COMPARATIVE STUDY BETWEEN LOAD DISTRIBUTION METHODS ON STRAIGHT BRIDGE DECKS WITH VARIATION IN THE NUMBER OF CROSSBEAMS - OPEN SECTION SUPERSTRUCTURE

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ABSTRACT

This work's objective is to analyze and dimension of Reinforced Concrete stringers using the Engesser-Courbon Method for moving loads and the Yield Line Theory, Areas of Influence and Grid Method for determining permanent loads. It also to evaluate the influence of the number and stiffness of crossbeams on load distribution. With the research, concluded that the Areas of Influence method has shown to be more effective on decks with high crossbeams stiffness. The Yield Line Theory offered more accurate results compared to the Areas of Influence Method. The number of crossbeams was what most impacted the difference in steel areas between the decks. When the crossbeams numbers went from 4 to 5 and from 5 to 6, there was a reduction in the steel areas of the reinforcements by 25%, 35% and 40% on average, between the steel areas of the vertical stirrups positive and negative longitudinal reinforcements respectively.

KEYWORDS: *Reinforced Concrete Bridges, Load Distribution, Method Engesser-Courbon, Grid method e Variation of Crossbeams.*

I. INTRODUCTION

Among the analyses necessary for bridge project, the transverse distribution of forces is essential to ensure safety and cost-efficiency in the project. Regarding moving loads, there are different load distribution methods studied over the years. Thus, Santos and Santos [1] evaluated the transverse distribution of forces using the Engesser-Courbon, Leonhardt, Guyon-Massonet methods, and the Finite Element Method (FEM) in two distinct structural models with two and five beams. They concluded that for the first model, the Engesser-Courbon Method was considered to favor safety, while for the second model, the Leonhardt load distribution method showed better overall performance, with Courbon and Guyon-Massonet methods performing better in the internal beams.

In addition to Santos and Santos [1], Cóco et al. [2] also studied the Courbon Method and compared its results with the Finite Element Method (FEM) for a curved trajectory deck. They concluded that for the sake of simplifying calculations, the Engesser-Courbon Method can be used in the analysis of curved trajectory deck structures with cross-sections similar to the tested case study. Cóco et al. [2] also evaluated that the Courbon Method showed better performance in the innermost beams of the deck, just like Santos and Santos [1].

The authors Donin and Machado [3] evaluated the transverse load distribution using the Engesser-Courbon, Guyon-Massonnet methods, and the Finite Element Method (FEM), while Jovem [4] studied the Engesser-Courbon, Leonhardt, Guyon-Massonet, Homberg-Trenks methods, and the Fauchart Process and compared them with the numerical models idealized in SAP2000 and CSiBridge V18. Donin and Machado [3] used a deck with 4 beams and 3 crossbeams, while Jovem [4] used decks with multiple beams composed of 3, 5, 7, and 8 main beams. Both studies concluded that the

FEM discretized models represented the bridge's behavior more satisfactorily, but the other methods obtained close results, validating the use of the Courbon method for the studied cases.

Santos and Santos [1] conducted the analysis through the bending moments found in the beams and slabs, while Cóco et al. [2] and Donin and Machado [3] did so using load trains and maximum bending moments. On the other hand, Jovem [4] obtained the results through influence lines and load distribution coefficients of the structures.

For the analysis of permanent loads, it is also necessary to assess the distribution of forces along the deck. This analysis varies according to the distribution methods used and the geometry of the slabs. According to Pinheiro [5], there are basically two methods for calculating the forces, namely: the Elastic Method and the Plastic Method. The Elastic Method provides a good approximation of service loads, while the Plastic Method gives a good approximation of the loads in the failure condition.

For the elastic method, one can highlight the Hillerborg equilibrium theory. According to Pinheiro [5], this theory consists of determining a distribution of moments that satisfy the boundary conditions and equilibrium, and then calculating the slab's resistance to these forces. In agreement with Pinheiro [5], Xavier and Melo [6] state that a satisfactory distribution of forces occurs when torsional moments are zero, characterizing the Simplified Hillerborg Strip Method.

The Plastic Method can be derived from the Theory of Plastic Hinges (failure lines), where slabs subjected to maximum loads form hinges, or plastic zones, which allow a redistribution of forces in the structure. The ABNT NBR 6118 [7] standard considers this theory for analyzing the ultimate limit state of the structure.

In their work, Xavier and Melo [6] compare the Hillerborg Method with the Theory of Plastic Hinges, using Rectangular Elastic Slabs, and obtained identical results. That is, based on the results obtained by Xavier and Melo [6], it is possible to conclude that the Plastic Hinges Method is well suited for rectangular slabs. The ABNT NBR 6118 [7] allows a simplified approach to this method, where the reactions at the supports of the structures can be calculated by assuming a uniform distribution of forces in the slabs over the structural elements that support them. This simplification is referred to, in this study, as the Method of Influence Areas.

This study aims to analyze, and project reinforced concrete beams for three distinct decks. Based on the results found by [1]-[4], it is evident that the Engesser-Courbon Method supports the results obtained through discretized models in FEM, which are considered more accurate. The analyses in [1]-[4] also show that the Courbon Method is easy to apply, making it feasible for use in this study for the analysis of live loads. The Plastic Hinges, Influence Areas, and Grid Methods for permanent loads will also be used in this research. The three deck models adopted are composed of 3 beams and 4, 5, and 6 crossbeams, respectively. The variation in the number of crossbeams will allow for the evaluation of the impact of crossbeam quantity on the beam project, using the different applied methods.

The aim of the study is to compare the results obtained using the Plastic Hinges and Influence Areas Methods in the effort envelopes and, consequently, in the areas of steel reinforcement. Furthermore, the Grid Method will be used to evaluate the beams in isolation, considering both the infinite stiffness and the actual stiffness of the beams. This analysis will not only compare the Load Distribution Methods but will also assess their implications on the final project of the superstructure. It will also investigate whether the number of crossbeams has a significant impact on the application of these methods and whether there are differences between the edge beams and the central beams of the deck.

The organization of this work was structured into sequential stages to ensure clarity and efficiency in the development of this article. Initially, in Section II, the methodology was defined, which included the geometric characterization of the defined deck, presented in Section 2.1. Section 2.2 covered the application of the methods for determining permanent loads used in the research: the Plastic Hinge Method (Section 2.2.1), the Influence Area Method (Section 2.2.2), and the Grid Model (Section 2.2.3). Section 2.2.4 presented the determining of moving loads acting on the deck, and finally, Section 2.2.5 outlined the considerations used for beam sizing.The next phase, Section III, focused on presenting the results considering the study's objectives. In Section 3.1, a comparison of the methods used to determine permanent loads was presented. Section 3.2 displayed the analysis of moving loads, while Section 3.3 presented the steel areas of the beams obtained after sizing. Finally, Section IV consolidated the conclusions derived from the results obtained.

II. METHODOLOGY

To carry out the proposed work, the first step was to determine the model to be analyzed. Based on the geometric characterization of the deck, the loads acting on the structure were defined, and the longitudinal load train was obtained according to the proposed analysis methods. Using the obtained load trains and influence lines, we identified the maximum and minimum forces acting on the structure, which led to the effort envelope and enabled the project of the beams according to ABNT NBR 6118 [7].

2.1. Geometric characterization of the deck

The idealized decks have a length of 36 meters, with a total width of 21.6 meters, consisting of 4 lanes occupying 12.6 meters, plus a shoulder of 2.50 meters and a sidewalk of 1.5 meters. The cross-section of the deck is composed of two central spans of 7.2 meters and 2 cantilevers of 3.6 meters at the ends. The beams (longitudinal and transverse) have equal dimensions, with a thickness of 0.30m and a height of 1.6m.

For the research execution, three decks were idealized, altering the number of support transverses between them. The deck 1 consists of 4 transverses, supported by 12 pillars, forming 3 spans of 12 meters each. The deck 2 consists of 5 transverses, supported by 15 pillars, forming 4 spans of 9 meters each. The deck 3 consists of 6 transverses, supported by 18 pillars, forming 5 spans of 7.2 meters each, as shown in Figure 1.

Figure 1. Structure of the three idealized decks

The deck was projected in reinforced concrete, embedded in the pillars, and considering the superstructure in an open section. As for the slabs, solid slabs with a constant thickness of 25 cm were considered, with a free lateral edge, supported longitudinal end, and the other slab connections embedded. New Jersey-type reinforced concrete barriers and 8 cm of pavement were adopted.

2.2. Permanent loads determining methods

Three distinct methods were applied to calculate the permanent loads affecting the structure: the Plastic Hinge Method, the Influence Area Method, and the Grid Model. The last method was carried out with the assistance of a computer program implemented in FORTRAN 90.

The load due to the structure's self-weight was calculated by multiplying the specific weight of the structural elements by the value of the cross-sectional area of these elements, considering the contribution of the slabs and the resurfacing.

2.2.1. Plastic Hinge Method

Through the calculation of the failure lines, the permanent loads acting on each beam were determined. First, the influence areas of the load distribution on the slabs over each beam were determined, as shown in Figure 2. The blue area is absorbed by the external beams, the green area by the central beam, and the white area by the cross beams.

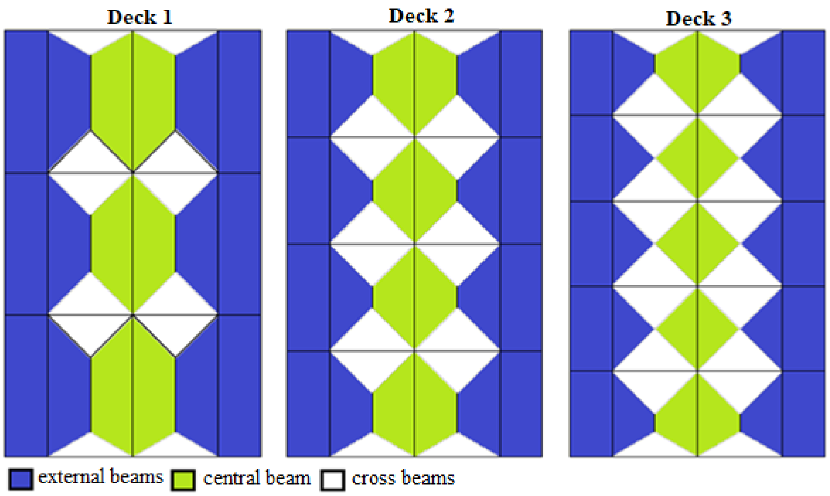

Figure 2. Failure lines and permanent load's areas distributed along the deck

The permanent loads acting on the longitudinal beams of the three idealized decks were calculated by summing the loads corresponding to the self-weight of the slab, paving, and beam, obtained with the assistance of ABNT NBR 6120 [8]. The load related to the reinforced concrete parapet was considered only on the edge beams. Table 1 shows the values of the permanent loads obtained for the three decks using the Plastic Hinge Method.

Table 1. Permanent load distributed along the longitudinal beams of the idealized decks, Plastic Hinge method

Deck	Section	External beams (kN/m)	Central beam (kN/m)
1	1 e 3	82.36	67.9
	2	80.04	63.26
2	1 e 4	79.48	62.13
	2 e 3	76.38	55.93
3	1 e 5	76.59	56.35
	2, 3 e 4	72.72	48.61

2.2.2. Influence Area Method

The Influence Area Method divides the slabs into strips parallel to the longitudinal beams, and each strip has an influence area corresponding to the region of the slab that contributes to the effort in the beam, as shown in Figure 3.

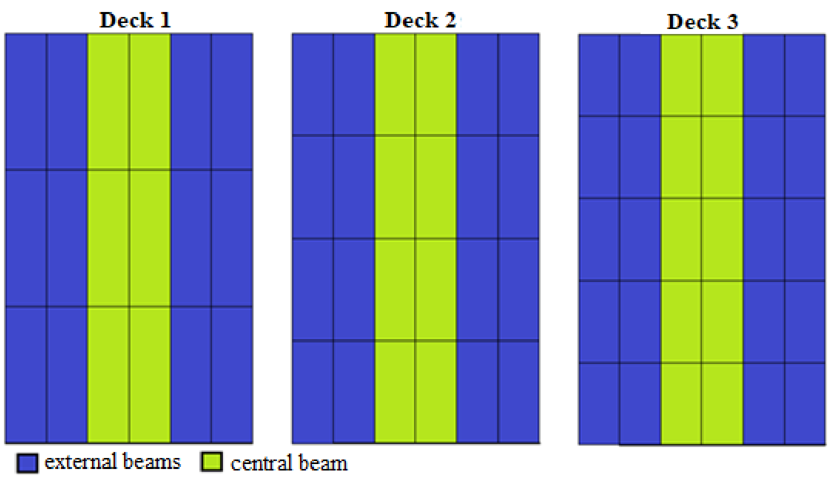

Figure 3. Influence area, external and central beams

Table 2 shows the permanent loads for each beam of the three idealized decks, noting that due to the geometry of the decks and the new influence areas found, there is no change in values by section as in the Plastic Hinge Method.

Table 2. Permanent load distributed along the longitudinal beams, Influence Area method

Table	External beams (kN/m)	Central beam (kN/m)
1	91.02	85.22
2	91.02	85.22
3	91.02	85.22

2.2.3. Grid Model

The structural analysis using the Grid Model was performed based on the Displacement Method (Stiffness Method), using results obtained from a computational code implemented in Fortran.

To validate the developed program, two grid examples were used, taken from the book Analysis of Reticulated Structures [9]. Due to the similarity between the results obtained through the computational program and the responses from the literature, it can be stated that the code is effective for the examples studied. Thus, it can be used as an efficient tool to evaluate the other grid structures in this study.

The analysis of permanent load through the Grid Model was carried out under two criteria. The first considered a very high moment of inertia value for the cross beams, a principle also used for the calculation of live loads through the Engesser-Courbon Method. The second adopted the actual torsional stiffness of the beams, defined by the relationship given in Equation 1.

$$\left(\frac{GJ}{EI}\right) \quad (1)$$

Where G is the transverse modulus of elasticity, J is the torsional constant, E is the longitudinal modulus of elasticity (tension/compression), and I is the longitudinal moment of inertia. Thus, by substituting the values used for the project, the following relationship was obtained $J=0,127I$.

2.2.4. Moving loads determining – Load train

The reference vehicle used to define the live loads on the structure was the one specified in ABNT NBR 7188 [10], class TB-450. The Engesser-Courbon Method was employed to determine the transverse load distribution and the longitudinal load train. To identify the points with the highest forces due to the positioning of the longitudinal load train, influence lines were drawn every 1 meter until the centerline of the beam, as the beams have similar sections. Finally, the force envelopes were calculated for each beam.

The calculation of the influence lines was carried out with the assistance of the FTOOL software. The maximum values and the areas under the influence line curves were determined using three different methods. The first consisted of obtaining an equation representing the curve using the Lagrange Method, performing manual calculations, and using the Geogebra software for visualization assistance. The second involved plotting the curve in Autocad and extracting the maximum points and areas under the curve from it. The third method involved obtaining the reference train using the FTOOL software. The results obtained from the FTOOL software were used to calibrate the other two methods.

2.2.5. Beams project

For the beam project, it was assumed that the slab and the beam form a T-shaped composite section. The structure was projected according to the NBR 6118 [7] standard. First, the calculation of the cooperating width of the T-section was performed. Next, the section project was carried out, obtaining the longitudinal steel area, and verifying the effective height. Then, the shear project was performed, the skin reinforcement was determined, and finally, the checks for cracking and immediate deflection and deferred deflection were conducted.

III. RESULTS E DISCUSSIONS

3.1. Comparison between the Analysis's Methods of Permanent Loads

The project load values obtained by the Plastic Hinge Method and the Influence Area Method are presented in Tables 3, 4, and 5 for decks 1, 2, and 3, respectively. In which 'Ext.' refers to the edge beams and 'Cent.' to the central beam. The units of the project loads for positive and negative bending moments are expressed in kN.m, while for shear force, the unit used is kN.

Table 3. Project Loads for Deck 1

		Plastic Hinge			Influence Area		
Beam	Section	Positive Bending Moment	Negative Bending Moment	Shear Force	Positive Bending Moment	Negative Bending Moment	Shear Force
Ext.	1	2353.24	-4189.66	2001.552	2410.22	-4322.1	2070.992
	2	2673.619	-3235.8	1906.433	2778.759	-3407.16	1998.693
	3	2353.24	-4189.66	2001.552	2410.22	-4322.1	2070.992
Cent.	1	1540.802	-2877.83	1358.769	1663.103	-3142.71	1497.789
	2	1681.418	-2346.4	1269.86	1891.838	-2689.26	1454.24
	3	1540.802	-2877.83	1358.769	1663.103	-3142.71	1497.789

Table 4. Project Loads for Deck 2

		Plastic Hinge			Influence Area		
Beam	Section	Positive Bending Moment	Negative Bending Moment	Shear Force	Positive Bending Moment	Negative Bending Moment	Shear Force
Ext.	1	1453.327	-2581.71	1649.159	1500.927	-2683.49	1719.439
	2	1684.661	-1951.1	1568.393	1757.461	-2074.86	1658.133
	3	1684.661	-1951.1	1568.393	1757.461	-2074.86	1658.133
	4	1453.327	-2581.71	1649.159	1500.927	-2683.49	1719.439
Cent.	1	906.577	-1670.35	1050.834	986.104	-1873.91	1191.394
	2	1010.541	-1339.66	979.853	1156.281	-1602.26	1159.473
	3	1010.541	-1339.66	979.853	1156.281	-1602.26	1159.473
	4	906.577	-1670.35	1050.834	986.104	-1873.91	1191.394

Table 5. Project Loads for Deck 3

		Plastic Hinge			Influence Area		
Beam	Section	Positive Bending Moment	Negative Bending Moment	Shear Force	Positive Bending Moment	Negative Bending Moment	Shear Force
Ext.	1	981.4952	-1766.29	1419.426	1019.295	-1847.21	1489.566
	2	1148.085	-1300.63	1339.934	1207.725	-1406.65	1430.797
	3	1178.836	-1293.81	1338.537	1232.036	-1406.65	1430.797
	4	1148.085	-1300.63	1339.934	1207.725	-1406.65	1430.797
	5	981.4952	-1766.29	1419.426	1019.295	-1847.21	1489.566
Cent.	1	589.7122	-1089.95	858.6802	665.3122	-1251.79	998.8202
	2	656.39	-846.528	789.0498	775.67	-1065.33	972.1838
	3	686.2534	-839.651	787.6638	792.7934	-1065.33	972.1838
	4	656.39	-846.528	789.0498	775.67	-1065.33	972.1838
	5	589.7122	-1089.95	858.6802	665.3122	-1251.79	998.8202

The difference in the final project loads between the Plastic Hinge Method and the Influence Area Method increased as the span of the longitudinal section decreased, and the structure became stiffer. This leads us to consider that the Influence Area Method is more effective for larger spans in relation to the cross-section.

The Plastic Hinge Method resulted in lower project loads compared to the Influence Area Method. The most significant difference in the final project load occurred in the central beam of the decks.

Therefore, considering the geometry of the deck, the boundary conditions used, and the fact that the Influence Area Method is an approximation based on the Plastic Hinge Theory, the Influence Area Method proved to be more effective for the end beams, as the results between the methods were closer in the analysis of these beams.

The permanent loads were also evaluated using the Grid Method, through the computational program implemented in FORTRAN language. To validate the developed code, two examples from the book Análise de estruturas reticuladas [9] were used. Table 6 compares the results for both examples, showing the three possible displacements at each node, obtained through the computational code and the results documented in the book [9].

Table 6. Displacements at the nodes for both examples

		Current work			Gere and Weaver		
Example	Nod	Disp. X	Disp.Y	Disp.Z	Disp. X	Disp.Y	Disp.Z
1	1	-0.007598538	0.00509489	-0.355075332	-0.0076	0.005095	-0.3551
	2 e 3	0.00	0.00	0.00	0.00	0.00	0.00
2	1	-0.00068351	-0.000287536	-0.07171067	-0.00068	-0.00029	-0.0702
	2	0.0005540423	0.0003528258	-0.120917063	0.000554	0.000353	-0.12092
	3 a 6	0.00	0.00	0.00	0.00	0.00	0.00

The results of the forces in the three directions for each element of the evaluated examples, obtained through the computational program, were also compared. Due to the accuracy of the results, it can be stated that the code is effective for the studied examples and, therefore, can be used to evaluate the other grid structures in this work.

The program generates results related to the forces at the ends of each element. Based on the end data, the internal forces corresponding to the bending moment and shear along the beam were manually calculated for the evaluation of the force envelopes for the three decks. As an example, the forces obtained through the computational program, related to the stringers of Deck 1, were obtained using the Grid Method assuming the infinite stiffness of the crossbeams. These results are presented in the first columns of Table 7, which compares the forces obtained by analyzing the beams of Deck 1, considering both real stiffness of the crossbeams (with J=0.127I) and the infinite stiffness assumption.

Table 7. Forces in the three directions at the end nodes of the elements of the longitudinal beams of Deck 1, with moments of inertia following the relation J=0.127I and infinite stiffness

	Infinite Stiffness of the Crossbeams						Actual Stiffness of the Crossbeams					
Elem.	Initial nod			Final nod			Initial nod			Final nod		
	x	y	z	x	y	z	x	y	z	x	y	z
1	0	-988.32	494.16	0	988.32	494.16	0	-988.32	494.16	0	988.32	494.16
2	0	-960.48	480.24	0	960.48	480.24	0	-960.48	480.24	0	960.48	480.24
3	0	-988.32	494.16	0	988.32	494.16	0	-988.32	494.16	0	988.32	494.16
4	0	-814.80	407.40	0	814.80	407.40	0	-814.80	407.40	0	814.80	407.40
5	0	-759.12	379.56	0	759.12	379.56	0	-759.12	379.56	0	759.12	379.56
6	0	-814.80	407.40	0	814.80	407.40	0	-814.80	407.40	0	814.80	407.40
7	0	-988.32	494.16	0	988.32	494.16	0	-988.32	494.16	0	988.32	494.16
8	0	-960.48	480.24	0	960.48	480.24	0	-960.48	480.24	0	960.48	480.24
9	0	-988.32	494.16	0	988.32	494.16	0	-988.32	494.16	0	988.32	494.16
10 a 17	0	0	0	0	0	0	0	0	0	0	0	0

Just like for Deck 1, Decks 2 and 3 also showed no change in the results when the analysis was performed considering the actual stiffness of the crossbeams. It is believed that this occurred because the structure was not loaded in the cross-sectional direction. In other words, for the analysis of the impact of torsional stiffness on the structure, the input data used were not sufficient, and new evaluations considering the crossbeams loaded as well are necessary. It is important to note that for all the analyses conducted in this study, the displacements were zero due to the boundary conditions adopted.

The project load values obtained through the Grid Model were compared with the responses determined with the aid of the FTOOL software, considering the stringer beam completely decoupled from the rest of the structure. Table 8 compares the final project loads obtained through the grid program and FTOOL for Deck 1. In this comparison, the units for the bending moment forces are expressed in kN·m, while for the shear force, the unit used is kN.

Table 8. Project loads obtained through FTOOL and Grid Model for Deck 1

		FTOOL			Grid Model		
Beam	Section	Positive Bending Moment	Negative Bending Moment	Shear Force	Positive Bending Moment	Negative Bending Moment	Shear Force
Ext.	1	2353.24	-4189.66	2001.552	2343.552	-4176.66	1998.276
	2	2673.619	-3235.8	1906.433	2686.555	-3261.73	1906.489
	3	2353.24	-4189.66	2001.552	2343.552	-4176.66	1998.276
Cent.	1	1540.802	-2877.83	1358.769	1521.272	-2851.79	1352.329
	2	1681.418	-2346.4	1269.86	1708.158	-2398.34	1272.42
	3	1540.802	-2877.83	1358.769	1521.272	-2851.79	1352.329

The project loads of the beams obtained through the Grid Model and FTOOL showed very close results. Although there are slight differences, especially in the central beams, the maximum percentage difference between the final tr loads was only 6%. It is also notable that the differences between the methods are much smaller concerning the shear forces. The convergence of the results suggests that both the isolated beam analysis by the FTOOL software and the grid-structured analysis using the computational program are effective methods, for the conditions studied, where only the stringers of the structure are loaded.

3.2. Moving loads analysis

The longitudinal train-type generated due to the action of moving loads was determined using the Engesser-Courbon Method. The influence lines (L.I.) were used to position the train-type in a way that generates maximum forces in the structure. The maximum value and the areas under the influence line curves were determined using three distinct methods.

Figure 4 (a) shows the bending moment influence line curve for Section 1 of the analyzed beam, obtained with the aid of the FTOOL software and used as reference to calibrate the other methods. Figure 4 (b) demonstrates the curve drawn in AutoCAD, using coordinates at one-meter intervals that give rise to the original curve. In Figures 4 (a) and (b), the negative bending moment is represented upwards, and the positive moment is drawn downwards from the ordinate axis.

The curves shown in Figures 4 (c), (d), (e), (f), and (g) are the curves obtained through Lagrange polynomial regression, with polynomial degrees of 12, 11, 10, 9, and 8, respectively, as the curve is formed by 13 points. These curves are drawn following the Cartesian coordinates; therefore, the negative values are expressed below the x-axis, and the positive values are above it.

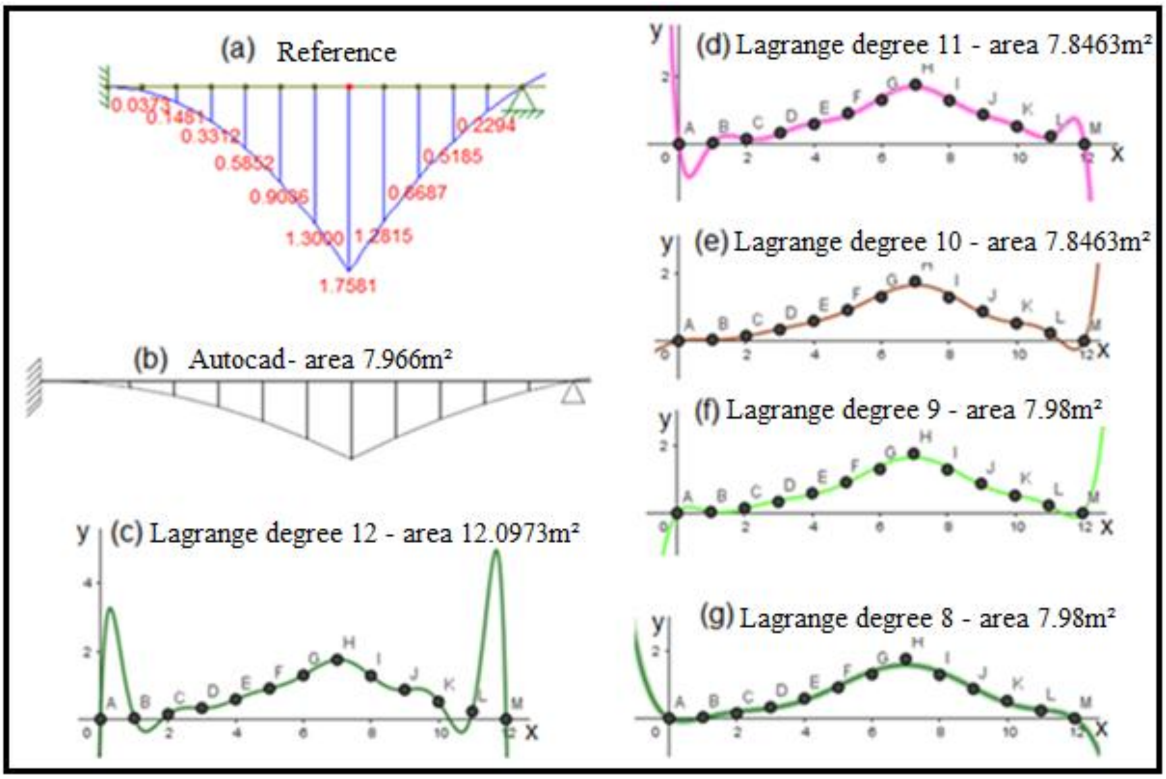

Figure 4. L.I. curves obtained through Lagrange method and AutoCAD for the section 1 of the external beam of the Deck 1

It is known that the higher the degree of the polynomial, the better the approximation of the points to the original curve. However, due to the large spacing between the coordinates, it was necessary to evaluate the curves generated by the Lagrange method and their respective areas to determine which polynomial degree would best represent the influence line curves. The highest degree polynomial was satisfactory for curves close to second-degree equations, but for the other curves, the best area was determined using polynomials of lower degrees.

Table 9 compares the absolute values of the moving loads obtained by the Lagrange Method, AutoCAD area, and FTOOL program for the analysis of Sections 1 and 2 of Deck 1, as Section 3 is similar to Section 1. Due to the closeness of the values, Decks 2 and 3 were calculated only using the FTOOL software.

Table 9. Moving loads values of the external beam of the deck1 obtained through the three methods

	Lagrange		**Autocad**		**FTOOL**	
Section	Maximum (kN.m)	Minimum (kN.m)	Maximum (kN.m)	Minimum (kN.m)	Maximum (kN.m)	Minimum (kN.m)
1	804.8439	255.2212	813.3413	252.6104	808.7	255.4
2	959.7605	259.7092	956.6898	254.0714	952.8	252.3

3.3. Steel areas of the main beams

Table 10 presents the longitudinal steel areas, given in cm², for the positive bending moment forces (A.A.M.P) and negative bending moment forces (A.A.M.N), as well as the steel areas for the transverse reinforcement (T.A), given in cm²/cm, corresponding to the shear forces for the beams of Deck 1. Table 10 also shows the comparison of the steel areas between the Plastic Hinge Method and the Influence Area Method for Deck 1.

Table 10. Steel áreas for the beams of the Deck 1 through Plastic Hinge and Influence Areas methods

		Plastic Hinge Method			Influence Area Method		
Beam	Section	A.A.M.P	A.A.M.N	T.A	A.A.M.P	A.A.M.N	T.A
Ext.	1	31.21 cm²	65.21 cm²	23.16 cm²/cm	32.35 cm²	67.85 cm²	24.52 cm²/cm
	2	35.95 cm²	47.66 cm²	22.1 cm²/cm	37.39 cm²	50.65 cm²	23.46 cm²/cm
	3	31.21 cm²	65.21 cm²	23.16 cm²/cm	32.35 cm²	67.85 cm²	24.52 cm²/cm
Cent.	1	20.45 cm²	41.62 cm²	13.94 cm²/cm	21.96 cm²	46.06 cm²	15.85 cm²/cm
	2	22.20 cm²	33.08 cm²	12.54 cm²/cm	25.02 cm²	38.54 cm²	15.22 cm²/cm
	3	20.45 cm²	41.62 cm²	13.94 cm²/cm	21.96 cm²	46.06 cm²	15.85 cm²/cm
A.A (longitudinal reinforcement steel areas) /T.A (steel areas for the transverse reinforcement)							

For the three projected decks, the steel area for the skin reinforcement was 5 cm², as the ABNT NBR 6118 [7] standard establishes that the steel area for skin reinforcement should not exceed 5 cm²/m per face and must be at least 0.10% of the concrete area of the beam web. Since all the beams have the same concrete area in the web, the adopted steel area for the skin reinforcement was also the same. The bars used were 6.3" spaced according to the smallest value between 20 cm, one-third of the beam's effective height, or 15 times the diameter of the adopted steel. As the effective height varied by beam section, this analysis also varied for each section.

In all the analyses performed, the verification of cracking, immediate deflection, and time-dependent deflection was satisfactory. In the case of Deck 1, which has a larger span, it was necessary to apply the camber as specified in the standard for the analysis of time-dependent deflection.

In order to compare the difference in the total steel area in each beam as the number of crossbeams varies, the average steel areas for each force in the beams were calculated. This average is presented in the graphs of Figures 5 and 6. Figures 5A and 6A represent the variation in the steel area of the outer beams for the Plastic Hinge Method and the Influence Area Method, respectively. Meanwhile, Figures 5B and 6B illustrate the same variation for the central beams of the studied decks.

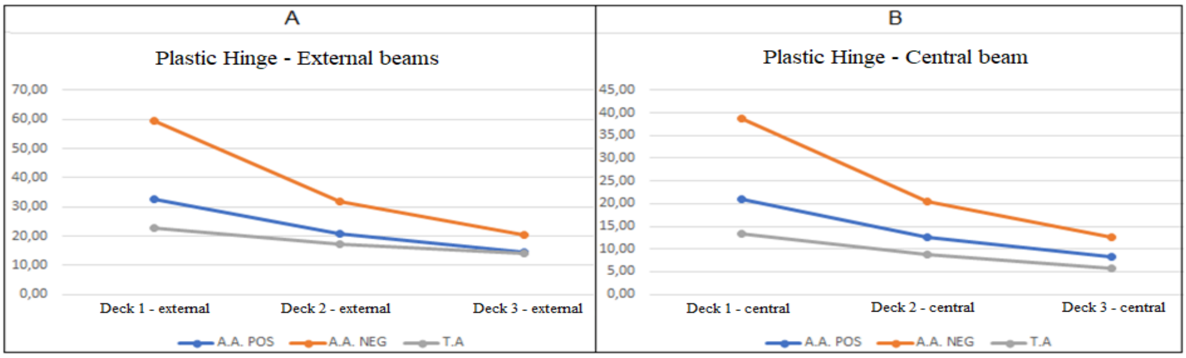

Figure 5. Steel areas variation according to the alteration of the decks for the Plastic Hinge Method

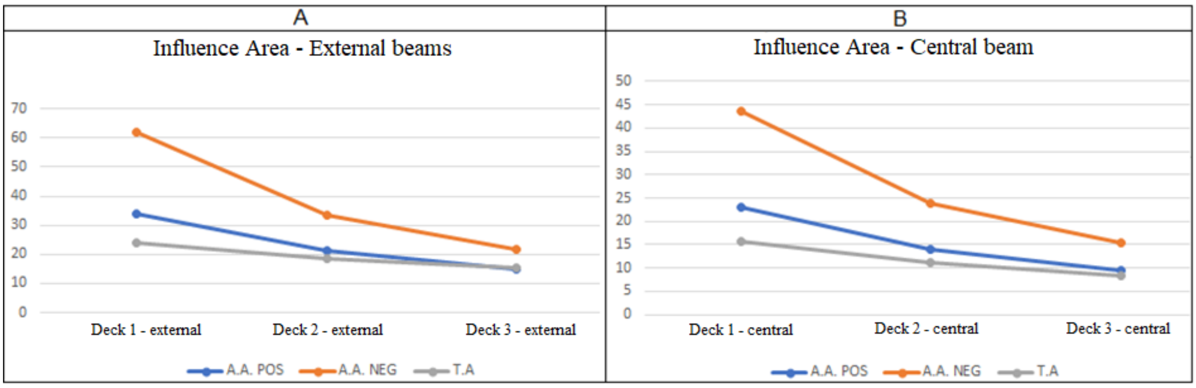

Figure 6. Steel areas variation according to the alteration of the decks for the Influence Area Method

The variation in steel areas for the two methods converged, with the steel areas for both longitudinal and transverse reinforcements decreasing as the number of crossbeams increased. Based on the slope of the graphs, it can be inferred that the greatest variation in steel area occurs when the number of crossbeams is changed from 4 to 5, that is, when moving from Deck 1 to Deck 2. Table 11 presents this percentage variation in steel areas between Decks 1 and 2 and between Decks 2 and 3 for the two evaluated methods, where D.P.M.P is the percentage difference in steel areas due to positive bending moments, D.P.M.N is the percentage difference in steel areas due to negative bending moments, and D.P.C is the percentage difference in steel areas due to shear forces.

Table 11. Percentage variation of steel areas among the decks 1 and 2, and 2 and 3.

		Plastic Hinge			Influence Area		
Beam	Decks	D.P.M.P	D.P.M.N	D.P.C	D.P.M.P	D.P.M.N	D.P.C
Ext.	1 e 2	36.89	46.23	23.71	37.00	45.80	23.51
	2 e 3	30.92	36.29	19.98	29.93	35.82	17.66
Cent.	1 e 2	40.36	47.24	34.98	38.93	45.25	29.22
	2 e 3	34.05	38.33	33.40	31.49	35.94	25.09
D.P. (Percentage difference of the longitudinal reinforcement's steel area) / D.P.T (Percentage difference of the transverse reinforcement's steel area)							

The closer the percentage difference in the steel area between the decks, the less economically throughble it is to increase the number of crossbeams. For all the decks, it is observed that the steel area least affected by the variation in the number of crossbeams is the transverse reinforcement. In contrast, the most affected area is the one projected for negative moments. Generally, although it is the steel area most impacted, the negative moment forces occur only at the supports. Most of the beams in the analysis are projected to withstand positive bending moments. Therefore, special attention must be given to these areas, which show an average percentage variation between 38% and 32% when changing from Decks 1 to 2 and from Decks 2 to 3, respectively.

IV. CONCLUSIONS

In general, the Engesser-Courbon Method proves to be excellent for an initial project of the load distribution, provided that there is a sufficiently rigid cross-sectional system, as it is quick to apply and easy to obtain results.

The Influence Area Method is simple to implement and proved to be more effective in decks with high transverse stiffness. This suggests that its use in combination with the Engesser-Courbon Method could provide a quick analysis of the structure, although it would be more conservative and require a greater evaluation of the transverse stiffness of the structure before application.

In turn, the Plastic Hinge Method proved to be an effective approach in structural analysis. However, further studies are needed to assess its impact on the steel areas in conjunction with the crossbeams.

Finally, the number of crossbeams was the factor that most affected the difference in steel areas between the decks. The fewer the crossbeams, the greater the steel area required to resist the forces in the stringers. With each additional crossbeam, there was an average reduction of 25%, 35%, and 40% in the steel areas for the positive longitudinal reinforcement, negative longitudinal reinforcement, and transverse reinforcement, respectively.

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