AN INVENTORY MODEL FOR INFLATION INDUCED DEMAND AND WEIBULL DETERIORATING ITEMS

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ABSTRACT

The objective of this model is to investigate the inventory system for perishable items under inflationary conditions where the demand rate is a function of inflation and two parameter Weibull distribution for deterioration is considered. The Economic order quantity is determined for minimizing the average total cost per unit time under the influence of inflation and time value of money. Here the deterioration starts after a fixed time interval. The influences of inflation and time-value of money on the inventory system are investigated with the help of some numerical examples.

KEYWORDS: *Inventory system, Inflation, Deterioration, Weibull distribution.*

AMS Classification No: $90B05$

I. INTRODUCTION

From a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. One of the important problems faced in inventory management is how to maintain and control the inventories of deteriorating items. Deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage etc. that result in decrease of usefulness of the original one. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) deteriorate remarkably overtime.

Whitin [12] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and Scharder [5] developed an EOQ model with an exponential decay and a deterministic demand. Thereafter, Covert and Philip [3] and Philip [7] extended EOQ models for deterioration which follows Weibull distribution. Wee [10] developed EOQ models to allow deterioration and an exponential demand pattern. In last two decades the economic situation of most countries have changed to an extent due to sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of time value of money. Data and Pal [4], Bose et al. [1] have developed the EOQ model incorporating the effects of time value of money, a linear time dependent demand rate. Further Sana [8] considered the money value and inflation in a new level. Wee and Law

(1999) [11] addressed the problem with finite replenishment rate of deteriorating items taking account of time value of money. Chang (2004) [2] proposes an inventory model for deteriorating items under inflation under a situation in which the supplier provides the purchaser a permissible delay in of payment if the purchaser orders a large quantity. Jaggi et al. (2006) [6] developed an inventory model for deteriorating items with inflation induced demand under fully backlogged demand. Thangam et al. (2010) [9] developed an inventory model for deteriorating items with inflation induced demand and exponential backorders.

In some real-life situations there is a part of the demand which cannot be satisfied from the inventory, leaving the system in stock-out. In these systems two situations are mainly considered: customers wait until the arrival of next order (completely backorder case) or customers leave the system (lost sale case).However, in many real inventory systems, some customers are able to wait for the next order to satisfy their demands during the stock-out periods, while the others do not wish or cannot wait and they have to fill their demands from the other sources. This situation is modeled by the consideration of partial backordering in the mathematical formulation of inventory models. Here we present an inventory model with partial backlogging, where the fraction of backlogged demand is a negative exponential function of the waiting time.

In this paper an attempt has been made to develop an inventory model with partial backorders for perishable items with two-parameter Weibull density function for deterioration and the demand rate is increasing exponentially due to inflation over a finite planning horizon. In shortage state during stock out it is assumed that all demands are backlogged or lost. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Optimal solution for the proposed model is derived and we have considered the time-value of money and inflation of each cost parameter.

II. ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

i. Single inventory will be used.
ii. Lead time is zero.
iii. The model is studied when shortages are allowed.
iv. Demand rate is exponentially increasing and is represented by $D(t) = d_0\, e^{i\,t}$ where d_0 is initial demand rate.
v. When shortages are allowed, it is also partially backlogged. The backlogging rate is variable and depends on the length of waiting time for the next replacement. The backlogging rate is assumed to be $\dfrac{1}{1+\delta(T-t)}$ where δ is the non negative constant backlogging parameter.
vi. Replenishment rate is infinite but size is finite.
vii. Time horizon is finite.
viii. There is no repair of deteriorated items occurring during the cycle.
ix. Deterioration occurs when the item is effectively in stock.
x. The time-value of money and inflation are considered.
xi. The second and higher powers of α and δ are neglected in this analysis of the model hereafter.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time t, $t \geq 0$.

$D(t) = d_0\, e^{i\,t}$ is the demand rate at time t.

$\theta : \theta = \alpha\beta\, t^{\beta-1}$, The two-parameter Weibull distribution deterioration rate (unit/unit time). Where $0 < \alpha << 1$ is called the scale parameter, $\beta > 0$ is the shape parameter.

Q = Total amount of replenishment in the beginning of each cycle.

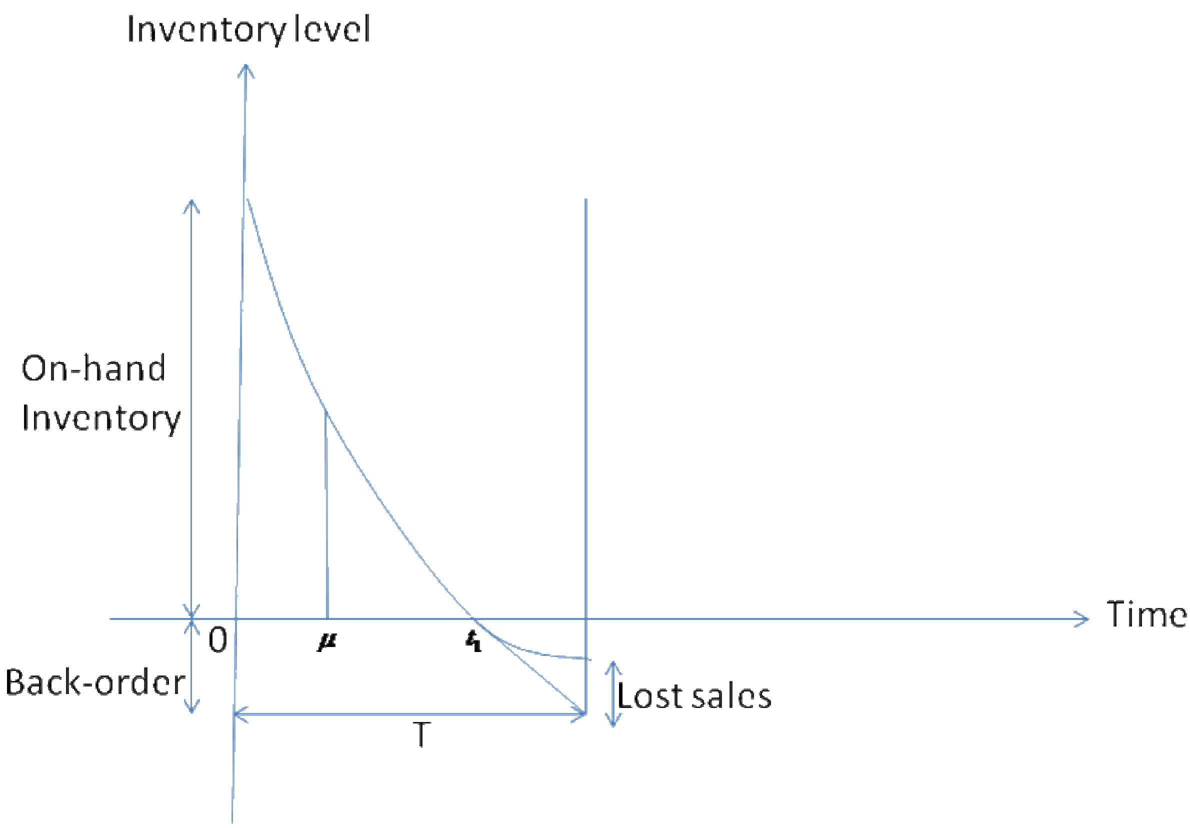
S = Inventory at time $t = 0$

T = Duration of a cycle.

μ =The life-time of items.
i = The inflation rate per unit time.
r = The discount rate representing the time value of money.
c_p = The purchasing cost per unit item.
c_d = The deterioration cost per unit item.
c_h = The holding cost per unit item.
c_0 = The opportunity cost due to lost sales per unit.
c_b = The shortage cost per unit.
K =The total average cost of the system.

III. FORMULATION

Let Q be the total amount of replenishment in the beginning of each cycle and after fulfilling backorders let S be the level of initial inventory. The objective of the model is to determine the optimal order quantity in order to keep the total relevant cost as low as possible. The optimality is determined for shortage of items. In the period $(0, \mu)$ the inventory level decreases due to market demand only but during the period (μ, t_1) the inventory stock further decreases due to combined effect of deterioration and demand. At t_1, the level of inventory reaches zero and after that the shortages are allowed to occur during the interval $[t_1, T]$. Here part of shortages is backlogged and part of it is lost sale. Only the backlogging items are replaced by the next replenishment. The behavior of inventory during the period $(0, T)$ is depicted in the following inventory-time diagram.



Here we have taken the total duration T as fixed constant. The objective here is to determine the optimal order quantity in order to keep the total relevant cost as low as possible.

If $I(t)$ be the on-hand inventory at time $t \geq 0$, then at time $t+\Delta t$, the on-hand inventory in the interval $[0, \mu]$ will be

$I(t+\Delta t) = I(t) - D(t)\,\Delta t$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.1) $$\frac{dI}{dt} = -d_0\, e^{it}; \quad 0 \leq t \leq \mu$$

For the next interval $[\mu, t_1]$, where the effect of deterioration starts with the presence of demand, i.e.,

$I(t+\Delta t) = I(t) - \theta(t)\, I(t)\,\Delta t - D(t)\Delta t$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.2) $$\frac{dI(t)}{dt} + \alpha\beta\, t^{\beta-1}\, I(t) = -d_0\, e^{it}; \quad \mu \leq t \leq t_1$$

Finally in the interval $[t_1, T]$, where the shortages are allowed

$$I(t+\Delta t) = I(t) - \frac{D(t)}{1+\delta(T-t)} \cdot \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.3) $$\frac{dI(t)}{dt} = -\frac{d_0 e^{it}}{1+\delta(T-t)}; t_1 \leq t \leq T$$

The boundary conditions are $I(0) = S$ and $I(t_1) = 0$.

On solving equation (3.1) with boundary condition we have

(3.4) $$I(t) = S + \frac{d_0}{i}(1 - e^{it}); \quad 0 \leq t \leq \mu$$

On solving equation (3.2) with boundary condition we have

(3.5) $$I(t) = d_0\, e^{-\alpha t^{\beta}} \left[t_1 - t + \frac{\alpha}{\beta+1}\left\{t_1^{\beta+1} - t^{\beta+1}\right\} + \frac{i}{2}\left\{t_1^2 - t^2\right\} \right]; \quad \mu \leq t \leq t_1$$

On solving equation (3.3) with boundary condition we have

(3.6) $$I(t) = d_0 \left[t_1 - t - \delta T\{t_1 - t\} + \frac{\delta + i}{2}\left\{t_1^2 - t^2\right\} \right]; \quad t_1 \leq t \leq T$$

Now the initial inventory is given by,

(3.7) $$S = d_0\, e^{-\alpha \mu^{\beta}} \left[(t_1 - \mu) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1}) + \frac{i}{2}(t_1^2 - \mu^2) \right] + \frac{d_0\,(e^{i\mu} - 1)}{i}$$

The total cost function consists of the following elements if the inflation and time-value of money are considered:

(i) **Purchasing cost per cycle**

(3.8) $$C_p S \int_0^T e^{-(r-i)t}\, dt = \frac{C_p S}{i-r}\left[e^{-(r-i)T} - 1\right]$$

(ii) **Holding cost per cycle**

(3.9) $$C_h \int_0^{\mu} I(t).e^{-(r-i)t}\, dt + C_h \int_{\mu}^{t_1} I(t).e^{-(r-i)t}\, dt$$

$$= C_h \left[S\mu + \frac{S(i-r)}{2}\mu^2 - \frac{d_0\, \mu^2}{2} - \frac{d_0 (i-r)\, \mu^3}{3} \right] + C_h\, d_0 \left[t_1 \left\{ t_1 + \frac{\alpha\, t_1^{\beta+1}}{\beta+1} - \frac{i\, t_1^2}{2} \right\} - \frac{t_1^2}{2} \right.$$

$$\left. - \frac{\alpha}{(\beta+1)(\beta+2)} t_1^{\beta+2} - \frac{i}{6} t_1^3 + \frac{(i-r)\, t_1^3}{6} - \frac{\alpha\, t_1^{\beta+2}}{\beta+1} + \frac{\alpha\, t_1^{\beta+2}}{\beta+2} - \mu \left\{ t_1 + \frac{\alpha\, t_1^{\beta+1}}{\beta+1} - \frac{i\, t_1^2}{2} \right\} + \frac{\mu^2}{2} \right.$$

$$+\frac{\alpha}{(\beta+1)(\beta+2)}\mu^{\beta+2}+\frac{i}{6}\mu^{3}-\frac{(i-r)\,t_1\,\mu^{2}}{2}+\frac{(i-r)\,\mu^{3}}{3}+\frac{\alpha\,t_1\,\mu^{\beta+1}}{\beta+1}-\frac{\alpha\,\mu^{\beta+2}}{\beta+2}\Bigg]$$

(iii) **Deterioration cost per cycle**

(3.10)
$$C_d\int_{\sigma}^{t_1}\alpha\beta\, t^{\beta-1}I(t)\,e^{-(r-i)t}\,dt$$
$$=C_d\ \alpha\beta\ d_0\left[\frac{t_1^{\beta+1}}{\beta(\beta+1)}-\frac{t_1\,\mu^{\beta}}{\beta}+\frac{\mu^{\beta+1}}{\beta+1}\right]$$

(iv) **Shortage cost per cycle**

(3.11)
$$C_b\int_{t_1}^{T}-I(t)e^{-(r-i)}dt$$
$$=-C_b d_0\Bigg[\frac{(i+\delta)t_1^2\,T}{2}+t_1\,T-\delta\,t_1\,T^2-\frac{T^2}{2}+\frac{\delta\,T^3}{2}-\frac{(i+\delta)\,T^3}{6}$$
$$+(i-r)\left\{\frac{t_1\,T^2}{2}-\frac{t_1\,T^3\delta}{2}+\frac{(i+\delta)t_1^2\,T^2}{4}-\frac{T^3}{3}+\frac{\delta\,T^4}{3}-\frac{(i+\delta)\,T^4}{8}\right\}$$
$$-t_1^2+\frac{t_1^2\delta\,T}{2}-\frac{(i+\delta)t_1^3}{3}+\frac{t_1^2}{2}-(i-r)\left\{\frac{t_1^3}{6}-\frac{t_1^3\,T\delta}{6}+\frac{(i+\delta)t_1^4}{8}\right\}\Bigg]$$

(v) **Opportunity cost due to lost sales per cycle**

(3.12)
$$C_0\int_{t_1}^{T}D(t)\left[1-\frac{1}{1+\delta(T-t)}\right]e^{-(r-i)t}dt$$
$$=C_o d_0\delta\left[\frac{T^2}{2}+\frac{t_1^2}{2}-T\,t_1+\frac{(2i-r)T^3}{6}-(2i-r)\left\{\frac{T\,t_1^2}{2}-\frac{t_1^3}{3}\right\}\right]$$

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

(3.13)
$$K(t_1)=\frac{1}{T}\{\text{Purchasing cost + Holding cost + Deterioration cost}$$
$$\text{+Shortage cost + Opportunity cost}\}$$

$$=\frac{1}{T}\Bigg[\frac{C_p S}{i-r}\left[e^{-(r-i)T}-1\right]+C_d\ \alpha\beta\ d_0\left[\frac{t_1^{\beta+1}}{\beta(\beta+1)}-\frac{t_1\,\mu^{\beta}}{\beta}+\frac{\mu^{\beta+1}}{\beta+1}\right]$$
$$+C_h\left[S\mu+\frac{S(i-r)}{2}\mu^2-\frac{d_0\,\mu^2}{2}-\frac{d_0(i-r)\,\mu^3}{3}\right]+C_h\ d_0\Bigg[t_1\left\{t_1+\frac{\alpha\,t_1^{\beta+1}}{\beta+1}-\frac{i\,t_1^2}{2}\right\}-\frac{t_1^2}{2}$$
$$-\frac{\alpha}{(\beta+1)(\beta+2)}t_1^{\beta+2}-\frac{i}{6}t_1^3+\frac{(i-r)\,t_1^3}{6}-\frac{\alpha\,t_1^{\beta+2}}{\beta+1}+\frac{\alpha\,t_1^{\beta+2}}{\beta+2}-\mu\left\{t_1+\frac{\alpha\,t_1^{\beta+1}}{\beta+1}-\frac{i\,t_1^2}{2}\right\}+\frac{\mu^2}{2}$$
$$+\frac{\alpha}{(\beta+1)(\beta+2)}\mu^{\beta+2}+\frac{i}{6}\mu^3-\frac{(i-r)\,t_1\,\mu^2}{2}+\frac{(i-r)\,\mu^3}{3}+\frac{\alpha\,t_1\,\mu^{\beta+1}}{\beta+1}-\frac{\alpha\,\mu^{\beta+2}}{\beta+2}$$
$$-C_b d_0\Bigg[\frac{(i+\delta)t_1^2\,T}{2}+t_1\,T-\delta\,t_1\,T^2-\frac{T^2}{2}+\frac{\delta\,T^3}{2}-\frac{(i+\delta)\,T^3}{6}$$

$$+(i-r)\left\{\frac{t_1\,T^2}{2}-\frac{t_1\,T^3\delta}{2}+\frac{(i+\delta)t_1^2\,T^2}{4}-\frac{T^3}{3}+\frac{\delta\,T^4}{3}-\frac{(i+\delta)\,T^4}{8}\right\}$$

$$\left.\left.-t_1^2+\frac{t_1^2\delta\,T}{2}-\frac{(i+\delta)t_1^3}{3}+\frac{t_1^2}{2}-(i-r)\left\{\frac{t_1^3}{6}-\frac{t_1^3\,T\delta}{6}+\frac{(i+\delta)t_1^4}{8}\right\}\right]\right]$$

$$\left.\left.C_o d_0\delta\left[\frac{T^2}{2}+\frac{t_1^2}{2}-T\,t_1+\frac{(2i-r)T^3}{6}-(2i-r)\left\{\frac{T\,t_1^2}{2}-\frac{t_1^3}{3}\right\}\right]\right]\right]$$

Now equation (3.13) can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.13), Matlab Software has been used to determine optimal t_1^* and hence the optimal cost $K(t_1^*)$ can be evaluated. Also level of initial inventory level S^* can be determined.

IV. EXAMPLES

Example- 4.1:
The values of the parameters are considered as follows:
$r=0.12, i=0.05, \alpha=0.001, \beta=2, \delta=0.1, \mu=0, T=1\ year, d_0=50\ units$
$c_h=\$3/unit/year, c_p=\$4/unit, c_d=\$8/unit, c_b=\$12/unit\ c_o=\$5/unit$.
Now using equation (3.13) which can be minimized to determine optimal $t_1^*=0.5313\ year$ and hence the average optimal cost $K(t_1^*)=\$\,189.074/unit$.
Also level of initial inventory level $S^*=26.92$ *units*.
Example- 4.2:
The values of the parameters are considered as follows:
$r=0.12, i=0.05, \alpha=0.001, \beta=2, \delta=0.1, \mu=0.3, T=1\ year, d_0=50\ units$
$c_h=\$3/unit/year, c_p=\$4/unit, c_d=\$8/unit, c_b=\$12/unit\ c_o=\$5/unit$.
Now using equation (3.13) which can be minimized to determine optimal $t_1^*=0.5094\ year$ and hence the average optimal cost $K(t_1^*)=\$\,197.597/unit$.
Also level of initial inventory level $S^*=27.27$ *units*.

V. CONCLUSION

Due to high inflation and sharp decline in the purchasing power of money the financial situation has been completely changed and hence we cannot ignore the effect of inflation and time value of money. In this paper, the inventory model has been developed considering both deterioration and inflation of the items with shortages over a finite planning horizon. Two-parameter Weibull distribution for deterioration is used. The model is studied for minimization of total average cost under the influence of inflation and time-value of money. Numerical examples are used to illustrate the result.

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