CHEMICAL EFFECTS ON ECOLOGY IN MESOSPHERIC DYNAMICS AND APPLICATIONS

Vivekanand Yadav and R. S. Yadav
Department of Electronics and communication Engineering
J K Institute for Applied Physics and Technology
University of Allahabad, Allahabad, India

ABSTRACT

The joint interactions of hydrodynamics, radiative transfer and photochemistry destabilize a hydrodynamic system that would otherwise stable. This observation is of important interest in itself. In this paper, the stability of baroclinic, axially symmetric vortex on an f-plane to axially symmetric disturbances is studied. It is found that with photochemistry and radiative transfer disturbances are unstable regardless of the value of the Richardson Number. The growth rates under conditions relevant to the mesospheric dynamics and applications are, however, very small.

KEYWORDS: *Stability, Photochemical and Radiative process, disturbance, axially symmetric.*

I. INTRODUCTION

In this paper we deal with the stability of a zonal vortex to an axially symmetric disturbance. As per Eady [1] and Kuo [2], in case of adiabatic motion the condition for stability is

$$\frac{1}{f}(f-\frac{\partial u}{\partial y})\frac{\frac{g}{T}(\frac{\partial T}{\partial z}+\frac{g}{c_p})}{\frac{\partial y^2}{\partial x}} > 1 \tag{1.1}$$

where $\frac{\frac{g}{T}(\frac{\partial T}{\partial z}+\frac{g}{c_p})}{\frac{\partial y^2}{\partial x}}$ is the Richardson number and f=2Ω where Ω is the vertical component of the earth's rotation rate. Condition (1.1) indicates that the earth's atmosphere is generally stable .It therefore appears that symmetric meridional motions, where they exist, must be drawn by external heat or momentum sources. Several Researchers (like Eliassen [3], Leovy [4] and Kuo [2], Steven [5], David.et.al [6] and Richard [12]) have studied such driven systems. The sources in these studies are usually taken to be fixed and unaffected by the circulations they drive, which results from the need for advections to maintain a steady state in the presence of these fixed sources and sinks.

In the present study, heat sources, resulting from perturbations in ozone and temperature, are not fixed, but are homogeneous functions of the perturbation velocity. We assume density variations to be negligible except where they are associated with gravity. Our equations are linear in the perturbation and coefficient function of the basic fields. We are restricted to disturbances whose vertical scale is small compared to the scale –heights for the various coefficients. The accuracy of the results of this paper is obtained by the help of MATLAB Simulation setup.

The paper has been devided into sections: stability with respect to axially symmetric disturbances - adiabatic case, Stability with respect to axially disturbance with photochemistry and radiative transfer, Influences of Vertical inhomogeneities, results and discussions, conclusion and future work.

II. STABILITY WITH RESPECT TO AXIALLY SYMMETRIC DISTURBANCES – ADIABATIC CASE

Following Spiegel and Veronis [10], in this paper, density variations are considered negligible when they are associated with gravity. Consider a basic field of the form

$$\bar{u}=\bar{u}_0+\bar{u}_{01}+\bar{u}_{02}z \tag{2.1}$$
$$\bar{T}=\bar{T}_0+\bar{T}_{01}y+\bar{T}_{02}z \tag{2.2}$$

Let our system be rotating at rate Ω=1/2f, with axis of rotation parallel to z-axis. We keep Ω constant, and following Phillips [11] f-plane approximation, introduce no curvature effects. Our disturbance equations are

$$\frac{\partial u}{\partial t}-(f-u_{01})v+u_{02}z=0 \tag{2.3}$$
$$\frac{\partial v}{\partial t}+fu=-\frac{1}{p_0}\frac{\partial P}{\partial y} \tag{2.4}$$
$$\frac{\partial w}{\partial t}=\frac{g}{\bar{T}_0}\theta--\frac{1}{p_0}\frac{\partial P}{\partial z} \tag{2.5}$$
$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.6}$$
$$\frac{\partial \theta}{\partial t}+\bar{T}_{01}v+\left(\bar{T}_{02}+\frac{g}{c_p}\right)w=0 \tag{2.7}$$

where θ, v, w and p are the disturbance temperature, zonal velocity, vertical velocity and pressure field and $\frac{\partial}{\partial x}=0\ by$ assuption.

By eqs. (2.4) and (2.5), we have

$$\frac{\partial^2 v}{\partial t\,\partial z}-f\frac{\partial u}{\partial z}=\frac{\partial^2 w}{\partial t\,\partial y}-\frac{g}{\bar{T}_0}\frac{\partial \theta}{\partial y} \tag{2.8}$$

From eq. (2.6), $$\frac{\partial^2 v}{\partial y^2}=-\frac{\partial^2 w}{\partial y\,\partial z} \tag{2.9}$$

Differentiating eq. (2.7) w.r.t to y two times, we have

$$\frac{\partial^3\theta}{\partial y^2\partial t}+\bar{T}_{01}\frac{\partial^2 v}{\partial y^2}+(\bar{T}_{02}+\frac{g}{c_p})\frac{\partial^2 w}{\partial y^2}=0 \tag{2.10}$$

Multiply eq. (2.10) by $\frac{g}{\bar{T}_0}$,

$$\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial y^2\partial t}+\frac{g}{\bar{T}_0}\bar{T}_{01}\frac{\partial^2 v}{\partial y^2}+\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\frac{\partial^2 w}{\partial y^2}. \tag{2.11}$$

By eq. (2.6),

$$\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial y^2\partial t}+\frac{g}{\bar{T}_0}\bar{T}_{01}(-\frac{\partial^2 w}{\partial y\partial z})+\frac{g}{\bar{T}_0}\bar{T}_{02}+\frac{g}{c_p})\frac{\partial^2 w}{\partial y^2}=0 \tag{2.12}$$

Taking ($\frac{g}{\bar{T}_0}\bar{T}_{01}$= -f$\bar{u}_{02.}$),

$$\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial y^2\partial t}+\frac{g}{\bar{T}_0}\frac{T_o}{g}f\bar{u}_{02}\frac{\partial^2 w}{\partial y\partial z}+\frac{g}{\bar{T}_0}\bar{T}_{02}+\frac{g}{c_p})\frac{\partial^2 w}{\partial y^2}=0 \tag{2.13}$$

$$\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial y^2\partial t}+f\bar{u}_{02}\frac{\partial^2 w}{\partial y\partial z}+\frac{g}{\bar{T}_0}(\bar{T}_{02}+\frac{g}{c_p})\frac{\partial^2 w}{\partial y^2}=0 \tag{2.14}$$

By eqs. (2.3) and (2.8), i.e., (f$\frac{\partial}{\partial z}eq.(2.3)-\frac{\partial}{\partial t}eq.(2.8)$), then we have

$$-f\,(f-\bar{u}_{01})\frac{\partial v}{\partial z}+f\bar{u}_{02}\frac{\partial w}{\partial z}-\frac{\partial^3 v}{\partial t^2\partial z}+\frac{\partial^3 w}{\partial t^2\partial y}-\frac{g}{\bar{T}_0}\frac{\partial^2\theta}{\partial t\partial y}=0 \tag{2.15}$$

Differentiating above eq. w. r. t. to y, we have

$$-f\,(f-\bar{u}_{01})\frac{\partial^2 v}{\partial y\partial z}+f\bar{u}_{02}\frac{\partial^2 w}{\partial z\partial y}-\frac{\partial^4 v}{\partial t^2\partial z\partial y}+\frac{\partial^4 w}{\partial t^2\partial^2 y}-\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial t\partial^2 y}=0$$

or $$\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial t\partial^2 y}=-f\,(f-\bar{u}_{01})\frac{\partial^2 v}{\partial y\partial z}+f\bar{u}_{02}\frac{\partial^2 w}{\partial z\partial y}+\frac{\partial^2}{\partial t^2}(-\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial^2 y})$$

By eq. (2.6),

$$or\ \frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial t\partial^2 y} = \text{-f (f-}\bar{u}_{01})\left(-\frac{\partial^2 w}{\partial z^2}\right) + f\bar{u}_{02}\ \frac{\partial^2 w}{\partial z\partial y} + \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial^2 y}\right)$$

$$or\ \frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial t\partial^2 y} = f(f-\bar{u}_{01})\left(\frac{\partial^2 w}{\partial z^2}\right) + f\bar{u}_{02}\ \frac{\partial^2 w}{\partial z\partial y} + \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial^2 y}\right) \qquad (2.16)$$

Putting the value of $\frac{g}{\bar{T}_0}\frac{\partial^3\theta}{\partial t\partial^2 y}$ from eq. (2.16) in eq. (2.14), we have

$$\text{f (f-}\bar{u}_{01})\left(\frac{\partial^2 w}{\partial z^2}\right) + f\bar{u}_{02}\ \frac{\partial^2 w}{\partial z\partial y} + \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial^2 y}\right) + f\bar{u}_{02}\ \frac{\partial^2 w}{\partial y\partial z} + \frac{g}{\bar{T}_0}\left(\bar{T}_{02} + \frac{g}{c_p}\right)\frac{\partial^2 w}{\partial y^2} = 0$$

$$\text{or}\quad \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial^2 y}\right) + \text{f (f-}\bar{u}_{01})\left(\frac{\partial^2 w}{\partial z^2}\right) + 2f\bar{u}_{02}\ \frac{\partial^2 w}{\partial y\partial z} + \frac{g}{\bar{T}_0}\left(\bar{T}_{02} + \frac{g}{c_p}\right)\frac{\partial^2 w}{\partial y^2} = 0 \qquad (2.17)$$

We may reduce eqs. (2.3) – (2.7) to the following equation for w:

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial^2 y}\right) + f(f-\bar{u}_{01})\frac{\partial^2 w}{\partial z^2} + 2f\bar{u}_{02}\ \frac{\partial^2 w}{\partial y\partial z} + \frac{g}{\bar{T}_0}\bar{T}_{02} + \frac{g}{c_p})\ \frac{\partial^2 w}{\partial y^2} = 0$$

where assuming the geostrophy of the basic field, we have taken $\frac{g}{\bar{T}_0}\bar{T}_{01}$= -f$\bar{u}_{02.}$ Let us assume for eq. (2.17), a solution of the form w=$\hat{w}(z)e^{\delta t^2+\gamma t+i\alpha y}$
where δ is constant and α *is real*, γ is may be complex and eq. (2.17) becomes

$$\{4\delta^2t^2 + \gamma^2 + 4\delta\gamma t + 2\delta + f(f-\bar{u}_{01})\}\ \frac{\partial^2\hat{w}}{\partial z^2} +$$

$$2\text{i}\alpha f\bar{u}_{02}\frac{\partial\hat{w}}{\partial z} - \alpha^2\{4\delta^2t^2 + \gamma^2 + 4\delta\gamma t + 2\delta + \frac{g}{\bar{T}_0}(\bar{T}_{02} + \frac{g}{c_p})\}\ \hat{w} = 0 \qquad (2.18)$$

Equation (2.18) is equivalent to

$$\frac{\partial^2 N}{\partial z^2} + QN = 0$$

where $\hat{w} = e^{-ipz}N$ and N is the function of z and t

$$\text{P} = \frac{\alpha f\bar{u}_{02}}{4\delta^2t^2+\gamma^2+4\delta\gamma t+2\delta+f(f-\bar{u}_{01})}$$

$$\text{Q} = \frac{\alpha^2}{\{4\delta^2t^2+\gamma^2+4\delta\gamma t+2\delta+f(f-\bar{u}_{01})\}}\left[\frac{(f\bar{u}_{02})^2}{\{4\delta^2t^2+\gamma^2+4\delta\gamma t+2\delta+f(f-\bar{u}_{01})\}} - \{4\delta^2t^2 + \gamma^2 + 4\delta\gamma t + 2\delta + \frac{g}{\bar{T}_0}(\bar{T}_{02} + \frac{g}{c_p})\}\right]$$

The solution for $\hat{w}$ is

$$\hat{w} = \text{A}e^{-iPz}(e^{.5Qz} + Be^{-.5Qz})$$

where A is arbitrary amplitude and B is determined by a boundary condition.

The stability conditions for this case are easily arrived at knowing that $\hat{w}(z)$ must be of the form $e^{i\mu z}$. We substitute $\hat{w}(z) = e^{i\mu z}$ into eq. (2.18) and solve for γ^2 to obtain the relation

$$\gamma^2 = [1 + \left(\frac{\alpha}{\mu}\right)^2]^{-1}[\left(\frac{\alpha}{\mu}\right)^2\{4\delta^2t^2 + 4\delta\gamma t + 2\delta + \frac{g}{\bar{T}_0}(\bar{T}_{02} + \frac{g}{c_p})\} + 2f\bar{u}_{02}\left(\frac{\alpha}{\mu}\right) +$$

$$\{4\delta^2t^2 + 4\delta\gamma t + 2\delta + f(f-\bar{u}_{01})\}] \qquad (2.19)$$

$$\gamma^2 = [1 + \left(\frac{\alpha}{\mu}\right)^2]^{-1}F\left(\frac{\alpha}{\mu}\right)$$

$$F\left(\frac{\alpha}{\mu}\right) = [\left(\frac{\alpha}{\mu}\right)^2\ \{\ 4\delta^2t^2 + 4\delta\gamma t + 2\delta + \frac{g}{\bar{T}_0}\ (\ \bar{T}_{02} + \frac{g}{c_p})\} + 2\ f\bar{u}_{02}\left(\frac{\alpha}{\mu}\right) + \{4\delta^2t^2 + 4\delta\gamma t + 2\delta + f(f-\bar{u}_{01})\}]$$

Now, $F\left(\frac{\alpha}{\mu}\right)$ is a quadratic in $\left(\frac{\alpha}{\mu}\right)$ and $\{4\delta^2t^2+4\delta\gamma t+2\delta+\frac{g}{\bar{T}_0}(\bar{T}_{02}+\frac{g}{c_p})\}$ is, general, positive, therefore, from Hall and Knight [7], the elementary theory of equation, a necessary and sufficient condition for F to be positive is

$$\{4\delta^2t^2+4\delta\gamma t+2\delta+f(f-\bar{u}_{01})\}\{4\delta^2t^2+4\delta\gamma t+2\delta+\frac{g}{\bar{T}_0}(\bar{T}_{02}+\frac{g}{c_p})\} \geq (f\bar{u}_{02})^2 \qquad (2.20)$$

or $\{4\delta^2t^2+4\delta\gamma t+2\delta+f(f-\bar{u}_{01})\}\mathrm{R_i} \geq 1$

where $$\mathrm{R_i} = \frac{\{4\delta^2t^2+4\delta\gamma t+2\delta+\frac{g}{\bar{T}_0}(\bar{T}_{02}+\frac{g}{c_p})\}}{(u_{02})^2} \qquad (2.21)$$

But, when F is positive, γ^2 is negative and the system is stable. Hence, eq. (2.21) is identical with condition (1.1), and the traditional results are seen to be preserved.

III. Stability with Respect to Axially Disturbance with Photochemistry and Radiative Transfer

The inclusion of photochemical and radiative process in the above problem involves replacing Equation (2.7) with the following equation of Lindzen and Goody [9]:

$$(\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)=-(\frac{\partial}{\partial t}+B)(v\bar{T}_{01}+w\bar{T}_{02}+\frac{g}{c_p}))-\eta(v\bar{\varphi}_{01}-w\varphi_{02}) \qquad (3.1)$$

Or, when η is constant,

$$\{\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)\}\theta=-(\frac{\partial}{\partial t}+a+B)(v\bar{T}_{01}+w\bar{T}_{02})-(\frac{\partial}{\partial t}+B)\frac{g}{c_p}w \qquad (3.2)$$

Assuming the geography of basic field, (3.2) becomes

$$\{\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)\}\theta=-(\frac{\partial}{\partial t}+a+B)\{v(-\frac{f\bar{T}_0}{g}\bar{u}_{02})+w\bar{T}_{02}\}-(\frac{\partial}{\partial t}+B)\frac{g}{c_p}w \qquad (3.3)$$

We are imposing boundedness at z=$\pm\infty$ and, since we are also taking our coefficient to be constant, our solutions for all fields are considered of the form $e^{\delta t^2+\gamma t+i\alpha y+i\mu z}$, where α *and* μ are real. From equations (2.3)-(2.6) and eq. (3.3) we obtain a quadratic dispersion relation for γ

$$(2\delta t+\gamma)^4[1+\left(\frac{\alpha}{\mu}\right)^2]+(a+B)(2\delta t+\gamma)^3[1+\left(\frac{\alpha}{\mu}\right)^2]+$$
$$(2\delta t+\gamma)^2[f\{f+(\frac{\alpha}{\mu})\bar{u}_{02}\}+(aB+\eta C)\{1+\left(\frac{\alpha}{\mu}\right)^2\}+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta\{1+\left(\frac{\alpha}{\mu}\right)^2\}]+$$
$$(2\delta t+\gamma)[(a+B)f(f+(\frac{\alpha}{\mu})\bar{u}_{02})+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta(1+\left(\frac{\alpha}{\mu}\right)^2)\}+\frac{g}{c_p}\frac{g}{\bar{T}_o}\left(\frac{\alpha}{\mu}\right)^2B]+$$
$$(aB+\eta C)\{f(f+(\frac{\alpha}{\mu})\bar{u}_{02})]=0 \qquad (3.4)$$

Where, we have set $\bar{u}_{01}=0$, for the sake of simplicity.

In this case, if $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]>0$, t=0 and $\delta=0$, the four roots of eq.(3.4) consist of a pair of complex roots with negative real parts corresponding to gravity waves in a rotating system with slight radiative photochemical damping . When $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]<0$ is negative, the two damped gravity-type waves remain.

Following Eliassen [3], when are neglected the terms $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$, then we obtain a quadratic in γ in instead of eq. (3.4).

$$(2\delta t+\gamma)^2[f\{f+(\frac{\alpha}{\mu})\bar{u}_{02}\}+(aB+\eta C)\{1+\left(\frac{\alpha}{\mu}\right)^2\}+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+$$
$$2\delta\{1+\left(\frac{\alpha}{\mu}\right)^2\}]+(2\delta t+\gamma)[(a+B)f(f+(\frac{\alpha}{\mu})\bar{u}_{02})+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+$$
$$2\delta(1+\left(\frac{\alpha}{\mu}\right)^2)\}+\frac{g}{c_p}\frac{g}{\bar{T}_o}\left(\frac{\alpha}{\mu}\right)^2B]+(aB+\eta C)\{f(f+(\frac{\alpha}{\mu})\bar{u}_{02})]=0. \qquad (3.5)$$

Let Q = $[f\{f+(\frac{\alpha}{\mu})\bar{u}_{02}\}+(aB+\eta C)\{1+\left(\frac{\alpha}{\mu}\right)^2\}+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta\{1+\left(\frac{\alpha}{\mu}\right)^2\}]$,

$$P=[(a+B)\,f\,(f+(\frac{\alpha}{\mu})\bar{u}_{02})+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta(1+\left(\frac{\alpha}{\mu}\right)^2)\}+\frac{g}{c_p}\frac{g}{\bar{T}_o}\left(\frac{\alpha}{\mu}\right)^2B].$$

Since we have $R_i \gg 1$, *then* Q and P are positive, equation (3.5) becomes

$$(2\delta t+\gamma)^2Q+(2\delta t+\gamma)P+(aB+\eta C)\{f\,(f+(\frac{\alpha}{\mu})\bar{u}_{02})]=0 \quad (3.6)$$

For $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]=0$ *and* $\delta=0, \gamma=0$ is the physically interesting root of (3.6), and for $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]<0$,this root become positive, implying instability. This instability depends on the presence of the joint radiative-photochemical restoring term (aB+ ηC).This explicity as follows:

Let $(\frac{\alpha}{\mu})_c$ be such that $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]=0$ and let $(\frac{\alpha}{\mu})=(\frac{\alpha}{\mu})_c+(\frac{\alpha}{\mu})'$. Near $(\frac{\alpha}{\mu})=(\frac{\alpha}{\mu})_c$

$$\gamma=-P^{-1}(aB+\eta C)\,\bar{u}_{02}(\frac{\alpha}{\mu})',$$

$$=-\frac{(aB+\eta C)}{(a+B)Q-a\frac{g}{fc_p}\frac{g}{\bar{T}_o}\left(\frac{\alpha}{\mu}\right)^2}\bar{u}_{02}(\frac{\alpha}{\mu}) \quad (3.7)$$

IV. INFLUENCES OF VERTICAL INHOMOGENEITIES

The coefficients in eqs.(2.3), (2.4), (2.5), (2.6) and (3.3) are taken to be constant. This implies a restriction to disturbances whose vertical scale is small compared to scale-height for the variation of the various coefficients. The scale- height for the variation of the photochemical parameters, B and C are very important.

V. RESULTS AND DISCUSSIONS

5.1 Stability with respect to axially symmetric disturbances –adiabatic case

For the solution of $\hat{w}$, $\hat{w}=A\,e^{-iPz}(e^{.5Qz}+Be^{-.5Qz})$, there are three possibilities for boundary conditions:

- **(a)** If w =0 at z=0 and H, ie, the fluid is confined by the two horizontal plates. In this case, B= -1 and $Q^{\frac{1}{2}}=\frac{n\pi}{H}$ .Since $\frac{n\pi}{H}$ is real, *then* γ^2 P and μ = P $\pm Q^{\frac{1}{2}}$ are also real.
- **(b)** For semi-infinite domain where w=0 at z=0, and bounded at z $=\pm\infty$. Also B=-1 and real μ satisfies the second boundary condition.
- **(c)** If there are no boundaries, boundedness at z = $\pm\infty$ only, then μ is real.

5.2 Stability with respect to axially disturbance with photochemistry and radiative transfer

(i)

$$(2\delta t+\gamma)^4[1+\left(\frac{\alpha}{\mu}\right)^2]+(a+B)\,(2\delta t+\gamma)^3[1+\left(\frac{\alpha}{\mu}\right)^2]+$$
$$(2\delta t+\gamma)^2\,[f\{f+(\frac{\alpha}{\mu})\bar{u}_{02}\}+(aB+\eta C)\{1+\left(\frac{\alpha}{\mu}\right)^2\}+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta\{1+\left(\frac{\alpha}{\mu}\right)^2\}]+$$
$$(2\delta t+\gamma)[(a+B)\,f\,(f+(\frac{\alpha}{\mu})\bar{u}_{02})+(\frac{\alpha}{\mu})\{f\bar{u}_{02}+(\frac{\alpha}{\mu})\frac{g}{\bar{T}_o}(\bar{T}_{02}+\frac{g}{c_p})\}+2\delta(1+\left(\frac{\alpha}{\mu}\right)^2)\}+\frac{g}{c_p}\frac{g}{\bar{T}_o}\left(\frac{\alpha}{\mu}\right)^2B]+$$
$$(aB+\eta C)\{f\,(f+(\frac{\alpha}{\mu})\bar{u}_{02})]=0 \quad (3.4)$$

For equation (3.4), t=0 and $\delta=0$

- **(a)** If for our basic state we have a zonal wind in geostriphic balance with a photochemical-radiative equilibrium temperature field, then the Richardson number is very large. When $[f+(\frac{\alpha}{\mu})\bar{u}_{02}]>0$,the four root of equation(3.4) consist of a pair of complex roots with negative real parts corresponding to gravity waves in a rotating systems with slight radiative-photochemical damping ,and two more roots with negative real parts corresponding to radiative – photochemical relaxations.

(b) When [f+ $(\frac{\alpha}{\mu})\bar{u}_{02}$] is negative, the two damped gravity type waves remain: one of the two remaining roots is now positive, ie, there is a new mode of instability.

(ii) $$\gamma = -\frac{(aB+\eta C)}{(a+B)Q - a\frac{g}{fc_p\bar{T}_o}\frac{g}{}\left(\frac{\alpha}{\mu}\right)^2}\bar{u}_{02}\left(\frac{\alpha}{\mu}\right) \quad (3.7)$$

Equation (3.7) suggests that the degree of instability is increased by increasing $\bar{u}_{02}$ and ratio $\frac{(aB+\eta C)}{(a+B)}$, but the dependence upon $\bar{u}_{02}$ is complicated by the effect of this quantity in the magnitude of Q when either t =0 or $\delta = 0$.

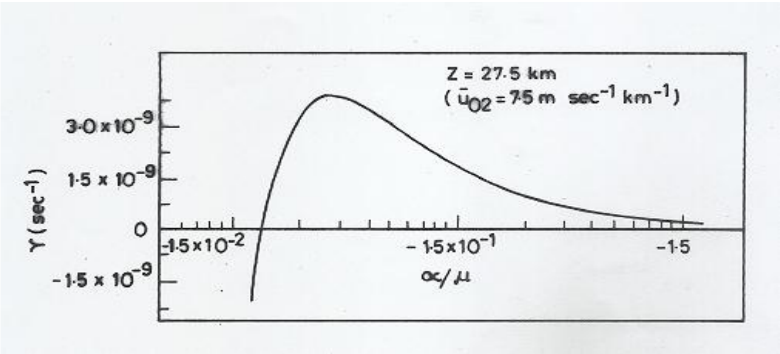

Fig .1 The growth rate, (γ) for axially symmetric disturbances vs $(\frac{\alpha}{\mu})$ is the ratio of meridional to vertical wave numbers for the disturbances for fixed value of z (=27.5 km) and $\bar{u}_{02}$ (= 7.5m $sec^{-1}km^{-1}$)

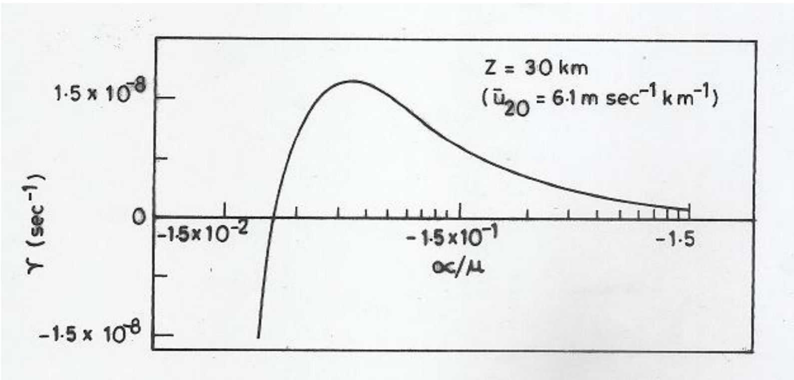

Fig .2. The growth rate, (γ) for axially symmetric disturbances vs $(\frac{\alpha}{\mu})$ is the ratio of meridional to vertical wave numbers for the disturbances for fixed value of z (=30 km) and $\bar{u}_{02}$ (= 6.1m $sec^{-1}km^{-1}$)

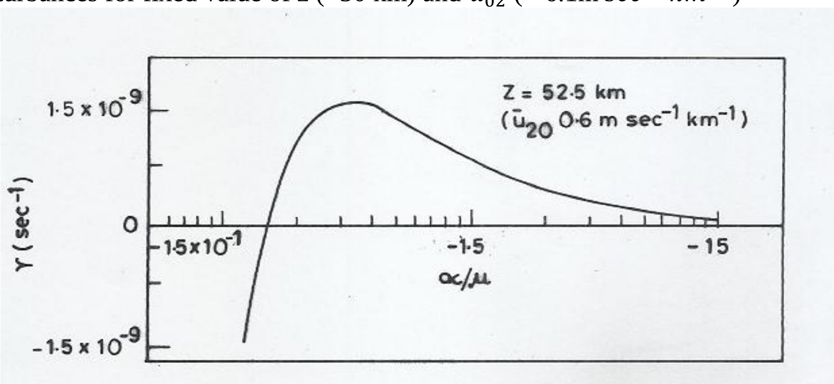

Fig. 3 The growth rate, (γ) for axially symmetric disturbances vs $(\frac{\alpha}{\mu})$ is the ratio of meridional to vertical wave numbers for the disturbances for fixed value of z (=52.5km) and $\bar{u}_{02}$ (= .6 m $sec^{-1}km^{-1}$)

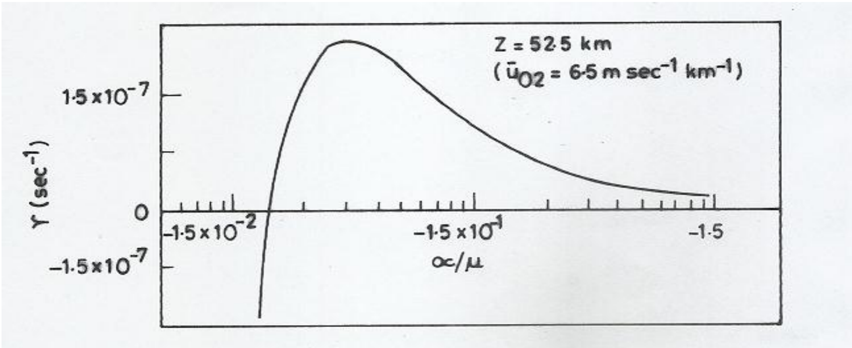

Fig .4 The growth rate, (γ) for axially symmetric disturbances vs ($\frac{\alpha}{\mu}$) is the ratio of meridional to vertical wave numbers for the disturbances for fixed value of z (=52.5km) and $\bar{u}_{02}$ (= 6.5 m $sec^{-1}km^{-1}$)

Figs 1, 2, and 3 shows as a function of ($\frac{\alpha}{\mu}$) for photochemical and radiative parameters typical of 27.5,30 and 52.5 km respectively, computed from eq. (3.5). For Figs. 1, 2 and 4 the shears are approximately the same, but ratio $\frac{(aB+\eta C)}{(a+B)}$ is increasing: in Figs. 3 and 4 $\frac{(aB+\eta C)}{(a+B)}$ is the same, but shear in Fig. 4 is much greater.
We see in Figs 1, 2, 3 and 4 for high shear the maximum growth rate for an unstable disturbance occurs for $\left(\frac{\alpha}{\mu}\right) \approx -0.03$.

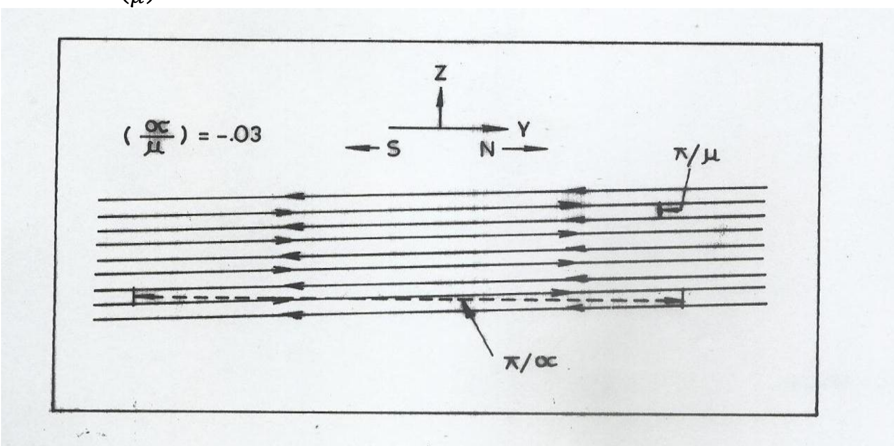

Fig. 5 Meridional circulation for $\left(\frac{\alpha}{\mu}\right) = -3 \times 10^{-2}$

III. Influences of Vertical inhomogeneities

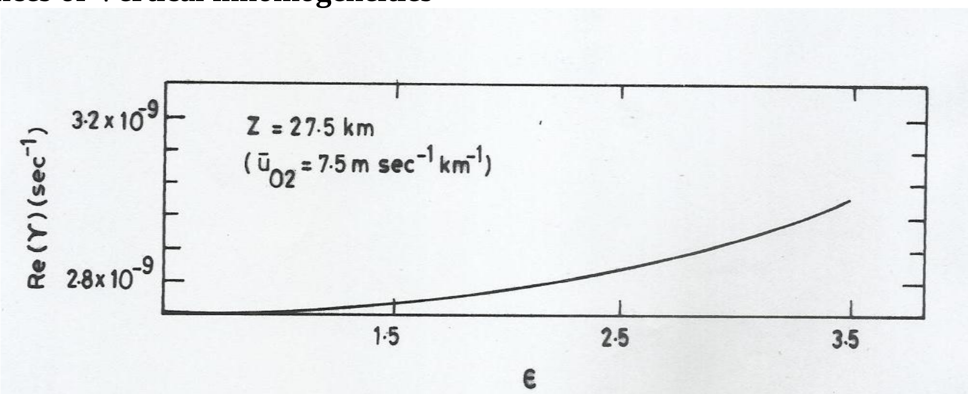

Fig .6 The growth rate of disturbances as a function of ε for fixed value of z (=27.5km) and $\bar{u}_{02}$ (= 7.5 m $sec^{-1}km^{-1}$)

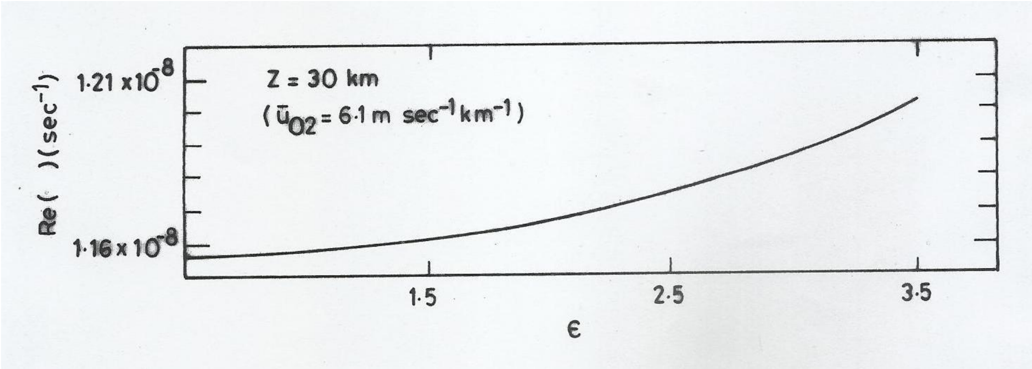

Fig .7 The growth rate of disturbances as a function of ε for fixed value of z (=30km) and $\bar{u}_{02}$ (= 6.1 m $sec^{-1}km^{-1}$)

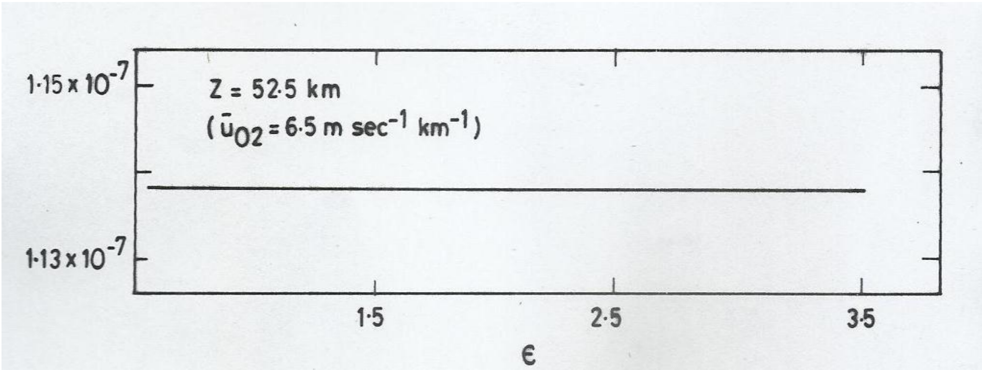

Fig .8. The growth rate of disturbances as a function of ε for fixed value of z (=52.5km) and $\bar{u}_{02}$ (= 6.5m km^{-1})

Fig.5 gives the meridional circulation associated with $\left(\frac{\alpha}{\mu}\right) = -0.03$, obtained from the continuity eq. (2.6). The motions are almost horizontal, vertically oscillating streams, rising slightly from south to north. In figs.6, 7 and 8 the real part of γ, maximized with respect to $\left(\frac{\alpha}{\mu}\right)$, and is plotted against $\epsilon = \frac{1}{\mu B}\frac{dB}{dz}$. The effect of ε is slight; γ increases slightly with ε indicating that larger disturbances scales are slightly more unstable.

VI. CONCLUSIONS

(i) Computing $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ to see whether they are small compared to fu and $(\frac{g}{T_o})\theta$, respectively, as assumed. The relevant ratios turn out to be [using equations (2.3)-(2.7) with $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ neglected]

$$\frac{\partial v}{\partial t} \text{/fu} = \frac{\gamma^2}{\text{f [f+} \left(\frac{\alpha}{\mu}\right)\bar{u}_{02}]} \text{ and } \frac{\partial v}{\partial t} \ /(\frac{g}{T_o})\theta = \frac{\gamma^2\left(\frac{\alpha}{\mu}\right)^2}{\text{f [f+} \left(\frac{\alpha}{\mu}\right)\bar{u}_{02}]}$$

Reference to figs. 1, 2, 3 and 4 (noting f$\sim 10^{-4} sec^{-1}$) shows that both these ratios are $\leq 10^{-6}$, and hence our approximations are valid.
(ii) The joint interactions of hydrodynamics, radiative transfer and photochemistry can destabilize hydrodynamics system that would otherwise be stable. In view of the very small growth rates of the instabilities described here, it is difficult to identify them with the observed phenomena. On the other hand, the photochemical and radiative process dissipates disturbances for some value of $\left(\frac{\alpha}{\mu}\right)$ while slightly amplifying disturbances with other values of $\left(\frac{\alpha}{\mu}\right)$ may prove important in selecting the form for disturbances.

VII. FUTURE WORK

(i) It will be interesting to deal with a disturbance whose characteristics height will be small compared to the height of the medium. The boundaries may be removed from the region of interest and their nature will be either very complicated or unknown to be studied in the future.
(ii) In view of the very small growth rates of the instabilities discussed in this paper, it is difficult to identify them with the observed phenomena .This will be a matter for future work.

(iii) The growth rate for baroclonic waves in mid-latitudes is much larger. Thus, we faced with the problem of the growth of these instabilities in the presence of the more rapidly growing baroclinic waves, which, mathematically, is an extremely difficult problem for further study.

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REFERENCES

[1] Eady, E.T., 1949: Long waves and cyclone waves. Tellus, 1, 258-277.
[2] H. L. Kuo, Forced and free meridional circulations in the atmosphere, J. Meteor., 13(1956), 406-413.
[3] A. Eliassen, Slow thermally of frictionally controlled meridional circular vortex, Astrophys. Norv. 5(1952), 19-60.
[4] C.Leovy, Simple models of thermally driven mesospheric circulation, J. Atmos. Sci., 21(1964), 327-341.
[5] Steven M. Smith, Seasonal variations in the correlation of mesospheric OH temperature and radiance at midlatitudes, Journal of Geophysical Research, vol. 117, a10308, 8 pp., 2012.
[6] David E. Siskind, Daniel R. Marsh, Martin G. Mlynczak, F. Javier Martin-Torres, and James M. Russell III, Decreases in atomic hydrogen over the summer pole: Evidence for dehydration from polar mesospheric clouds, Geophysical Research Letters, vol. 35, l13809, doi: 10.1029/2008gl033742, 2008.
[7] H. S. Hall and S. R. Knight, Heigher Algebra, London McMillan, (1884) 557pp.
[8] R. S. Lindzen, Radiative and photochemical process in strato and mesospheric dynamics,
Ph.D.Thesis, Harvard University, (1964) 210, pp.
[9] R. S. Lindzen and R. Goody, Radiative and photochemical process in mesospheric dynamics: part I, model for radiative and photochemical process, J. Atmos. Sci., 22(1965), 341-348.
[10] Spiegel, E.A and G.Veronis, 1960: On the Boussinesq approximation for a compressible fluid. Astrophys, 131,442-449.
[11] Phillips, N.A., 1963: Geostrophic motion. Rev Geophys., 1,123-176.
[12] Richard Lindzen and Richard Goody, Radiative and photochemical processes in Mesospheric Dynamics: Part I, Journal of the atmospheric Sciences, Vol, 22 No.4, July 1965.

AUTHORS

Vivekanand Yadav is currently pursuing D.Phil at University of Allahabad, Allahahbad-211002(India) and obtained his B.E in ECE from Dr.B.R.A.University agra,India. Obtained M.Tech(EC) at HBTI Kanpur from UPTU, Lucknow,India.Area of interest are Filter Design,Digital Signal Processing and Atmosphheric Dynamics.

R. S. Yadav is Presently working as Reader in the University of Allahabad,Allahabad-211002,India. Obtained D.Phil from University of Allahabad,Allahabad-211002,India.Area of interest are Digital Electronics and Atmosphheric Dynamics.