COMPARATIVE STUDY OF NON-LOCAL MEANS AND FAST NON –LOCAL MEANS ALGORITHM FOR IMAGE DENOISING

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ABSTRACT

Visual information transmitted in the form of digital images is becoming a major method of communication in the modern age. All digital images contain some degree of noise. Removing noise from the original signal is still a challenging problem for researchers. In this paper, the non-local denoising approach presented by Buades et al. is compared and analyzed by Fast non- local means algorithm. The original non-local means method is based on Self Similarity concept. Non local means denoising algorithm has disadvantage that remarkable denoising results are obtained at high expense of computational cost due to the enormous amount of weight computations. In order to accelerate the algorithm a new one that reduces the computation cost for calculating the similarity of neighborhood window was developed, known as Fast non- local means algorithm. In this algorithm , an approximate measure about the similarity of neighborhood windows, an efficient Summed Square Image (SSI) scheme and Fast Fourier transform (FFT) were used to accelerate the calculation of this measure. Furthermore, results obtained by simulation using Matlab7.0 shows that Fast non- local means algorithm is fifty times faster than original non –local means algorithm .We finally demonstrate the potential of the both algorithms through comparisons. We also present denoising results obtained on real images.

KEYWORDS: *Image denoising, Non-local means, Summed square image, FFT*

I. INTRODUCTION

1.1. Digital Image Processing

Most of the common image processing functions available in image analysis systems can be categorized into the following four categories:
1. Pre-processing
2. Image Enhancement
3. Image Transformation
4. Image Classification and Analysis

Most denoising algorithms make two assumptions about the noisy image. These assumptions can cause blurring and loss of detail in the resulting denoised images. The first assumption is that the noise contained in the image is white noise. This means that the noise contains all frequencies, low and high. Because of the higher frequencies, the noise is oscillatory or non-smooth. The second assumption is that the true image (image without the noise) is smooth or piecewise smooth [7].This means the true image or patches of the true image only contain low frequencies.

Previous methods attempt to separate the image into the smooth part (true image) and the oscillatory part (noise) by removing the higher frequencies from the lower frequencies. However, not all images are smooth. Images can contain fine details and structures which have high frequencies. When the high frequencies are removed, the high frequency content of the true image will be removed along with the high frequency noise because the methods cannot tell the difference between the noise and true image [7].This will result in a loss of fine detail in the denoised image. Also, nothing is done to remove the low frequency noise from the image. Low frequency noise will remain in the image even after denoising.

Numerous and diverse denoising methods have already been proposed in the past decades, just to name a few algorithms: total variation[8], bilateral filter or kernel regression[2][9] and wavelet-based techniques.[3][10][11][12]. All of these methods estimate the denoised pixel value based on the information provided in a surrounding local limited window.

Unlike these local denoising methods, non-local methods estimate the noisy pixel is replaced based on the information of the whole image. Buades developed a non-local image denoising algorithm by making use of the information encode in the whole image [8]. When modifying a pixel, the algorithm first computes the similarity between a fixed window centered around it and the windows centered around the other pixels in the whole image, then it takes the similarity as a weight to adjust this pixel. This method has shown remarkable and convincing results, but the efficiency is low for its pixel-wise window matching. The computational complexity of the NLM algorithm and Fast NLM algorithm are about $O(n^4)$ and $49 * 441 * n^2$ respectively, in which n^2 is the number of pixels of the image. However, even with the NLM algorithm, it still takes about 1 minute to denoise a $640 * 480$ image on a common PC. In evidence, the high computational complexity makes it unfeasible to tackle with practical issues.

Fast NLM denoising algorithm [4] was developed in the frame of nonlocal method by developing a fast calculation method for the comparison of windows' similarity. Exploiting a Summed Squares Image (SSI) scheme and Fast Fourier Transform (FFT), the per-pixel neighborhood matching is converted into the SSI pre-computing and efficient FFT. Computational complexity analysis and experiments indicate that fast non-local algorithm is about 50 times faster than the original non-local algorithm, and yet produces similar results. Using Fast NLM algorithm, it normally takes less than 1 second to denoise a normal size image (e.g. $640 * 480$), and less than 10 seconds to denoise a 500 Megabyte photograph, which enables the algorithm widely applicable to practical situations.

The rest of this paper is organized as follows. In section 2, we introduce the non-local means algorithm. Section 3 introduces fast non-local algorithm for image denoising. Section 4 provides comparison and some discussion about above mentioned non-local means and fast non-local algorithm. The last Section concludes the whole paper.

1.2. Related Work

A. Buades, B. Coll, and J. Morel proposed a non-local algorithm for image denoising[5][6]. The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. In spite of the sophistication of the recently proposed methods, most algorithms have not yet attained a desirable level of applicability. All show an outstanding performance when the image model corresponds to the algorithm assumptions but fail in general and create artifacts or remove image fine structures. The main focus of this paper is, first, to define a general mathematical and experimental methodology to compare and classify classical image denoising algorithms and, second, to propose a nonlocal means (NL-means) algorithm addressing the preservation of structure in a digital image. The mathematical analysis is based on the analysis of the "method noise," defined as the difference between a digital image and its denoised version. The NL-means algorithm is proven to be asymptotically optimal under a generic statistical image model. The denoising performances of all considered methods are compared in four ways; mathematical: asymptotic order of magnitude of the method noise under regularity assumptions; perceptual-mathematical: the algorithms artifacts and their explanation as a violation of the image model; quantitative experimental: by tables of $L2$ distances of the denoised version to the original image. The most powerful evaluation method seems, however, to be the visualization of the method noise on natural images.

A. Efros and T. Leung[13] proposed Texture synthesis by non parametric sampling.Textures can often more easily be described as a composition of subtextures than as a single texture. The paper proposes a way to model and synthesize such "composite textures", where the layout of the different subtextures is itself modeled as a texture, which can be generated automatically. This procedure comprises manual or unsupervised texture segmentation to learn the spatial layout of the composite texture and the extraction of models for each of the subtextures. Synthesis of a composite texture includes the generation of a layout texture, which is subsequently filled in with the appropriate subtextures. This scheme is refined further by also including interactions between neighboring subtextures.

II. NON- LOCAL MEANS ALGORITHM

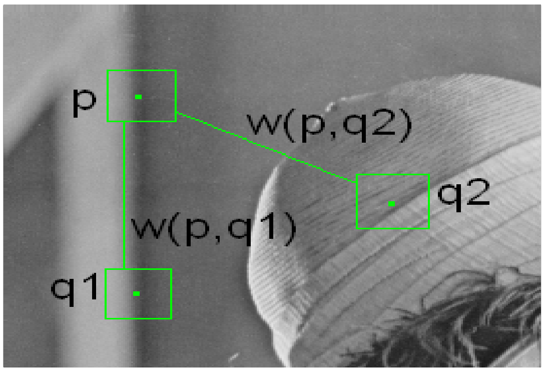

Figure 1: Example of self-similarity in an image. Pixels *p* and *q1* have similar neighborhoods, but pixels *p* and *q2* do not have similar neighborhoods. Because of this, pixel *q1* will have a stronger influence on the denoised value of *p* than *q2*.

The Self-Similarity concept was originally developed by Efros and Leung for texture synthesis [13]. The NLM method proposed by Buades [6] is based on the same concept. This concept is better explained through an example given in figure 1. The figure shows three pixels *p, q1*, and *q2* and their respective neighborhoods. It can be seen that the neighborhoods of pixels *p* and *q1* are much more similar than the neighborhoods of pixels *p* and *q2*. In fact, to the naked eye the neighborhoods of pixels *p* and *q2* do not seem to be similar at all. In an image adjacent pixels are most likely to have similar neighborhoods. But, if there is a structure in the image, non-adjacent pixels will also have similar neighborhoods. Figure 1 illustrates this idea clearly. Most of the pixels in the same column as p will have similar neighborhoods to *p*'s neighborhood. In the NLM method, the denoised value of a pixel is determined by pixels with similar neighborhoods.

2.1. Non-local Means Method

Each pixel *p* of the non-local means denoised image is computed with the following formula:

$$NL(V)(p) = \sum_{q \epsilon V} w(p,q)V(q) \tag{1}$$

where *V* is the noisy image, and weights *w(p,q)* meet the following conditions$0 \leq w(p, q) \leq 1$ and $\sum_q w(p,q) = 1$. Each pixel is a weighted average of all the pixels in the image.

The weights are based on the similarity between the neighbourhoods of pixels *p* and *q* [1, 2]. For example, in Figure1 above the weight *w(p,q1)* is much greater than *w(p,q2)* because pixels *p* and *q1* have similar neighbourhoods and pixels *p* and *q2* do not have similar neighbour hoods. In order to compute the similarity, a neighbourhood must be defined. Let Ni be the square neighbourhood centred about pixel *i* with a user-defined radius *Rsim*. To compute the similarity between two neighbourhoods take the weighted sum of squares difference between the two neighbourhoods or as a formula

$$d(p,q) = \| V(N_p) - V(N_q) \|_{2,F}^2 \text{ [1, 2].} \tag{2}$$

Here F is the neighbourhood filter applied to the squared difference of the neighbourhoods and will be further discussed later in this section. The weights can then be computed using the following formula:

$$w(p, q) = \frac{1}{Z(p)} e^{\frac{-d(p,q)}{h}} \tag{3}$$

Z(p) is the normalizing constant defined as

$$Z(p) = \sum_q e^{\frac{-d(p,q)}{h}} \quad [1,2]. \tag{4}$$

h is the weight-decay control parameter.
As previously mentioned, *F* is the neighborhood filter with radius *Rsim*. The weights of *F* are computed by the following formula:

$$\frac{1}{R_{sim}} \sum_{i=m}^{R_{sim}} 1/(2 \neq i \mid 1)^2 \tag{5}$$

where *m* is the distance the weight is from the center of the filter. The filter gives more weight to pixels near the center of the neighborhood, and less weight to pixels near the edge of the neighborhood. The center weight of *F* has the same weight as the pixels with a distance of one [5]. Despite the filter's unique shape, the weights of filter *F* do sum up to one.

Equation (1) from above does have a special case when $q = p$. This is because the weight *w(p, p)* can be much greater than the weights from every other pixel in the image. By definition this makes sense because every neighborhood is similar to itself. To prevent pixel *p* from over-weighing itself let *w(p ,p)* be equal to the maximum weight of the other pixels, or in more mathematical terms

$$w(p, p) = max\ w(p, q) \mid p \neq q \mid \tag{6}$$

III. FAST NON-LOCAL MEANS DENOISING ALGORITHM

Fast non-local means algorithm [19] is based on Summed Squared Image (SSI)[14] and fast Fourier transform (FFT), together with an approach for estimating the standard deviation of noise. An improvement of image quality towards the original algorithm is to ignore the contributions from dissimilar windows. Even though their weights are very small at first sight, the new estimated pixel value can be severely biased due to the many small contributions. Using a pre-classification technique, only weights for the most meaningful pixels are computed. This pre-classification is a fast way to exclude dissimilar windows, which eventually results in a smaller computation time and even in a better overall denoising quality. Fast non-local means algorithm is further optimized by taking advantage of the symmetry in the weights and by using a lookup table to speed up the weight computations.
For the convenience of acceleration, fast non-local means algorithm adopted the Euclidean distance [4] to compare two neighbourhoods,

$$S(i,j) \quad = ||N_i - N_j||^2, \tag{7}$$

$$= \sum_{l=0}^{M-1} \sum_{m=0}^{M-1} [I_i(l,m) - I_j(l,m)]^2 \tag{8}$$

Where $I_i(l,m)$ and $I_j(l,m)$) represent the corresponding pixels in N_i and N_j respectively. In fact, $I_j(l,m)$) in equation (8) can be represented in the global coordinates on the mirrored image as:
$I_j(l - x_j', m - y_j')$ with $x_j' = 3M/2 + x_j$, $y_j' = 3M/2 + y_j$ (see Fig. 2).

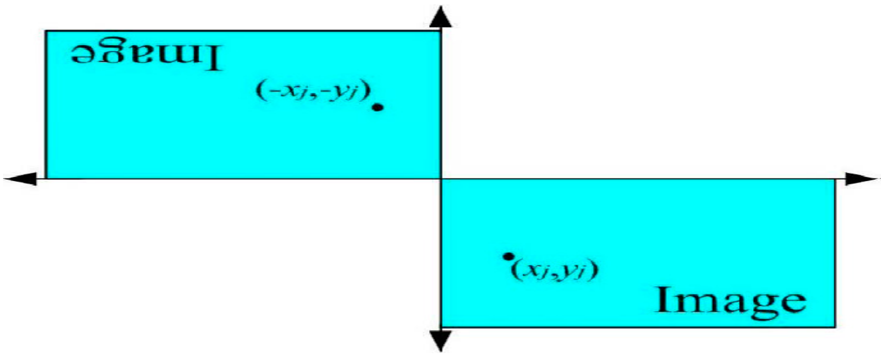

Figure 2. Mirrored image

So equation (7) is transformed into:

$$\mathrm{S}(i,j) = \sum_{l=0}^{M-1}\sum_{m=0}^{M-1}[I_i(l,m) - I_j(l-x_j', m-y_j')]^2,$$
$$= N_i^2 + N_j^2 - N_i * N_j \quad (9)$$

Where $N_i^2 = \sum_{l=0}^{M-1}\sum_{m=0}^{M-1}[I_i(l,m) - I_j(l,m)]^2$,

$N_j^2 = \sum_{l=0}^{M-1}\sum_{m=0}^{M-1}[I_i(l,m) - I_j(l-x_j', m-y_j')]^2$,

and $N_i * N_j = 2\sum_{l=0}^{M-1}\sum_{m=0}^{M-1}((l,m).I_j(l-x_j', m-y_j'))^2$
denotes the convolution between N_i *and* N_j . In above formula, $N_i * N_j$ can be figured out quickly with multiplications under the fast Fourier transform, while N_i^2 and N_j^2 can be fast calculated as well using the Summed Squared Image (SSI) proposed in subsection 3.1. Note that if the compare window size is $M * M$, in (7), computing the similarity of the two compare window requires M^2 pixel operations, while, in algorithm, it is figured out once which is achieved by means of FFT.

3.1. Summed Square Image (SSI)

The principle of *SSI* resembles Integral image which has been used in face detection [14]. For each pixel in the image, integral image maintains the summed value of all the pixels in the upper left part of the original image. Here it was extended to SSI. Similar to the definition of integral image, for each pixel (x_0, y_0), SSI stores its sum for the squared values of the upper left pixels,

$$\mathrm{SSI}(x_0, y_0) = \sum_{x<x_0, y<y_0} I^2(x,y) \quad (10)$$

SSI can be obtained in linear time proportional to the image size, each pixel in the original image is processed only once, so the computational complexity for computing SSI is $O(n^2)$, in which n^2 is the size of the image. By means of SSI, the sum of squares can be easily obtained for each pixel in any rectangles of the image within constant time. For example in Fig. 3, to calculate the sum of squares in rectangle D, only 3 addition operations are required,

$$\mathrm{SD} = \mathrm{SA{\cup}B{\cup}C{\cup}D} + \mathrm{SA} - \mathrm{SA{\cup}C} - \mathrm{SA{\cup}B}$$
$$= \mathrm{SSI}(x_1, y_1) + \mathrm{SSI}(x_0, y_0) - \mathrm{SSI}(x_0, y_1) - \mathrm{SSI}(x_1, y_0) \quad (11)$$

Therefore, N_i^2 and N_j^2 can be computed quickly with equation (11) . Accurate analysis in section 4 will show that fast non-local algorithm is much faster than the original one.

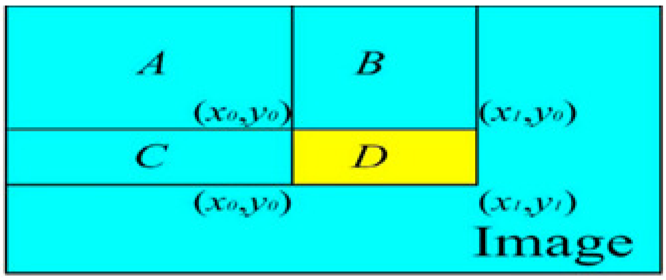

Figure 3. Using SSI to compute the summed squared pixels in the rectangle D

IV. RESULTS AND DISCUSSIONS

The most time-consuming part of the non-local algorithm [5] is the calculation of the Euclidian distance between similar windows in the image. For each pixel in the image, it takes $M^2 \cdot n^2$ square calculations and the whole computational complexity is $M^2 \cdot n^4$ (M^2 denotes the size of similar windows and n^2 represents the number of pixels in the noised image).
It is well known that convolution becomes multiplication under Fourier transform, thus only n^2 multiplications need to be taken for each pixel in the image in fast non-local means algorithm. Thus the total computational complexity is n^4 in fast non-local means algorithm, which is M^2 times faster than the original algorithm.
As suggested in Buades' paper [5], M is set to be 7, and a $21 * 21$ search window is used instead of the whole image. In such simplification, instead of requiring 49 pixel operations when computing the

similarity of two windows in [8], the similarity can be figured out once by FFT and Summed Squares Image (SSI), thus the complexity of fast non-local algorithm is $441 * n^2$. Clearly, comparing to the original non-local means $49 * 441 * n^2$, fast non-local algorithm is about fifty times faster. This makes the performance of fast non-local algorithm acceptable to common users as is demonstrated in Table 1.

Table 1. Performance results.

Image size	Original NL	Fast NL algorithm	Ratio
512*512	28.16 secs	0.35 secs	80.5
1024*768	85.45 secs	1.44 secs	58.6
2592*1944	551.1 secs	9.55 secs	57.7

Fast non-local denoising results are comparable to that of the original non-local algorithm in terms of mean squared error (MSE). Table 2 compares the mean squared error (MSE) for different standard deviation of the added noise between Fast non-local means method and the original non-local means algorithm.

Table 2. MSE comparison.

Stand deviation σ	5	10	15	20	25
Fast NL Method	14.6	32	55	81	110
Original NL means method	12	30	52	68	106

Finally in figure 4-5, we present some qualitative results obtained by simulation using Matlab 7.0[20][21]. Figure 4 shows a original and noisy image and its nice reconstruction by Non-Local Mans and Fast Non-Local Means algorithms.

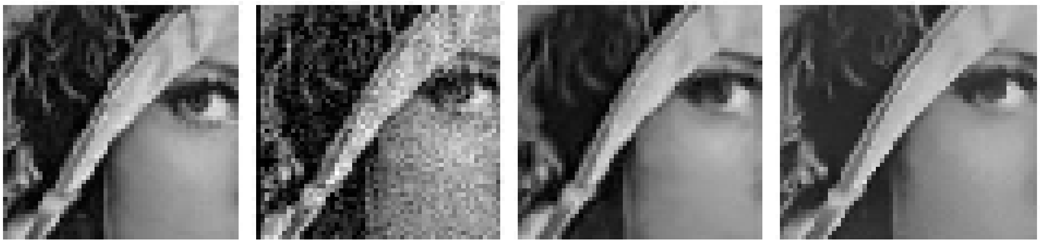

Figure 4. Comparison between non-local method and fast non-local denoising method result. From left to right, original image, noised image, result of non-local (MSE 60) and result of fast non-local (MSE 72)

The aim of figure 5 is to compare the quality of denoised image obtained by Non-Local Mans and Fast Non-Local Means algorithms by visual infection. Figure 6 shows the Fast Non-Local Means algorithm result of noisy Lina with standard deviation 25. All simulation were carried out using Matlab 7.0[20].

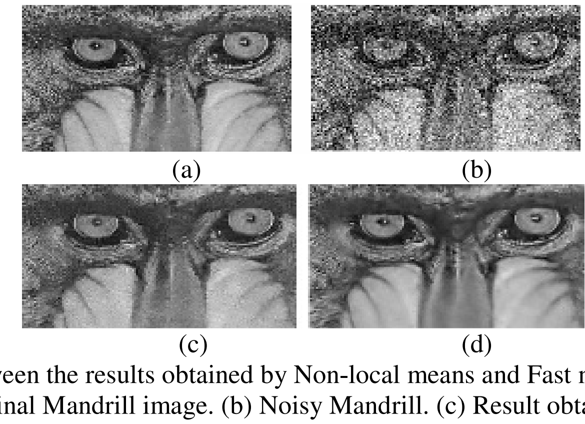

Figure 5. Comparison between the results obtained by Non-local means and Fast non-local means algorithm for noisy Mandrill. (a) Original Mandrill image. (b) Noisy Mandrill. (c) Result obtained by Non-local means algorithm. (d) Result by Fast non-local means algorithm.

Figure 6. Fast non- local means denoising result of noised Lena (standard deviation 25)

V. CONCLUSIONS

In this paper, Fast non-local denoising algorithm [4][19] is compared to original non-local means algorithm[5]. Then an in-depth talk about the non-local means algorithm for removing noise from digital image was given. Afterward a overview of how fast non-local means algorithm denoised the digtal image using summed square image[14] and fast Fourier transform technique was presented. Then based on simulation results obtained by Matlab 7.0[20] and in terms of MSE and efficiency, a comparison between non-local means and fast non-local means algorithms was presented for image denoising. Using these comparisons it was proven that the efficiency of fast non-local means denoising algorithm is about fifty times of the original NL algorithm and the denoising results are still comparable to the results of the original algorithm both in MSE and perception. Thus fast non-local means denoising algorithm is feasible to tackle with practical problems.

VI. FUTURE WORK

In future the Matlab code can be converted to VHDL code and implemented on FPGA kit in order to develop ASIC (application specific IC) for image transformation and analysis. ASIC can be made for doing the specific work of image denoising , so a person who do not know any algorithm for image denoising are also capable of doing it.

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