LINEAR BIVARIATE SPLINES BASED IMAGE RECONSTRUCTION USING ADAPTIVE R-TREE SEGMENTATION

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ABSTRACT

This paper presents a novel method of image reconstruction using Adaptive R-tree based segmentation and Linear Bivariate Splines. A combination of Canny and Sobel edge detection techniques is used for the selection of Significant Pixels. Significant pixels representing the strong edges are then stored in an adaptive R-tree to enhance and improve image reconstruction. The image set can be encapsulated in a bounding box which contains the connected parts of the edges found using edge-detection techniques. Image reconstruction is done based on the approximation of image regarded as a function, by a linear spline over adapted Delaunay triangulation. The proposed method is compared with some of the existing image reconstruction spline models

KEYWORDS: *Adaptive R-tree, Image Reconstruction, Delaunay Triangulations, Linear Bivariate splines*

I. INTRODUCTION

Image reconstruction using regular and irregular samples have been developed by many researchers recently. Siddavatam Rajesh et. al. [1] has developed a fast progressive image sampling using B-splines. Eldar et. al [2] has developed image sampling of significant samples using the farthest point strategy. Muthuvel Arigovindan [3] developed Variational image reconstruction from arbitrarily spaced samples giving a fast multiresolution spline solution. Carlos Vazquez et al, [4] has proposed interactive algorithm to reconstruct an image from non-uniform samples obtained as a result of geometric transformation using filters. Cohen and Matei [5] developed edge adapted multiscale transform method to represent the images. Strohmer [7] developed a computationally attractive reconstruction of bandlimited images from irregular samples. Aldroubi and Grochenig, [9] have developed non-uniform sampling and reconstruction in shift invariant spaces.

Delaunay triangulation [10] has been extensively used for generation of image from irregular data points. The image is reconstructed either by linear or cubic splines over Delaunay Triangulations of adaptively chosen set of significant points. This paper concerns with triangulation of an image using standard gradient edge detection techniques and reconstruction using bivariate splines from adaptive R-tree segmentation. The reconstruction is done based on the approximation of image regarded as function, by a linear spline over adapted Delaunay triangulation. The reconstruction algorithm deals with generating Delaunay triangulations of scattered image points, obtained by detection of edges using Sobel and Canny edge detection algorithms.

Section 3 describes the significant pixel selection method here we used Sobel and canny edge detection and delaunay triangulation. In Section 4 the modeling of the 2D images using the Linear Bivariate splines is elaborated. The linear spline is bivariate and continuous function which can be evaluated at any point in the rectangular image domain in particular for non uniform set of significant samples. Section 5 deals with Adaptive R-tree based Segmentation. The edges so found are not fully connected owing to the various kinds of masks applied. The connectivity of the edges changes according to the mask applied. and the proposed novel reconstruction algorithm is discussed in Section 6. Algorithm Complexity has been stated in the Section 7. Section 8 presents the Significant Performance Measures. The experimental results and conclusion by using the proposed method are discussed in Section 9.

II. RELATED WORK

To perform image reconstruction, the significant pixels need to be found. For this image segmentation is performed[13]. Image segmentation can be considered as a cluster procedure in feature space. Each cluster can be encapsulated in a bounding box which contains the connected parts of the edges found using edge-detection techniques like canny, sobel or a combination of both. The boxes can further be stored in R-trees using suitable child – parent relationship[14]. The R-tree was proposed by Antonin Guttman in 1984[15] and has found significant use in both research and real-world applications[16]. We will explore to use a combination of canny and sobel edge detection techniques and then store the edges in R-tree to perform image segmentation. Image segmentation is a process of grouping an image into homogenous regions with respect to one or more characteristics. It is the first step in image analysis and pattern recognition which has been extensively studied for a few decades due to its applications in computer vision such as: medical imaging (locate tumor), Object detection in satellite image, face/fingerprint recognition, traffic monitoring, online image search engine etc. Image segmentation responsible for extracting semantic foreground objects[17] correctly from a given image, the performance of the subsequent image analysis procedures like retrieval will strongly dependent on the quality of the segmentation.

III. SIGNIFICANT PIXEL SELECTION

We use algorithm proposed by Rajesh Siddavatam et.al [8] which involves following steps : Let M be a mXn matrix representing a grayscale image

2.1. Initialization

X=0 - Matrix representing the x coordinates of the points used for triangulation
Y=0 - Matrix representing the y coordinates of the points used for triangulation
count - Represents the number of points obtained for triangulation
Xs Data Set (Sobel Filter)
Xc Data Set (Canny Filter)

2.2 Edge detection using Sobel and Canny filters.

Figure 1. Edge Detection 1- Sobel

Figure 2. Edge Detection 2 – Canny

2.3 Algorithm 1: Significant Points for Strong edges

Input: Original Lena Image I(x,y)
Step 1: for k=1, 3, 5, 7.....................2n-1
Step 2: Locate a point P(x,y) such that
Step 3: P (x,y) ∈ Xs,
Step 4: Add P(x,y) to matrices X and Y
Step 5: count = count+1
Step 6: end
Output: I(X, Y) ∈ Xs

2.4 Algorithm 2: Significant Points for Weak edges

Input: I (X, Y)
Step 1: for k= 1, 4,7,11.....................3n-2
Step 2: Locate a point P(x,y) such that
Step 3: P(x,y) ∈ Xc and P(x,y) ∈ Xs
Step 4: Add P(x,y) to matrices X and Y
Step 5: count = count+1
Step 6: end
Output: I(X, Y) ∈ Xc ∪ Xs

Figure 3. Lena with 4096 sample points

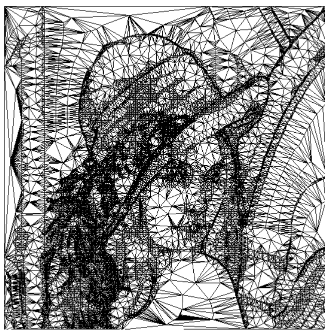

Figure 4. Lena Triangulation for most significant 4096 Sample points

2.5 Overview of Delaunay Triangulation

In the Delaunay triangulation method [11], the location of the global nodes defining the triangle vertices and then produce the elements by mapping global nodes to element nodes. Element definition from a given global set can be done by the method of Delaunay Triangulation. The discretization domain is divided into polygons, subject to the condition that each polygon contains only on global node , and the distance of an arbitrary point inside a polygon from the native global node is smaller than the distance from any other node . The sides of the polygon thus produced are perpendicular bisectors of the straight segments connecting pairs of nodes .

2.6 Delaunay Triangulation (First pass)

To further improve the triangulations, in every triangle a point is inserted at the centroid of the triangle and triangles are formed including that point. This algorithm is useful for even those images having low gradient at the edges or weak edges.
TRI=Delaunay triangulation for data points (X,Y)

2.7 Retriangulation Algorithm

Input: TRI(X, Y)
Step 1: T=Dataset (TRI)
Step 2: for m=1, 2, 3, 4,5,6,7....................................N
Step3: C(x,y)=Centroid of Triangle TN
Step4: add C(x,y) to data set (X,Y)
Step 5: count = count+1
Step 6: end
Step 7 : TRI = delaunay(X,Y)
Output: Updated TRI(X, Y)

IV. LINEAR BIVARIATE SPLINES

The Linear Bivariate Splines are used very recently by Laurent Demaret et al [6]. The image is viewed as a sum of linear bivariate splines over the Delaunay triangulation of a small recursively chosen non uniform set of significant samples Sk from a total set of samples in an image denoted as Sn. The linear spline is bivariate and continuous function which can be evaluated at any point in the rectangular image domain in particular for non uniform set of significant samples denoted as Sk from a total set of samples in an image denoted as Sn. If we denote Ω as the space of linear bivariate polynomials, for the above set Sk Sn , the linear spline space ΩL, containing all continuous functions over the convex hull of Sk denoted as [Sk].
Definition: If for any triangle where T(Sk) is the delaunay triangulation of Sk is in Ω defined as

$$\Omega_L = \{x : x \in [S^k]\} \forall \Delta \in T(S^k) \mid x \in \Omega\} \quad (1)$$

then any element in ΩL is referred to as a linear spline over T(Sk). For a given luminance values at the points of S, {I(y): y S} there is a unique linear spline interpolant L(S, I) which gives

$$\mathrm{L(S, I)(y) = I(y)} \ \ \forall\ \mathrm{y} \in \mathrm{S} \quad (2)$$

where I(y) denotes the image I with y samples that belong to S. Using the above bivariate splines and the concept of Significant Sample point selection algorithm discussed above the original image can be approximated and the reconstruction of the image can be done as per the algorithm given below.

V. ADAPTIVE R-TREE BASED SEGMENTATION

Using Canny- Sobel mix edge detection technique we have found the edges of the image. The edges so found are not fully connected owing to the various kinds of masks applied. The connectivity of the edges changes according to the mask applied. Thus each connected edge is encapsulated in a bounding box of the least possible size. Hence the 2D image is spatially segmented into a set of bounding boxes each with varying dimensions up to the size of the image as shown in Figure 5, based on usual R-tree segmentation.

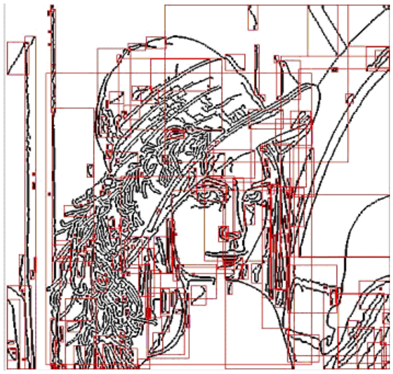

Figure 5. R-tree segmentation of Lena

The usual R-tree approach has a major fault. It gives us much more bounding boxes than that are required. As we are going to follow random sampling to find vertices for Delaunay triangulation apart from the significant pixels from significant pixel selection algorithm of section 2.The R-Tree approach gives us two types of random pixels. One type which is part of the high density edges and others which are located in isolated edges depending on the test image. In case of normal random sampling we will get approximately less no of pixels (or vertices) for triangulation in the isolated regions resulting in haziness near the isolated edges.
To avoid the same, we can take two types of pixels for efficient reconstruction:

4.1 Non-uniform pixels

These pixels are derived by randomly selecting a fixed number of pixels from the image edges (mixture of canny and sobel) edges, the same are responsible for uniform reconstruction throughout the image due to presence of many vertices in the high edge density region. Some of the random samples are also from the isolated significant edges.

4.2 Isolated Edge pixels

These are the pixels from the isolated edges which will now be permanent in order to get better reconstruction due to higher number of Delaunay triangulations in isolated regions. The area of each bounding box is tested against the threshold value between 100 to 1000 square pixels. If the area of the concerned bounding box is greater than the threshold value, then it is treated as a normally significant edge. The segmentation algorithm is run again on the significant edged image to give the bounding boxes encapsulating only the normally significant edges and the pixels from the smaller bounding boxes in the isolated region are made permanent in order to give high density for efficient triangulation. Thus we get only the normally significant edges and the highly significant edges on which further calculations are done. These significant edges are stored in an adaptive R-tree as shown in the Figure 6, of Lena on which the reconstruction algorithm is implemented.

4.3 R tree to Adaptive R-tree algorithm

1. $\forall P(x,y) \in Xs \ \cup Xc)$

Obtain set of edge pixels of image(canny+sobel), Xs & Xc
2.For all the pixels, Wherever connectivity breaks a encapsulating bounding box is drawn for the corresponding edge.
3.Compute area of bounding boxes;
Input: Updated TRI(X, Y)
if $\forall P(x,y) \notin Xs \ \cup Xc)$

draw a BBox

for m=1, 2, 3, 4,5,6,7....................................N
compute area of $BBox_m$
if $area_m$ > threshold (a set minimum value)
add P(x,y) to matrices X3, Y3
end
TRI = delaunay(X3,Y3)
Output: Updated TRI(X, Y)

Enclosed edge is marked as highly significant.

4. Store the highly significant edge pixels for triangulation and remove them from the image.
5. This will remove pixel overlapping while doing random sampling.
6. Redraw the bounding boxes to make adaptive r tree.

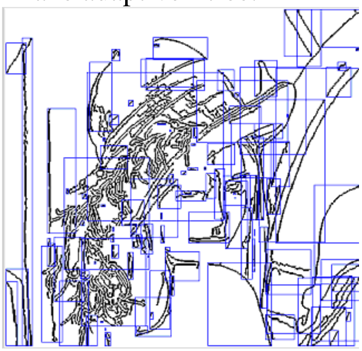

Figure 6. Adaptive R-tree segmentation of Lena

VI. RECONSTRUCTION ALGORITHM

The following steps are used to reconstruct the original image from set of pixels from Significant pixel selection algorithm of section 2 defined as significant (Sig) and isolated significant pixels defined as (Iso-sig) from adaptive R-tree Algorithm of section 4. Figure 15 shows the Flowchart of the proposed algorithm.

5.1 Input

1. Let S_N = data set
2. z_O: luminance
3. S_O: set of regular data for initial triangulation

Step1. Use Significant pixel selection algorithm to find a set of new significant pixels (SP)
Step2: Add adaptive R-tree pixels set to the above set.
Step3. Use Delaunay triangulation and Linear Bivariate Splines to produce unique set of triangles and image.
Step4. Get SIG = sig + Iso-sig
Step5. Repeat steps 1 to 3 to get the image IR (y)
Step6. Return SIG and IR (y)

5.2 Output:

SIG and Reconstructed Image IR (y)

VII. ALGORITHM COMPLEXITY

In general, the complexity of the non-symmetric filter is proportional to the dimension of the filter n2, where n * n is the size of the convolution kernel. In canny edge detection, the filter is Gaussian which is symmetric and separable. For such cases the complexity is given by n+1 [12]. All gradient based algorithms like Sobel do have complexity of O(n). The complexity of well known Delaunay algorithm in worst case is O(n^ceil(d/2)) and for well distributed point set is ~ O(n). N is number of points and d is the dimension. So in 2D, Delaunay complexity is O (N) is any case.

Step 1: Sobel Edge Detector: O(n)
Step 2: Canny Edge Detector: O(n)
Step 3: Filtering (rangefilt) : O(n)
Step 4: Delaunay triangulation: O(n)
Step 5: Retriangulation: O(3n+2)=O(n)
Step 6: Adative R-Tree based segmentation: O(4n+2)= O(2n+1)=O(n)
Step 7: Image Reconstruction: O(n)
Hence the total complexity of the proposed algorithm is O(n) which is quite fast and optimal.

VIII. SIGNIFICANCE MEASURES

Peak Signal to Noise Ratio

A well-known quality measure for the evaluation of image reconstruction schemes is the Peak Signal to Noise Ratio (PSNR),

$$PSNR = 20 * \log 10\ (b / RMS) \qquad (3)$$

where b is the largest possible value of the signal and RMS is the root mean square difference between the original and reconstructed images. PSNR is an equivalent measure to the reciprocal of the mean square error. The PSNR is expressed in dB (decibels). The popularity of PSNR as a measure of image distortion derives partly from the ease with which it may be calculated, and partly from the tractability of linear optimization problems involving squared error metrics.

IX. RESULTS

All the coding is done using MATLAB. The original image and its reconstruction results along with the error image are shown for LENA and PEPPERS images. The reason for a better PSNR = 30.78 dB for our proposed method as shown in Table 1 is due to the fact that the missing/isolated edges in Figure 6, are due to adaptive R-tree algorithm of section 4, and these pixel sets will now participate in greater majority in reconstruction than the normal random pixels from pixel selection algorithm of section 2. The proposed reconstruction algorithm is compared with the Progressive Image Sampling [1] and Farthest Sampling Point Selection FPS reconstructions [2] and it is found that the proposed reconstruction algorithm is much superior in visual quality to the adaptive FPS based reconstructions of [2] considering the same number of 4096 sample points. Also from Table 1, we can say that our method is quite competitive with the other existing methods. Table 2 shows the PSNR of different images

Figure 7. Original Lena Image

Figure 8. Adaptive FPS[2] 4096 sample points
(PSNR = 18.08 dB)

Figure 9. Reconstructed Lena Image 4096 samples
(PSNR = 29.22 dB)

Figure 10. Reconstructed Lena Image 4096 non-uniform samples
(PSNR = 30.78 dB)

Figure 11. R-tree Segmentation of Peppers Image

Figure 12. Adaptive R-tree of Peppers

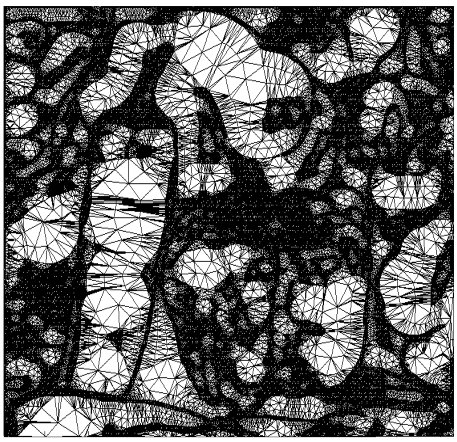

Figure 13. Triangulation of Peppers

Figure 14. Reconstructed Peppers (PSNR = 29.89 dB)

Table 1: Comparative evaluation of our new method

Test Case	Method	PSNR (dB)
Lena 512x512	Proposed Adaptive R-Tree	**30.78**
	Significant pixel selection [8]	29.22
	Progressive Image Sampling [1]	21.45
	Farthest Point Sampling(FPS)[2]	18.08
Peppers 512x512	Proposed Adaptive R-tree	**29.89**
	Significant pixel selection [8]	29.01
	Progressive Image Sampling [1]	22.06
	Farthest Point Sampling(FPS)[2]	18.18

Table 2: PSNR of Different Images

Image	PSNR(dB)
Lena	**30.78**
Peppers	**29.89**
Bird	**28.72**
Fruits	**29.92**
Goldhill	**28.82**
Mandrill	**28.94**
Club House	**28.68**

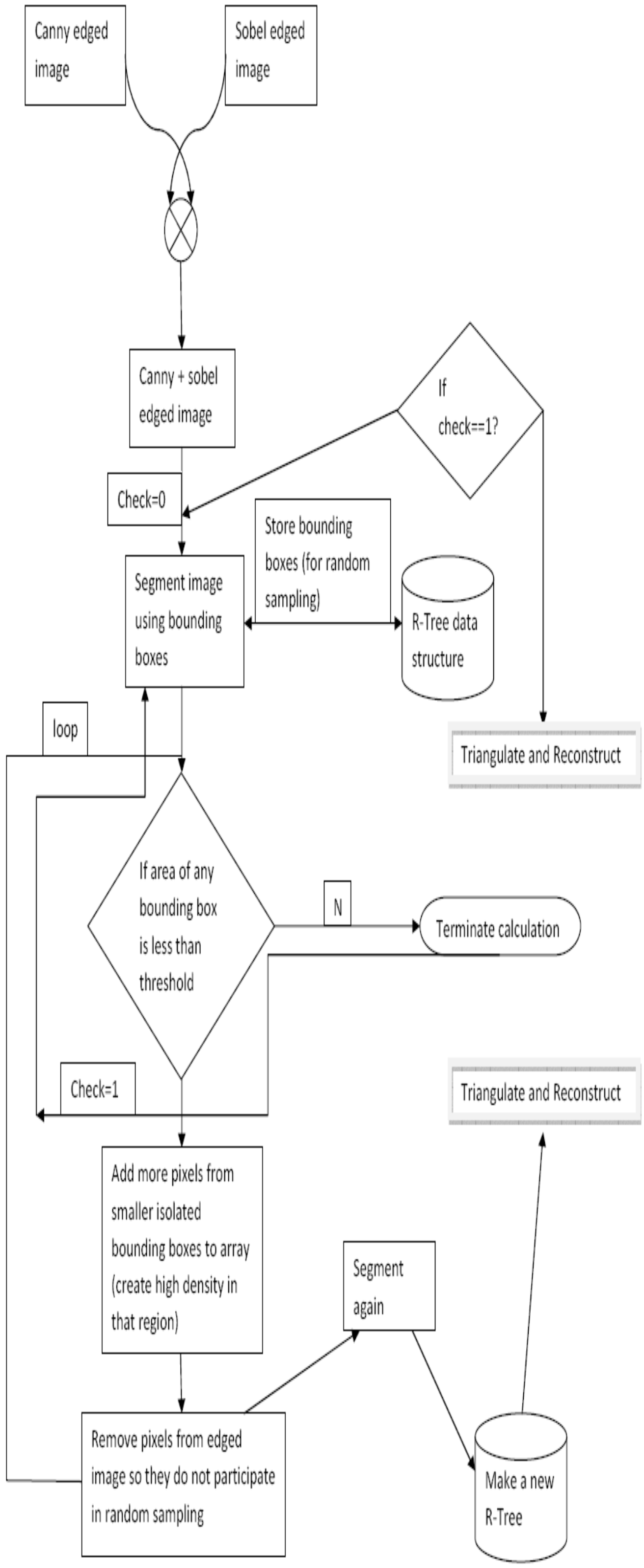

Figure 15. Flowchart of the Proposed Algorithm

X. CONCLUSION

In this paper, a novel algorithm based on Adaptive R-tree based significant pixel selection is applied for image reconstruction. Experimental results on the popular images of Lena and Peppers are presented to show the efficiency of the method. Set of regular points are selected using Canny and

Sobel edge detection and Delaunay triangulation method is applied to create triangulated network. The set of significant sample pixels are obtained and added in the preceding set of significant pixels samples at every iteration. The gray level of each sample point is interpolated from the luminance values of neighbour significant sample point.

XI. FUTURE SCOPE

The proposed algorithm is native for image reconstruction and can be further used for large images for progressive image transmission. As with the help of segmentation, the Region of Interest (ROI) can be easily find out. This can help the large images to transmit progressively.

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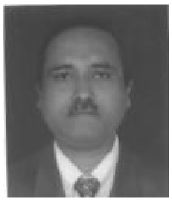

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