FINDING CRITICAL BUCKLING LOAD OF RECTANGULAR PLATE USING INTEGRATED FORCE METHOD

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ABSTRACT

A method which couples equilibrium equations and compatibility conditions that are developed based on equilibrium equations by using a systematic concatenation procedure is proposed here for the plate buckling analysis. A RECT_9F_12D plate bending element having 9 force unknowns and 12 displacement degrees of freedom is used with the necessary matrix formulation based on the Integrated Force Method (IFM). The geometric stiffness matrix required for buckling analysis is explicitly derived. Matlab software is used to develop compatibility conditions whereas other calculations are carried out in a program developed in VB.NET. A rectangular plate under uniaxial loading is analysed under 7 different boundary conditions. A case of biaxial loading of simply supported plate with loading ratio equals to one is also attempted using the proposed formulation. Results are obtained by considering either 2 x 2 discretization of quarter plate or 4 x 2 discretization of half plate depending upon the type of symmetry available based on support conditions. Results are compared with the available classical solutions to demonstrate the effectiveness of the proposed method; a good agreement is indicated.

KEYWORDS: *Buckling Problems, Hybrid Plate Element, Integrated Force Method, Matlab*

I. INTRODUCTION

Linear Elastic Stability Analysis (LESA) is an approach in which calculation is done for the critical intensity of applied in-plane loading. The internal distribution of orthogonal moment induced and possible nodal displacements at any point in the isotropic plate are considered as independent variable in secondary linear analysis, which is operated after calculation of critical loading. Actually practical aspects associated with uncertain buckling based collapse involves a nonlinear aspect of instability which is associated with post buckling behavior having large amount of inelastic deformations. Even in this connection, LESA thoroughly describes the complete circumstances of failure, which are of design importance for number of thin structural forms generally used in Naval and Aeronautical structures. It also furnishes the fundamental basis for large technical content of practical aspects of design methodology, even where nonlinear phenomena must be taken into account to define accurately the magnitude of load that causes failure. Thus, the complete form of the solution is frequently provided by Linear Analysis procedure only. Following are the three major approaches by which classical buckling problem of plates can be formulated.

1.1 EQUILIBRIUM METHOD

In this method, it is assumed that plate has buckled slightly, for which differential equation is written using the buckled form, where bending and stretching are included simultaneously. Alternatively, the method converts the complete problem into eigen value problem, in which one can evaluate the multiplying factor (λcr) to the external line load applied parallel to neutral plane. The solution involves the homogeneous equation w(x, y), with few arbitrary constants (C_0, C_1, $C_{2\ldots\ldots}$Cn) which are evaluated using boundary conditions. Equating the determinant of coefficients to zero, a polynomial equation is developed and the critical load is calculated from the following equation [1]:

$$Pcr = \lambda cr\ P_o \tag{1}$$

The non trivial solution λ = 0 for the characteristics equation corresponds to unbuckled state while λ ≠ 0 relates to buckled form.

1.2 Energy Method

As per energy method, whenever due to loading a plate passes from stable to unstable equilibrium stage, it goes through neutral state of equilibrium, which is characterized by conservation of energy. It also emphasizes that a plate changes from flat to curved shape without gaining and losing energy. The corresponding energy equation is written as [2],

$$\Delta W_i + \Delta W_e = Wi^* + (\lambda)\, We^* = 0 \tag{2}$$

Since the small bending is caused without stretching or contracting of the middle surface, the work done by the external compressive force We^* is due to inplane displacement produced by bending. Further, it is assumed that during buckling the intensity of the external forces remains the same. The work of the external forces is usually given as a function of the load parameter λ, as discussed above. It is evident that Eq. (1) can be used only when the expression for the deflection surface contains merely one undetermined coefficient. More often one can formulate the buckling problem using variational principle. In which, plate is said to be in state of equilibrium to which an infinitesimally small disturbance is applied. Since the work of the external and internal forces must vanish, Eq. (2) can be written in the form as

$$\pi_0 + \Delta\pi = 0 \tag{3}$$

Where π_0 denotes the total potential of the plate load system pertinent to the stable stage of equilibrium and $\Delta\pi$ is the increment in the total potential, representing the neighboring state of equilibrium, in which the middle surface is slightly curved, due to the small increase in the load. It is evident that for the stable equilibrium condition the total potential must vanish ($\Delta\pi = 0$).

Expressing the incremental part of total potential, $\Delta\pi$ by the taylor's series of expansion and by minimizing and differentiating higher order terms with respect to arbitrary constants (C_0, C_1, $C_{2\ldots\ldots}$Cn), smallest non zero solution of λ is calculated which is same as λ_{cr}.

1.3 Dynamic Method

The stability problem can be formulated by dynamic approach [3], in which the analogy of dynamic equilibrium aspect is directly implemented with numerical ease. The characteristic of stable state of equilibrium is such that even after applying small external incremental force the complete system oscillates but it returns to initial position of equilibrium. Another way, if buckling shape and the free vibration modal shapes are the analogous then the lowest natural frequency represents directly the lowest critical load for different variant conditions. Thus in developing differential equation of transverse vibration, the effect of the in-plane forces must be considered. Thus the equation of motion will contain the load factor λ. The smallest value of λ, producing lateral deflection that increases without limit, is the critical load factor.

In addition to above classical methods, researchers have attempted plate buckling problems by using numerical methods such as finite difference method [4] and finite element method [5-7]. Some of the problems which are difficult to solve by classical methods have also been attempted by using finite element method. Singh et al. [8] presented elastic buckling behavior of simply supported and clamped thin rectangular isotropic plates having central cutouts subjected to uni-axial partial edge compression. It was concluded that the buckling strength of square plates is highly influenced by partial edge compression, as compared to plate subjected to uniform edge compression. Monfared [9] investigated buckling of circular and rectangular plates with different boundary conditions under

sinusoid and axial compressive loading using differential equivalent direct method and FEM based ANSYS software. Good agreement was found between the analytical and numerical predictions for the critical buckling loads.
In the present paper, an approach known as Integrated Force Method (IFM) which has been successfully applied by Patnaik [10] and Patnaik and Yadagiri [11] for static and dynamic analysis of discrete and continuum structures is extended to deal with plate buckling problems of rectangular geometry. The method combines the Equilibrium Equations (EEs) and the Compatibility Conditions (CCs) that are developed based on EEs by using a systematic concatenation procedure. By using this approach, one can calculate the internal moments and then the nodal displacements of isotropic and orthotropic plate bending problems [12-13]. Authors have also developed the formulation based on the IFM for the buckling analysis of a variety of framed structures [14]. Here the same approach is extended to deal with plate buckling problems after development of geometric stiffness matrix for a rectangular element.
In the current work, after giving the formulation of element equilibrium matrix, element flexibility matrix, global compatibility matrix and geometric stiffness matrix for a rectangular element having 9 force unknowns and 12 displacement degrees of freedom, the type of uniaxially and biaxially loaded plate problems considered in the paper are described. The steps required for finding the solution are discussed with reference to a simply supported plate and then the results obtained using the proposed integrated force based methodology for 8 different square plate problems are compared with the available classical solutions [15].

II. FORMULATION FOR BUCKLING ANALYSIS

In integrated force method, the element forces {F} and the external load vector {P} are related as [12]

$$\begin{bmatrix} [B] \\ \hline [C][G] \end{bmatrix} \{F\} = \{P\} \tag{4}$$

$$\text{Or} \quad [S]\{F\} = \{P\} \tag{5}$$

where [B] is the basic equilibrium matrix of size m x n, [C] is the compatibility matrix of size (n - m) x n and [G] is the concatenated flexibility matrix of size n x n, with m being the force degrees of freedom and n being the displacement degrees of freedom.
The nodal displacement vector {δ} is related to the element force vector {F} as follows:

$$\{\delta\} = [S^{-1}]^T[G]\{F\} \tag{6}$$

The eigen based stability analysis equation is obtained by usual perturbation theory, which is given by

$$[S]\{F\} = \lambda[K_g][J][G]\{F\} \tag{7}$$

$$\text{Or } [[S] - \lambda[S_b]]\{F\} = \{0\}$$

where $[K_g]$ is the geometric stiffness matrix and λ is the stability parameter. The matrix $[S_b]$ is referred as the 'IFM stability matrix' and [J] consists of number of rows taken from $[S^{-1}]^T$ matrix. After calculating the eigen vector λ of size m, each is substituted in Eq. (7) for the calculation of {F}. Nodal displacements {δ} are then worked out for each vector by substituting {F} in Eq. (6).
The stability based IFM procedure comprises of the development of four matrices in which the equilibrium matrix [B] links internal forces to external loads, compatibility matrix [C] governs the deformations, flexibility matrix [G] relates deformations to forces and geometric stiffness matrix [Kg] is known as a eigen supportive operator for dynamic and buckling analysis. Both the equilibrium and compatibility matrices of the IFM are unsymmetrical having full row rank irrespective of type of problem, whereas the material constitutive matrix, flexibility matrix and geometric stiffness matrices are symmetrical.

2.1 ELEMENT EQUILIBRIUM MATRIX

The element equilibrium matrix written in terms of forces at grid points represents the vectorial summation of 'n' internal forces {F} and 'm' external loads {P}. The nodal EE in matrix notation can be stored as rectangular matrix $[B_e]$ of size m x n. The variational functional is evaluated as a portion of IFM functional which yields the basic element equilibrium matrix $[B_e]$ as follows [12]:

$$U_p = \int_D \left\{ M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} + M_{xy} \frac{\partial^2 w}{\partial x \partial y} \right\} d_x d_y$$

$$= \{M\}^T \{\epsilon\} \, ds \qquad (8)$$

where, $\{M\}^T = (M_x, M_y, M_{xy})$ are the internal moments and $\{\epsilon\}^T = \left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x \partial y}\right)$ represents the curvatures corresponding to each internal moment.

Consider a four-noded, 12 ddof (w_1 to Θ_{12}) rectangular element of thickness t with dimensions as 2a x 2b along the x and y axes as shown in (Figure 1). The force field is chosen in terms of nine independent forces as;

$$\{F\} = (F_1, F_2 \ldots \ldots F_9) \qquad (9)$$

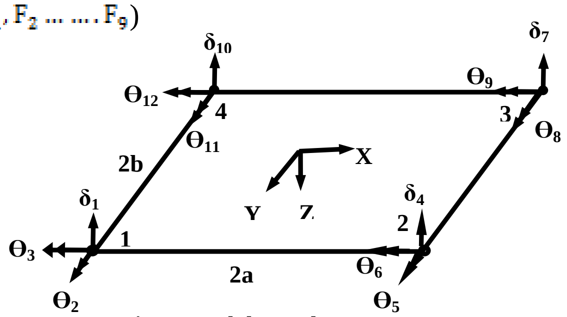

Fig. 1 Nodal Displacements

Relations between internal moments and independent forces are written as

$M_x = F_1 + F_2 x + F_3 y + F_4 xy$

$M_y = F_5 + F_6 x + F_7 y + F_8 xy$

$M_{xy} = F_9$

Arranging in matrix form,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & x & y & xy & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x & y & xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \{F_e\}$$

Or

$$\{M\} = [Y]\{F_e\} \qquad (10)$$

where $\{F_e\} = [F_1, F_2, F_3 \ldots\ldots\ldots\ldots F_9]^T$

The displacement field satisfies the continuity condition and the selected forces also satisfy the mandatory requirement. Polynomial function for lateral displacement for rectangular element is written as follows:

$$w_{(x,y)} = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 x + \alpha_6 y^2 + \alpha_7 x^2 + \alpha_8 x^2 y + \alpha_9 x y^2 + \alpha_{10} y^2 + \alpha_{11} x^2 y + \alpha_{12} x y^2 \qquad (11)$$

Or it can be written as $w_{(x,y)} = [A]\{\alpha\}$ (12)

where [A] is a row of size 1 x 12, which is a function of x and y and {α} is a vector of size 12 x 1, which consists of constants to be calculated. Substituting coordinates, one can find constants and finally the interpolation matrix [N] following the usual finite element procedure. Here each component of [N] is associated with nodal displacements δ_1, Θ_2, Θ_3 ---------- Θ_{12} as shown in **Fig. 1**. Eq. (12) is now expressed in terms of nodal displacements as follows:

$$w_{(x,y)} = [N]\{\delta\} \quad (13)$$

By arranging all force and displacement functions properly, one can discretize the Eq. (8) to obtain the elemental equilibrium matrix as follows.

$$U^e = \{\delta\}^T[B^e]\{F\} \quad (14)$$

$$\text{where } [B^e] = \int_s [Z]^T[Y]ds \quad (15)$$

Here [Z] = [L][N] where [L] is the differential operator matrix, [N] is the displacement interpolation function matrix and [Y] is the force interpolation function matrix. Substituting Eq. (11) in Eq. (15) and integrating within necessary limits a non symmetrical equilibrium matrix $[B^e]$ can be obtained. The matrix $[B^e]$ should have full row rank as mathematical property.

2.2 Element Flexibility Matrix $[G^E]$

The element flexibility matrix for isotropic material is obtained by discretizing the complementary strain energy which gives [13],

$$[G^e]= \int_s[Y]^T[D][Y]\,dxdy \quad (16)$$

where, [Y] is force interpolation function matrix and [D] is material property matrix. Substituting values in Eq. (16) and integrating one can calculate $[G^e]$ matrix. The size of $[G^e]$ is fdof x fdof and is a symmetrical matrix.

2.3 Global Compatibility Matrix [C]

The compatibility matrix is obtained from the deformation displacement relation ($\{\beta\} = [B]^T\{X\}$). In DDR all the deformations are expressed in terms of all possible nodal displacements and the 'r' compatibility conditions are developed in terms of internal forces i.e., F_1,------ F_{2n}, where '2n' is the total number of internal forces in a given problem. The concatenating or global compatibility matrix [C] can be evaluated by multiplying the compatibility matrix [C] and the global flexibility matrix [G].

2.4 Geometric Stiffness Matrix [Kg]

Figure 2 shows a rectangular plate having thickness 't' which is subjected to in-plane compressive forces $\sigma_x t$ and $\sigma_y t$ acting along neutral plane. The geometrical stiffness matrix for the element for the force in x – direction can be obtained from

$$[K_{g\,(x\text{-}x)}^{e}] = \int_S [N'_x]^T[N'_x]\,\sigma_x t\,dxdy \quad (17)$$

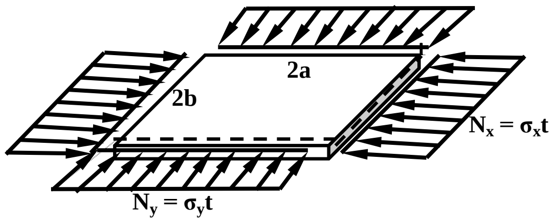

Fig. 2 Plate Under In-plane Forces

Where, N'_x is a vector of size (12 x 1) developed by differentiating with respect to x, $\sigma_x t$ is the inplane line load acting along neutral plane. Carrying out the operations as per Eq. (17), the geometric stiffness matrix for an element is obtained which is a symmetric matrix of size 12 x 12. The terms of the upper triangular part of the geometric stiffness $[Kg^e]$ are given below

$$K_g(1,1) = \frac{46b}{105a}, K_g(1,2) = \frac{b}{15}, \quad K_g(1,3) = \frac{11b^2}{105a}, \quad K_g(1,4) = -\frac{46b}{105a},$$
$$K_g(1,5) = \frac{b}{15}, \quad K_g(1,6) = -\frac{11b^2}{105a}, K_g(1,7) = -\frac{17b}{105a}, K_g(1,8) = \frac{b}{30},$$
$$K_g(1,9) = \frac{13b^2}{210a}, \; K_g(1,10) = \frac{17b}{105a}, \; K_g(1,11) = \frac{b}{30}, \quad K_g(1,12) = -\frac{13b^2}{210a},$$
$$K_g(2,2) = -\frac{8ab}{45}, K_g(2,3) = 0, \quad K_g(2,4) = -\frac{b}{15}, \; K_g(2,5) = \frac{-2ab}{45},$$
$$K_g(2,6) = 0, \quad K_g(2,7) = \frac{b}{30}, \quad K_g(2,8) = \frac{-ab}{45}, \; K_g(2,9) = 0,$$
$$K_g(2,10) = \frac{b}{30}, \quad K_g(2,11) = \frac{4ab}{45}, \quad K_g(2,12) = 0, \quad K_g(3,3) = \frac{4b^3}{105a},$$
$$K_g(3,4) = -\frac{11b^2}{105a}, \; K_g(3,5) = 0, \quad K_g(3,6) = -\frac{4b^3}{105a}, \; K_g(3,7) = \frac{13b^2}{210a},$$
$$K_g(3,8) = 0, \quad K_g(3,9) = \frac{b^3}{35a}, \quad K_g(3,10) = \frac{13b^2}{210a}, \; K_g(3,11) = 0,$$
$$K_g(3,12) = -\frac{b^3}{35a}, K_g(4,4) = \frac{46b}{105a}, \; K_g(4,5) = -\frac{b}{15}, \quad K_g(4,6) = -\frac{11b^2}{105a},$$
$$K_g(4,7) = \frac{17b}{105a}, \; K_g(4,8) = -\frac{b}{30}, \; K_g(4,9) = \frac{b^3}{35a}, \quad K_g(4,10) = -\frac{17b}{105a},$$
$$K_g(4,11) \; -\frac{b}{30}, \quad K_g(4,12) = \frac{13b^2}{210a}, K_g(5,5) = \frac{8ab}{45a}, \quad K_g(5,6) = 0,$$
$$K_g(5,7) = -\frac{b}{30}, \quad K_g(5,8) = \frac{4ab}{45}, \; K_g(5,9) = 0, \quad K_g(5,10) = \frac{b}{30},$$
$$K_g(5,11) = -\frac{ab}{45}, \; K_g(5,12) = 0, \quad K_g(6,6) = \frac{13b^2}{210a}, \quad K_g(6,7) = \frac{4b^3}{105a},$$
$$K_g(6,8) = 0, \quad K_g(6,9) = -\frac{b^3}{35a} \; K_g(6,10) = -\frac{13b^2}{210a}, \; K_g(6,11) = 0,$$
$$K_g(6,12) = \frac{b^3}{35a}, \quad K_g(7.7) = \frac{46b}{105a}, \quad K_g(7,8) = -\frac{b}{15}, \quad K_g(7,9) = -\frac{11b^2}{105a},$$
$$K_g(7.10) = -\frac{46b}{105a}, K_g(7,11) = \frac{b}{15}, \quad K_g(7,12) = -\frac{11b^2}{210a}, K_g(8.8) = \frac{8ab}{45a},$$
$$K_g(8,9) = 0, \quad K_g(8,10) = \frac{b}{15}, \quad K_g(8.11) = \frac{-2ab}{45}, \quad K_g(8,12) = 0,$$
$$K_g(9,9) = \frac{4b^3}{105a}, \quad K_g(9.10) = \frac{11b^2}{105a}, \; K_g(9,11) = 0, \quad K_g(9,12) = \frac{4b^3}{105a},$$
$$K_g(10.10) = \frac{46b}{105a}, K_g(10,11) = \frac{b}{15}, \; K_g(10,12) = -\frac{11b^2}{105a}, \; K_g(11,11) = \frac{8ab}{45a},$$
$$K_g(11,12) = 0, \text{ and } K_g(12,12) = \frac{4b^3}{105a}$$

III. PLATE BUCKLING EXAMPLES

Total seven examples of uniaxially loaded plate are considered here to validate the proposed method under different boundary conditions i.e., Simply supported (S), Clamped (C), Free (F) and their combinations as shown in Figure 3. All the plates are subjected to inplane force along x-x direction. Each plate is having geometrical dimensions as 4000 mm x 4000 mm x 200 mm. The modulus of elasticity is considered as 2.01 x 10^{11} N/m^2 and Poisson ratio is considered as 0.23. Figures 4 and 5 shows different discretization schemes considering either one- or two- way symmetry depending upon the support conditions. One example of biaxially loaded plate is also included here as shown in Figure 2.

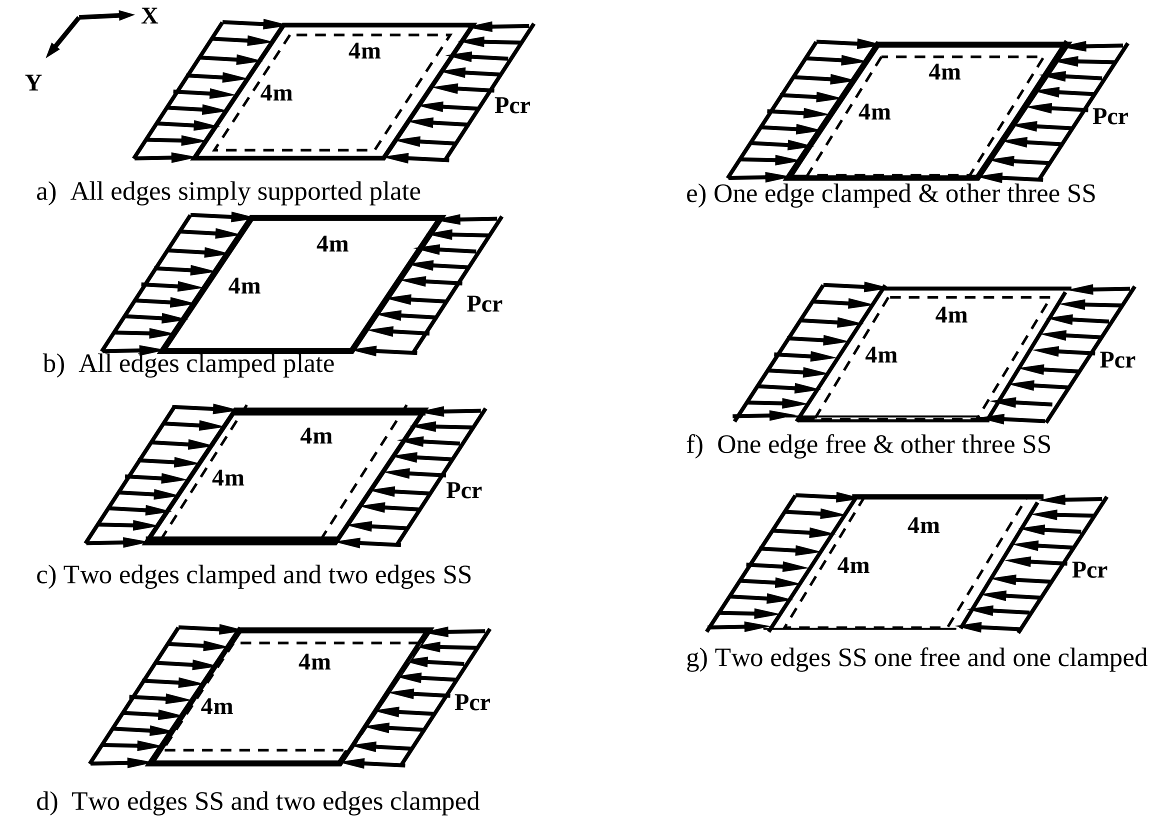

Fig. 3 Plates with Different Boundary Conditions

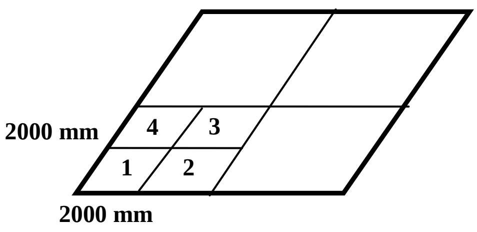

Fig. 4 Discretization using Two-way symmetry

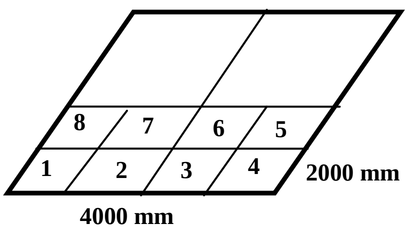

Fig. 5 Discretization using One-way Symmetry

IV. STEPS AND RESULTS

For buckling analysis through IFM, steps used are explained here with reference to a simply supported plate.

STEP 1 DEVELOP GLOBAL EQUILIBRIUM MATRIX [B]: A four-nodded rectangular element (2a x 2b) with 12 ddof and 9 fdof is used for discretizing the problem in to four elements. The equilibrium matrix $[B^e]$ is obtained by using the method described above. Assembled global equilibrium matrix, for quarter symmetry of S-S-S-S, will be of size 12 x 36.

STEP 2 DEVELOP GLOBAL COMPATIBILITY CONDITIONS [C]: The compatibility matrix for all discretized elements is obtained from the displacement deformation relations (DDR) i.e. $\beta = [B]T\{\delta\}$. In the DDR, 36 deformations which correspond to 36 force variables are expressed in terms of 12 displacements ($\delta 1$, $\Theta 2$ -------- $\Theta 12$) (Figure 6). The problem requires 24 compatibility conditions [C] that are obtained by using auto-generated Matlab based computer program by giving input as upper part of the global equilibrium matrix.

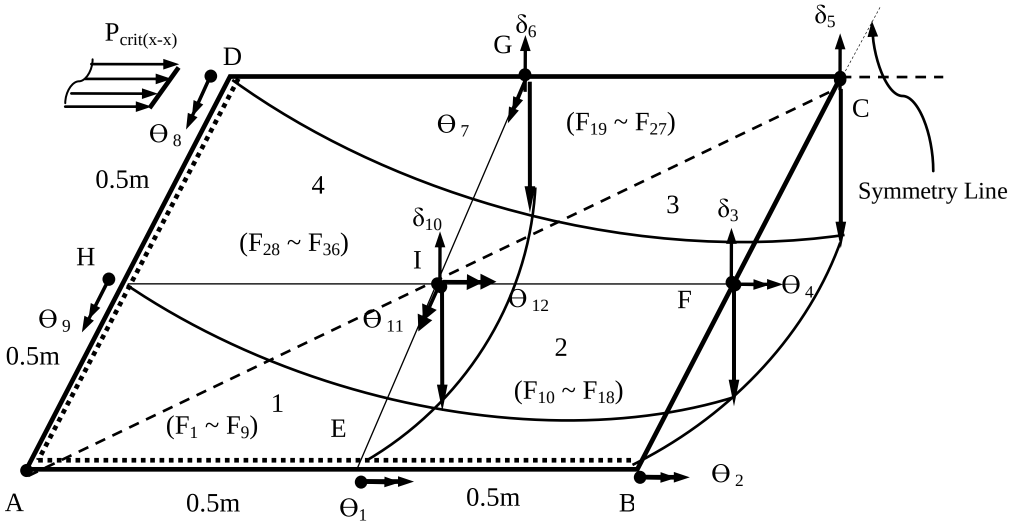

Fig. 6 Force and Displacement Unknowns

STEP3 DEVELOP GLOBAL FLEXIBILITY MATRIX [G]: The flexibility matrix for the problem is obtained by diagonal concatenation of the four flexibility matrices as;

$$[G] = \begin{bmatrix} G_{e^1} & & & \\ & G_{e^2} & & \\ & & G_{e^3} & \\ & & & G_{e^4} \end{bmatrix} \tag{18}$$

STEP 4 DEVELOP GLOBAL GEOMETRIC STIFFNESS MATRIX [K_{GC}]: The global geometric stiffness matrix is worked out by assembling the four elemental geometric matrices [K_g^1] to [K_g^4]. Using the standard stiffness based assembly procedure a global geometric stiffness matrix is developed of size 36 x 36.

STEP 5 CALCULATE BUCKLING LOAD (P_{CRIT}): Concatenating global CC matrix of size (24 x 36) is obtained after normalizing with respect to components of [Be] of size (12 x 36). It is developed by multiplying [C] matrix of size 24 x 36 by global flexibility matrix of size 36 x 36. Substituting all necessary matrices in Eq. (7), one can get solution for eigen vector of size (12 x 1) corresponding to 12 global displacements of quarter plate. The Buckling Load Ratio (BLR) is then calculated using ratio of IFM based critical load and exact solution [15] as reported in Table 1.

STEP 6 CALCULATE FORCE MODE SHAPE {F}: The internal unknowns (F_1, F_2…… F_{36}) are auto calculated in Matlab based eigen value analysis i.e. [F, Pcrit] = eig(Smatrix, KJG) where [F] is the matrix of size 36 x 36, Pcrit is the diagonal matrix of size 36 x 36. KJG is the product of global geometric stiffness matrix [K_g], Jmatrix is transpose of [Sinv] and G is the global flexibility matrix [G] of size (36 x 36). Taking sixth column of [F] matrix (corresponding to minimum critical load) and substituting in Eq. (10) moments at points I, F, C, G are worked out Figure. 6. The values are normalized with respect to point C and are depicted in Table 1.

Table1. Results for S_S_S_S Plate Case

Buckling Load Ratio (BLR)	**Normalized Moments with reference to Point C**		**Normalized Displacement with reference to δ_5**	
1.0608	Point I		Θ_1	-0.6651
	M_x	0.2854	Θ_2	-1.136
	M_y	0.09244	δ_3	0.992
	M_{xy}	0.8668	Θ_4	-0.7718

	Point F		δ_5	1.4313
	M_x	0.9935	δ_6	1.0000
	M_y	0.6321	Θ_7	0.8506
	M_{xy}	3.5133	Θ_8	-1.0611
	Point C		Θ_9	-0.7956
	M_x	1.000	δ_{10}	0.6553
	M_y	1.000	Θ_{11}	0.5076
	M_{xy}	1.000	Θ_{12}	0.6377
	Point G			
	M_x	0.5862		
	M_y	0.6931		
	M_{xy}	3.6198		

Following the above procedure, results obtained for critical load for the remaining six uniaxially plate problems are compared with the classical solution [15] in Table 2. Result obtained for a biaxially loaded plate, having load ratio of 1, is also included in Table 2.

Table 2. Buckling Load Ratio (BLR)

Case (Uniaxial)	**BLR = IFM/Exact [15]**
C_C_C_C	1.0534
C_S_C_S	1.0252
S_C_S_C	1.0354
C_S_S_S	1.0487
F_S_S_S	1.0634
F_S_C_S	1.0143
Case (Biaxial)	**BLR**
S_S_S_S	1.0112

V. CONCLUSIONS

- The development of compatibility conditions is the most crucial part of the IFM formulation. It is facilitated in the present work by developing an algorithm in VB.NET and linking it to the Matlab software. The generation of field and boundary compatibility conditions for any large scale continuum problem, with finer discretization, may require more time where the suggested approach may prove very efficient.
- For problems involving large number of unknowns, if the numerical difference between the components of the equilibrium and compatibility part is more, it may change the displacement to uncertain values. So before proceeding further a normalization of the compatibility is strongly recommended. It may be noted, however, that it does not make much difference in the calculation of internal force vector.
- A number of plate buckling problems are attempted under uniform compressive loading in x direction. A variety of support conditions are considered. The result for critical buckling load is found to differ by 1 to 6.34 % from the available classical solution. The maximum percentage difference is found as 6.34% in a case of axially compressed thin square plate having one edge free and three edges simply supported. In case of fully simply supported plate subjected to biaxial loading, with 2 x 2 discretization of quarter plate, the value of critical buckling load using IFM is found to differ from the exact value by 1.12%.
- Thus, the IFM can be considered as a viable alternative to the popular displacement based finite element method for finding the in-plane critical load of rectangular plates. Also, an extension of this method to buckling analysis of orthotropic rectangular plate problems is straight forward.

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