proposed a robot power system stabilizer using genetic algorithm and a first-order lead-lag compensator [12]. Ntogramatzidis, Zanasi and Cuoghi (2012) presented a range of design techniques for the analysis of standard compensators (lead, lag, PID) in terms of the steady-state performance, the stability margins and the crossover frequencies [13].

Nagshi, Rahmani, Vahidi and Hosseinian (2013) introduced a combination of static VAR and a lag-lead controller to enhance the dynamic stability of power systems. They achieved better dynamic performance and reduced steady-state error [14]. Mahmoud (2013) described short steps to design a phase-lead compensator for the mass-spring-damper system to achieve the desired level of its phase margin. He used a first-order phase lead compensator [15]. Hassaan, Al-Gamil and Lashin (2013) used the sum of absolute error criterion to tune a lag-lead compensator used with a first-order process plus an integrator process. Their tuned compensator could reduce the maximum overshoot from 67.3 % to 2.44 % and the settling time from 12 to 0.65 seconds [16]. Bansal and Dewan (2014) investigated the application of PID control a series compensator to stabilize a gimbal system. They were in favor of using a PID controller against using a simple compensator [17]. Sedaghati et. al. (2014) used a 2-stage lead-lag compensator compared with another types of controllers to provide the damping required to stabilize power systems [18]. Morishita, Suzuki and Iwamoto (2014) described obtaining robustness of a power system stabilizer of a lead-lag compensator using the H∞ control. They optimized the parameters of the compensator using a particle swarm optimization with evolution function considering the closed-loop H∞ norm and a desired response [19].

II. ANALYSIS

Process:

This process is a second order process consisting of two simple-poles and a process gain. It has the transfer function:

$$G_p(s) = K_p / \{(1 + T_1 s)(1 + T_2 s)\} \quad (1)$$

Where: $K_p = 1$

$T_1 = 0.1s$

and $T_2 = 50$ s

The process transfer function of Eq.1 can be written in the standard form;

$$G_p(s) = K_p\, \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \quad (2)$$

Where: $\omega_n = 1 / \sqrt{(T_1 T_2)} = 0.4472$ rad/s

And $\zeta = 0.5(T_1 + T_2) / \sqrt{(T_1 T_2)} = 11.203$

The process has the time response to a unit step input shown in Fig.1.

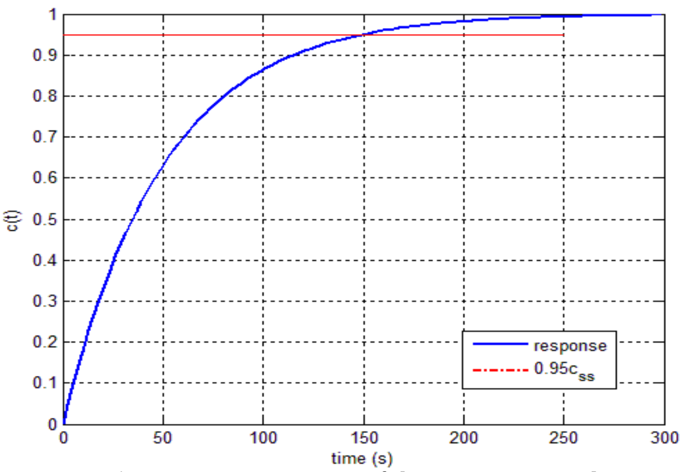

Figure1. Step response of the uncompensated process.

The process has a zero maximum overshoot and a 150 seconds settling time (indicating the very slow time response characteristic of the process).

Lag-lead First-order Compensator:

The first-order lag-lead compensator has a transfer function $G_c(s)$ given by [20]:

$$G_c(s) = K_c(1 + T_z s) / (1+T_p s) \quad (3)$$

Where: T_z = the compensator simple zero

T_p = the compensator simple pole

K_c = the compensator gain

The compensator has 3 parameters: T_z, T_p and K_c which have to be tuned to satisfy the required specifications of the closed-loop control system defined by:

- The steady-state characteristics.
- The maximum percentage overshoot.
- The maximum percentage undershoot.
- The settling time.

Control System Transfer Function:

Assuming that the control system is a unit feedback one, its transfer function with $G_c(s)$ and $G_p(s)$ in its forward path for $K_p = 1$ is:

$$M(s) = (b_0 s + b_1) / (a_0 s^3 + a_1 s^2 + a_2 s + a_3) \quad (4)$$

where:

$b_0 = \omega_n^2 K_c T_z$

$b_1 = \omega_n^2 K_c$

$a_0 = T_p$

$a_1 = 1 + 2\zeta\omega_n T_p$

$a_2 = 2\zeta\omega_n + \omega_n^2 T_p + \omega_n^2 K_c T_z$

$a_3 = \omega_n^2 + \omega_n^2 K_c$

System Step Response:

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response c(t) as function of time.

III. LAG-LEAD COMPENSATOR TUNING

The sum of square of error (ISE) is used as an objective function, F of the optimization process. Thus:

$$F = \int [c(t) - c_{ss}]^2 \, dt \quad (5)$$

where c_{ss} = steady state response of the system.

The performance of the control system is controlled using four functional constraints:

(i) The maximum percentage overshoot, OS_{max}:

$$OS_{max} = 100(c_{max} - c_{ss}) / c_{ss}$$

With OS_{maxdes} is the desired value of the maximum percentage overshoot, the first functional constraint is:

$$c_1 = OS_{max} - OS_{maxdes} \quad (6)$$

(ii) The settling time, T_s:

$$c_2 = T_s - T_{sdes} \quad (7)$$

Where T_s is the time after which the response enters a band of ± 5 % of c_{ss} and stays within it. This functional constraint controls the speed of response of the system time response. Tsdes is the desired setling time of the closed loop system.

(iii) The steady-state error, e_{ss}:

We define the steady-state error for a unit step input as:

$$e_{ss} = 1 - c_{ss}$$

Where using Eq.3, the steady-stagte response is:

$$c_{ss} = b_1 / a_3$$

With ess_{des} is the desired stead-state error, the third functional constaint is:

$c_3 = e_{ss} - e_{ssdes}$ (8)

(iv) A stability constraint:

The control system is a fourth order one which depending on the transfer functiom parameters may be unstable. Therefore, a constraint or more is required to be imposed to guarantee a stable system during the optimization process. The constraints for the case study in hand are assigned using Routh-Hurwitz criterion of the system characteristic equation:

- Positive compensator parameters.
- Certain function relating the compensator parameters. This function gives the third functional constaint of the optimization process. That is:

  $c_4 = a_0a_3 - a_1a_2$ (9)

Parameters Limits:

- A lower limit of 0.001 is set for the compensator parameters: T_z, T_p and K_c.
- An upper limit of 100 is set for the three compensator parameters.

Tuning Results:

The MATLAB command "*fmincon*" is used to minimize the optimization objective function given by Eq.5 subjected to the functional inequality constraints given by Eqs. 6, through 9 to provide the first-order lag-lead compensator parameters subjected to the limits mentioned in section 3. The results are as follows:

Compensator parameters:

T_z = 30.1813 s
T_p = 14.6565 s
K_c = 80.063

A MATLAB code was written as a .m file to apply the technique discussed in this paper leading to obtaining the optimum compensator parameter and proving its time response to a unit step input [21]. The time respone of the compensated system to a unit step input is shown in Fig.2.

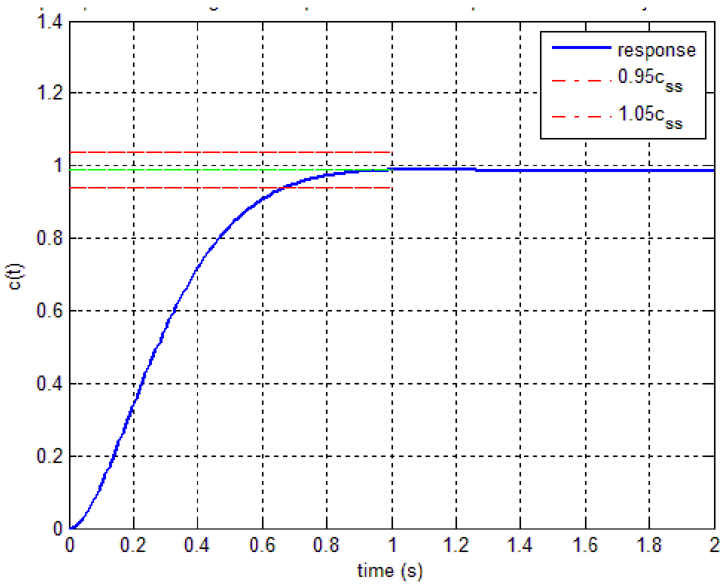

Figure 2. Step response of the lag-lead compensated system.

Characteristics of the control system using the tuned first-order lag-lead compensator:

- Maximum percentage overshoot: 0 %
- Maximum percentage undershoot: 0 %
- Settling time: 0.666 s

IV. CONCLUSIONS

- The suggested tuning technique of first-order lag-lead compensators used with a very slow process is superior.

- Using the proposed tuning technique, it was possible to reduce the settling time from 150 seconds to only 0.666 seconds.
- There was no overshoot and undershoot.
- The steady-state error associated with a unit step input was only 0.0099.
- The optimization tuning approach used in this work is simple, straight forward, and provides the compensator parameters in a very small time using MATLAB.

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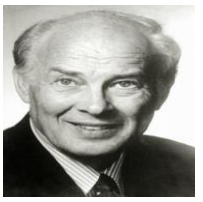
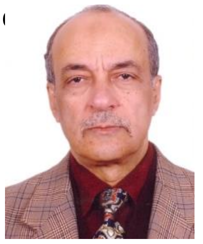
DEDICATION

I dedicate this work to my Ph.D. supervisor Prof. **John Parnaby** who was the chairman of the Manufacturing Systems Engineering Department of Bradford University during the 1970's. Prof. Parnaby died on 5th January 2011 in UK.



BIOGRAPHY



Galal Ali Hassaan Emeritus professor of System Dynamics, Faculty of Engineering, EGYPT.

- Has got hid Ph.D. from Bradford University, UK in 1979 under the Supervision of Prof. John Parnaby.
- Research on Automatic Control, Mechanical Vibrations and Mechanism Synthesis.
- Served in a large number of universities in Africa and Asia.
- Published 10's of papers in international journals and conferences.
- Wrote books on Experimental Systems Control and History of Mechanical Engineering.

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