BIFURCATION BUCKLING OF PRESSURIZED CONICAL VESSELS

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ABSTRACT

Metallic thin shell structures are used in different branches of industry, and the components of these structures such as dished ends show different modes of failure depending upon the geometrical and loading conditions. These are mainly gross plastic deformation under static load, loss of stability (buckling), fatigue crack initiation at highly stressed locations under cyclic loading (especially in the low cycle regime), progressive plastic deformation (ratcheting) and creep at high temperatures. In this paper, failure modes of a conical pressure vessel subjected to internal pressure has been investigated. Also, to study the effect of vessel geometry, a set of conical cylindrical vessels with the cap-cone apex half angles of 20 to 85 degrees, internal radius of 500 to 1000 mm and thickness of 1 to 10 mm has been selected. The failure modes of these vessels which include gross plastic deformation and bifurcation buckling have been taken into account. In this work, a new plastic criterion has been established which is based on the plastic work dissipation in the vessel by increasing the internal pressure. This plastic criterion can be used for structures subjected to single or a combination of loading condition. The calculated plastic limit loads, which have been obtained using the plastic criterion, are determined purely by the inelastic response of the vessel, and they are not altered by initial elastic behavior. In addition, a design graph for designing pressure vessels with conical heads subjected to internal pressure has been presented via comprehensive parametric study. The results show that when the ratio of the internal pressure to the limit pressure (Load Factor) approaches 0.5, the material yielding initiates and with further increase in the Load Factor, the plastic regions develop. Also, by increasing the ratio of cylinder radius to wall thickness (R/t), plastic buckling failure becomes more dominant.

KEYWORDS: *Plastic work criterion, Bifurcation buckling, Gross plastic deformation, Plastic load*

I. INTRODUCTION

Cone-cylinder intersections are found in many shell structures. Examples include steel silos and tanks with a conical roof, conical water tanks with a cylindrical shell support, large tubular members, transition cones between two cylinders or pipes with different diameters and pressure vessels with a conical end closures. Internal pressurization is often an important loading condition for these intersections. For the intersection of a large end of a cone and a cylinder (Fig. 1), internal pressure causes large circumferential compressive stresses in the intersection by either axi-symmetric collapse involving excessive inward gross plastic deformations or non-symmetric bifurcation buckling featuring periodical waves around the circumference. In the majority of the practical cases, gross plastic deformation under static loading is the fundamental failure mode that has been widely discussed in international standards such as PD5500 unfired fusion welded pressure vessels [1], ASME boiler and pressure vessel code, section III and IV [2] and EN13445-3: unfired pressure vessels [3]. Teng [4] has conducted a comprehensive study on the gross plastic deformation of the pressure vessels with conical caps under internal pressure and Mackenzie et al. [5,6,7] have proposed a plastic work criterion for evaluating gross plastic deformations in metallic shell structures with tori-spherical caps. In addition, a buckling failure of a cone-cylinder intersection was reported and analyzed in detail by Teng and Zhao [8]. Also, Blachut and K. Magnucki [9] have studied the stress concentrations at the junction of cylinder-ellipsoidal end closures in detail.

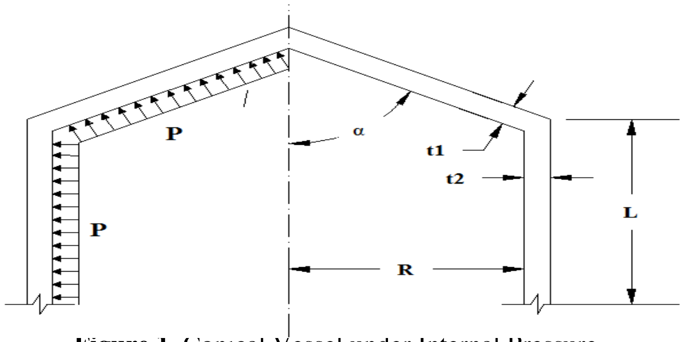


Figure 1. Conical Vessel under Internal Pressure

Their results established efficient choices for wall thicknesses in the vessel. Stability issues are also reviewed in their paper. In particular, attention has been paid to the stability of cylinder under external pressure and to the stability of end closures. Apart from buckling and plastic loads, the ultimate load carrying capacity, for internally pressurized heads is also examined. Results of a parametric study into buckling and first ply failure of bowed out cylinders, subjected to static external pressure, are also discussed by Blachut [10]. He has shown that substantial pressure increase can be achieved through barrelling. Since the experimental tests to study the mechanical behaviour and buckling and bifurcation of vessels are very expensive, there is a general trend to use numerical methods. The results of numerical solutions and experimental tests are compared in the buckling of multi-segment pressure hulls subjected to uniform hydrostatic pressure [11]. It has been shown that numerical collapse pressures agreed well with those obtained based on experiments. Numerical and experimental studies have also been carried out in buckling of steel ellipsoidal domes loaded by static external pressure [12]. The adverse effects of the variations in shape and wall thickness are discussed, and finite element predictions are made for geometrically imperfect domes. It has been shown that the correlation between the two sets of results is good. Numerically and experimentally obtained results are related to the current design codes: ASME Boiler and Pressure Vessel Code, Sec. VIII, Division 2 (described as ASME VIII), PD5500, and ECCS recommendations. The buckling of short, and relatively thick, mild steel conical shells subjected to the combined action of external pressure and axial compression have also been studied using numerical analysis and experimental tests [13]. Results suggest that the concept of equivalent cylinders is not applicable to short master-cones. Using the test results, combined stability plots have been derived for the master-cone and equivalent cylinders which provide not only the failure envelopes but also the yield envelopes and spread of plastic zones. Two issues of: (1) buckling performance of caps, with shallowness parameter, and (2) the effect of boundary conditions on the buckling strength have also been addresses in a research work by Blachut [14]. A number of finite-element studies on the collapse and buckling behavior of uniform thickness cone-cylinder intersections under internal pressure have also been carried out by Teng [15] who developed simple design approximations.

In this paper we have investigated the gross plastic deformation of the conical shells using plastic work criterion and also the buckling behavior of the conical vessels under internal pressure has been studied. Furthermore, the initiation and development of the buckling wave by increasing load has been studied and the summary of a comprehensive numerical study has been provided in the form of a designing graph. To do so, the model of the vessel and its geometry has been identified. Then, the method to obtain the plastic work criterion has been discussed. A 3D Finite Element model has been developed and analysis has been carried out. Using these solutions, the plastic work criterion was applied in order to investigate the plastic limit load. Also this model has been used to investigate the development of the bifurcation buckling wave in the structure. Finally, the effect of geometry on the failure modes of cylindrical vessel with conical cap has been studied using Finite Element solutions. The summary of a comprehensive parametric study has been provided.

II. MODEL DESCRIPTION

According to Fig. 1, a cylindrical vessel with conical cap under internal pressure has been selected. The geometrical properties of the model have been shown in Table 1.

Table 1. Geometrical Properties of the Model

Symbol	Definition	Quantity
α	Cone apex half angle [degrees]	20,40,60, 80 and 85
t_1	Thickness of the conical head [mm]	1,2,5 and 10
t_2	Thickness of the cylinder [mm]	1,2,5 and 10
R	Radius of the cylinder [mm]	500 and 1000
L	Length of the cylinder [mm]	488

III. THE PLASTIC WORK CRITERION

The increment of work done, dW, per unit volume on a infinite small element of material during a small strain increment, is the total of increment in elastic work *(dW^e)* and the increment in plastic work *(dW^p)* [5]:

$$dW = dW^e + dW^p \tag{1}$$

This may be written in terms of stress and elastic and plastic strain components as:

$$dW = \sigma_{ij}\left(d\varepsilon_{ij}^e + d\varepsilon_{ij}^p\right) \tag{2}$$

where $d\varepsilon_{ij}^e$ and $d\varepsilon_{ij}^p$ are elastic and plastic strain increments respectively, and σ_{ij} are the stress components. The nature of the elastic and plastic works are obtained by integrating through the volume for the strain loading path. The elastic work (equivalent to the elastic strain energy of the structure) is therefore given by:

$$W^e = \int \sigma_{ij} d\varepsilon_{ij}^e \tag{3}$$

The plastic work or the energy dissipated in the structure is given by:

$$W^p = \int \sigma_{ij} d\varepsilon_{ij}^p \tag{4}$$

The elastic work can be evaluated approximately from the Von Mises equivalent stress, σ_e, and equivalent elastic strain, ε_e by integrating through the volume of the model.

$$W^e = \int \sigma_e d\varepsilon_e \tag{5}$$

where:

$$\sigma_e = \frac{1}{\sqrt{2}}\left\{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 + 6\tau_{yz}^2 + 6\tau_{zx}^2\right\}^{1/2} \tag{6}$$

$$d\varepsilon_e = \frac{1}{\sqrt{2}}\left\{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right\}^{1/2} \tag{7}$$

Assuming the Von Mises yield criterion, the plastic work can be expressed in terms of equivalent stress and equivalent plastic strain as:

$$W^p = \int \sigma_e d\varepsilon_p \tag{8}$$

where:

$$d\varepsilon_p = \frac{\sqrt{2}}{3}\left\{\left(d\varepsilon_x^p - d\varepsilon_y^p\right)^2 + \left(d\varepsilon_y^p - d\varepsilon_z^p\right)^2 + \left(d\varepsilon_z^p - d\varepsilon_x^p\right)^2 + 6d\varepsilon_{xy}^2 + 6d\varepsilon_{yz}^2 + 6d\varepsilon_{zx}^2\right\}^{1/2} \tag{9}$$

A structure made from a ductile material may be subject to several loads. Taken together, these constitute a set of loads, $\{P_1, P_2, P_3, \ldots\}$ or **P**. Here it is assumed that the loads are applied to the structure in a proportional manner. During loading, the applied load **P** ranges from zero load to a maximum load set $\mathbf{P}_{\max}$. The value of the applied load set **P** at any point in the analysis characterized by *Load Factor*, λ, such that $P = \lambda P_{\max}$, where $0 \le \lambda \le 1$.

IV. FINITE ELEMENT IMPLEMENTATION

A Finite Element Analysis has been performed using the ANSYS program [16]. The 3D model has been developed using ANSYS 8-noded structural solid elements. The mesh of the model consisted of 23400 elements having 2 elements through thickness and is refined at the region next to the intersection, because, this region is likely to undergo the buckling failure mechanism. In order to model the evolution of the buckling deformation, the model was analyzed with 3D ANSYS 8-noded shell93 elements. The mesh of the 3D model consists of 12312 elements. For the GPD analysis, displacements of the nodes in the far part of the cylinder are restricted in all the directions except radial direction. The end boundary conditions of the cylindrical shell in the buckling analysis were defined to represent continuity in the shell (free for radial displacement and restrained against meridian rotation and axial and circumferential displacement). The length of the cylinder (*L*) is assumed to be 4ζ (where ζ is the linear elastic meridian half-wavelength and it is equal to $2.44\sqrt{Rt}$) to ensure that the boundary effects at the far end of the cylinder would not affect the behavior near the cone-to-cylinder joint [7]. In addition, a bilinear hardening material model was used in the analysis with material properties: elastic modulus $E = 198.5kN/mm^2$, yield stress $\sigma_y = 265N/mm^2$, Poisson's ratio $\nu = 0.294$ and plastic modulus $E_p=0.014E$.

V. RESULTS

5.1. Gross plastic deformation analysis

The plastic work criterion was applied in order to investigate the plastic limit load. Doerich and Rotter [17] have described two methods, based on modifications of the Southwell plot, of obtaining very accurate evaluations of the plastic limit load, irrespective of whether a fairly complete plastic strain field has developed or not. These two methods allow plastic collapse limit loads to be reported with great precision. Using the methods developed by Rotter, the plastic load limit and maximum applied pressure for each vessel has been obtained. In the solution method, the variation of the Load Factor, λ, with Plastic Work has been obtained. When the Load Factor approaches 0.5, the material yielding initiates and with further increase in the Load Factor, the plastic regions develop. When the structural response becomes steady (Fig. 2), the unlimited plastic flow establishes in the vessel. Based on the results presented in Fig. 2, the plastic Load Factor associated to the plastic work criterion is 0.71.

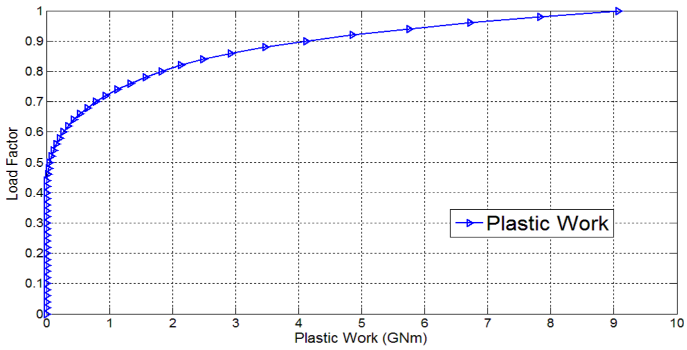


Figure 2. Variation of the Plastic Work with Load Factor for the vessel

5.2. Bifurcation buckling analysis

In order to investigate the development of the bifurcation buckling wave in the structure, according to Fig. 3, several consecutive nodes have been chosen in the intersection region of the cylindrical body and conical head.

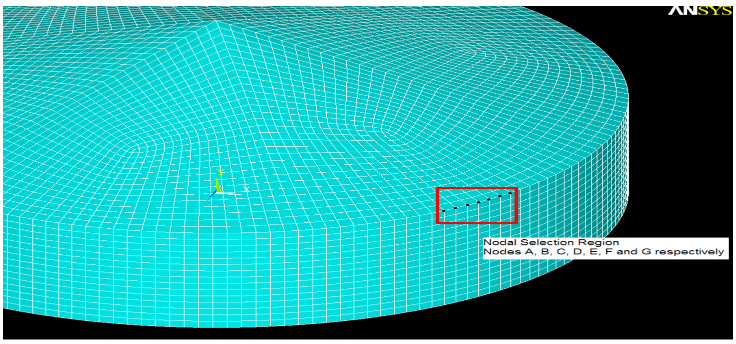
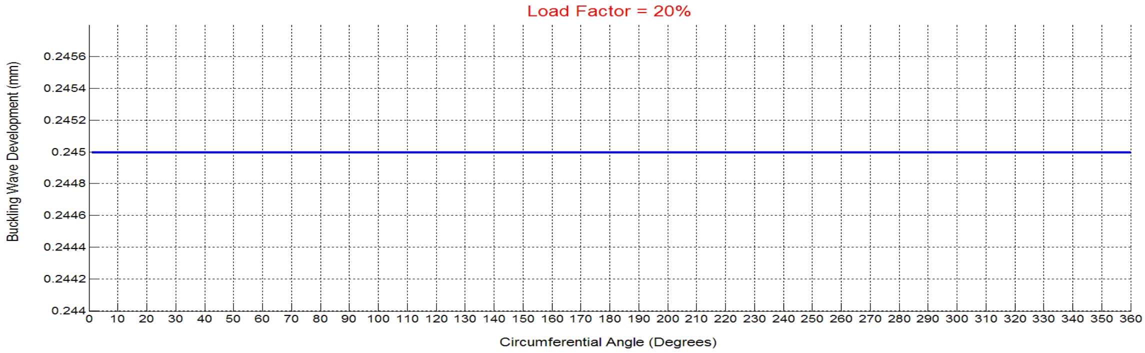
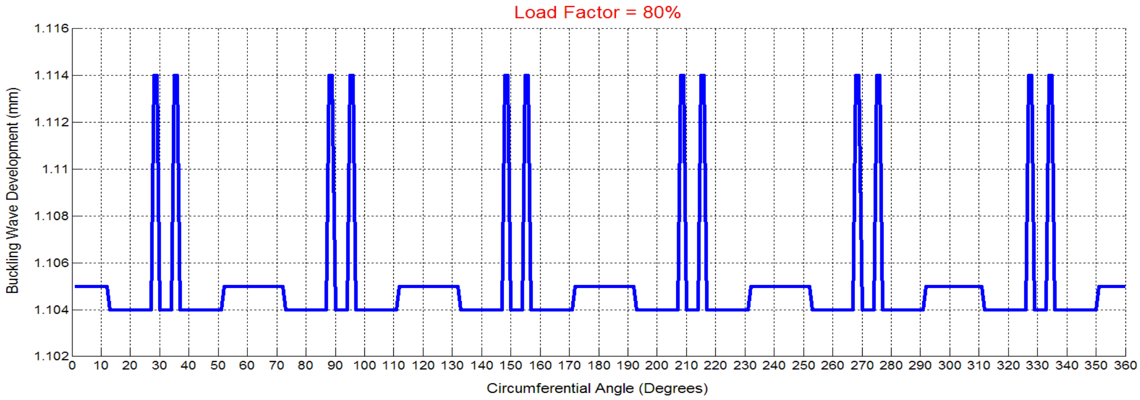


Figure 3. The selected nodes along the intersection region of the vessel

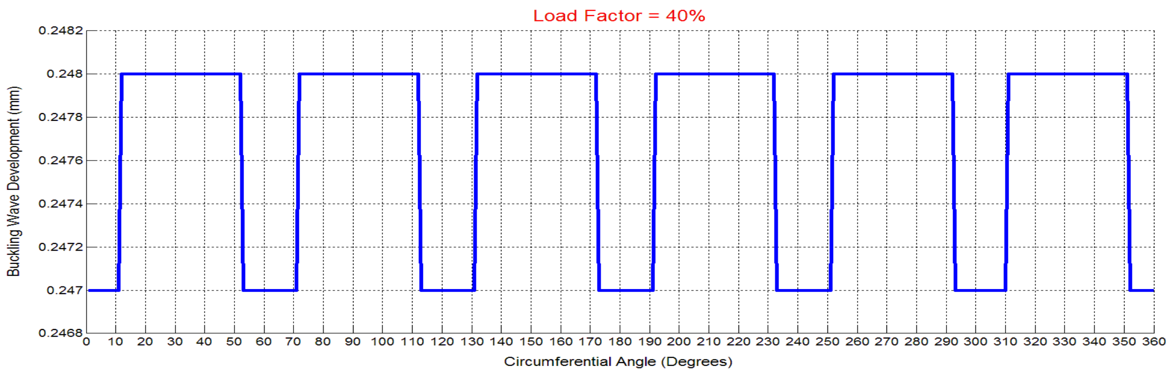
In Fig. 4, the buckling wave's status according to the applied load is shown and the formation and fully development of the bifurcation buckling wave in the intersection region has been shown in Fig. 5.



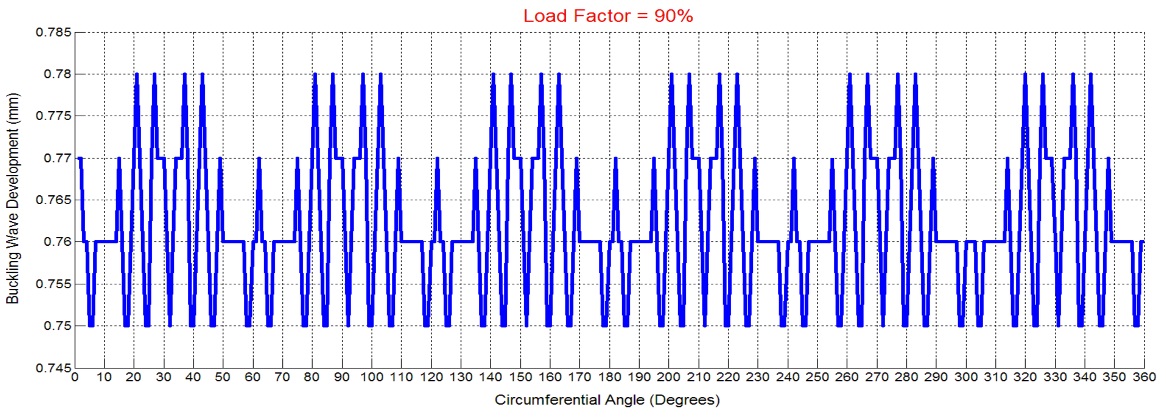
a: Load factor = 20%



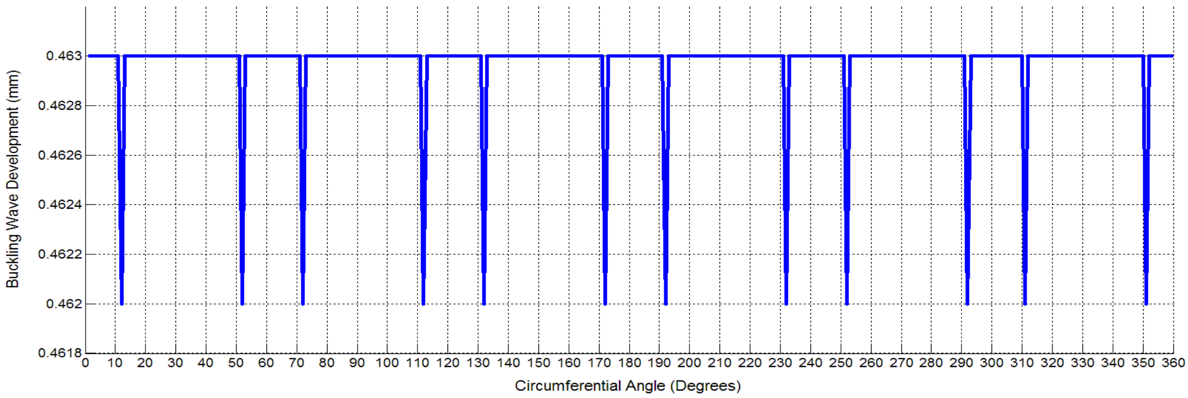
b: Load factor = 80%



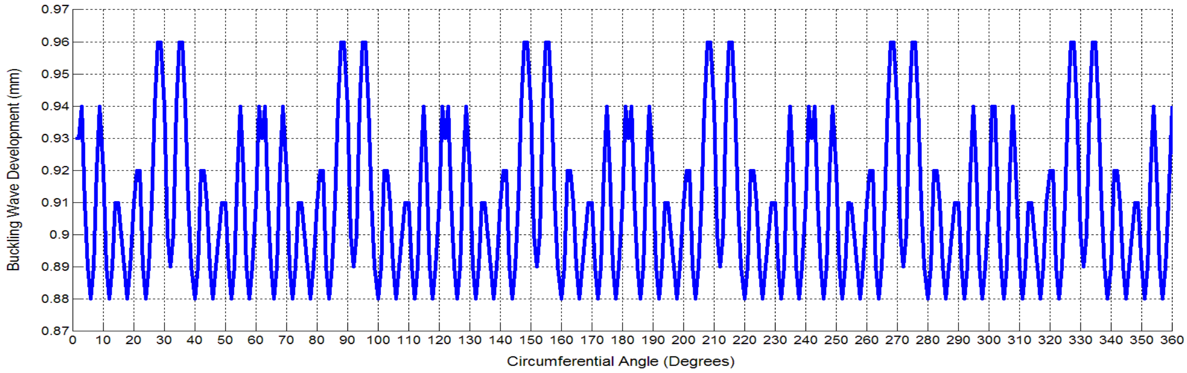
c: Load factor = 40%



d: Load factor = 90%



e: Load factor = 60%



f: Load factor = 100%

Figure 4. Status of buckling waves in the intersection region in different load factors

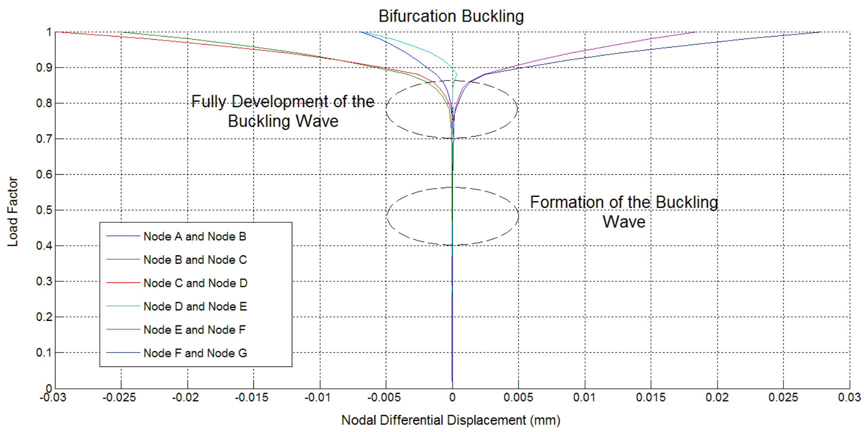


Figure 5. Bifurcation buckling formation

Abdi et al. [18] have investigated the influence of geometrical parameters such as thickness, knuckle radius, and the ratio of minor axis to the major axis of dome ends, on the weight and the critical buckling pressure of hemispherical, ellipsoidal, and tori-spherical dome ends. Senalp [19] has also investigated the effects of perturbation forces to buckling in pressure vessel heads. The pressure vessel heads in concern were confined to tori-spherical geometry with thin walls. In this paper, the effect of geometry on the failure modes of cylindrical vessel with conical cap has been studied using Finite Element solutions. The summary of a comprehensive parametric study has been provided in Fig. 6. According to Fig. 6, the effect of geometrical parameters of the vessel on the behavior of gross plastic deformation and buckling modes has been shown. It can be seen that by increasing the ratio of cylinder radius to wall thickness (*R/t*), plastic buckling failure becomes more dominant. For example, for *R/t=200*, plastic buckling occurs only in cone apex half angle of 70 t0 75 degrees. However, for *R/t=700*, plastic buckling occurs in cone apex half angle of 30 t0 80 degrees. This graph is applicable as a design basis for designing pressure vessels with conical heads subjected to internal pressure. This figure shows different failure mechanisms of the vessel as the geometrical parameters such as cone apex half angle and cylinder radius and thickness ratio.

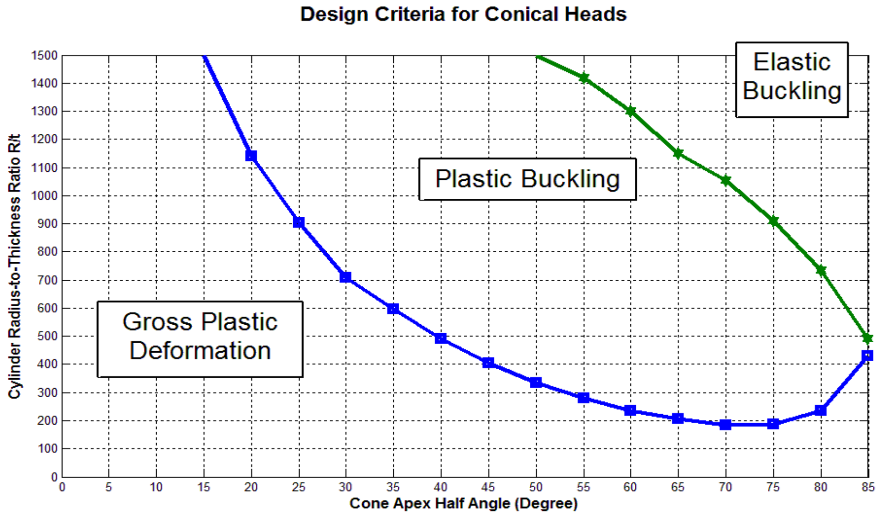


Figure 6. Design criterion for vessels with conical head

VI. Discussion and Conclusion

In this paper, the plastic work criterion was applied in order to investigate the gross plastic deformation of a cylindrical vessel with conical cap subjected to internal pressure. The plastic work criterion is utilized to determine the plastic limit loads of the pressure vessels. This criterion has some unique advantages in comparison with other plastic criteria in evaluating plastic limit loads of the vessels. Unlike other plastic criteria, this approach does not need selecting any deformation parameter. In addition, due to the nature of the plastic work criterion that predicts the plastic limit loads of the vessels purely based on the plastic response of the structures; the proposed criterion is easily applicable for multiple loading conditions.

Due to the internal pressure, the compressive hoop stresses arise in the intersection region and these stresses may lead to the buckling of the vessel. In order to evaluate the dominant failure mechanism of the vessel, the buckling analysis has been carried out using the Finite Element Based software ANSYS. According to Teng [7], when the structural response of the similar adjacent nodes in the intersection part of the vessel starts to diverge from each other, the buckling mode establishes in the vessel and the corresponding pressure is regarded as the buckling pressure. For the model with the geometrical and the material properties selected in this study, the gross plastic deformation failure mechanism occurred in the intersection region of the vessel prior to the formation of the bifurcation buckling mode. The results show that when the ratio of the internal pressure to the limit pressure (Load Factor) approaches 0.5, the material yielding initiates and with further increase in the Load Factor, the plastic regions develop.

Also, a comprehensive parametric study has been conducted in order to investigate the effect of different geometrical parameters on the behavior of gross plastic deformation and bifurcation buckling mode. Based on the results, a design curve has been obtained which can be used to predict the pattern of the failure mechanism of the vessel. This curve can be used as a design criterion for designing conical vessels under internal pressure. As it can be seen in this graph (see Fig. 6) for example, for a vessel with cylinder radius to thickness ratio of 500, by increasing the cone apex half angle beyond 40 degrees, the failure mode changes from gross plastic deformation to plastic buckling. By increasing this angle further (beyond 85 degrees) the failure mechanism changes to elastic buckling. Also, it can be seen that by increasing the ratio of cylinder radius to wall thickness (R/t), plastic buckling failure becomes more dominant. For example, for $R/t=200$, plastic buckling occurs only in cone apex half angle of 70 t0 75 degrees. However, for $R/t=700$, plastic buckling occurs in cone apex half angle of 30 to 80 degrees.

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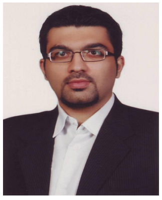

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