EIGEN VALUES OF SOME CLASS OF STRUCTURAL MATRICES THAT SHIFT ALONG THE GERSCHGORIN CIRCLE ON THE REAL AXIS

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ABSTRACT

In this paper, we have presented a simple approach for determining eigenvalues for some class of structural matrices. It has been shown that if all the principle diagonal elements of the given structural matrices are increased by $\pm\varepsilon$, it is as good as the Gerschgorin circle drawn for the given matrix is shifted by $\pm\varepsilon$ amount with respect to the origin. The main advantage of the proposed method is that there is need to use time-consuming iterative numerical technique for determining the eigenvalues. The proposed approach is expected to be applicable in various computer sciences like Pattern Recognition, Face Recognition identification of geometrical figures and also in control system application for obtaining the stability of the system.

KEYWORDS: *Eigenvalues, Gerschgorin theorem, structural matrices, trace of the matrix.*

I. INTRODUCTION

The concept of stability plays very important role in the analysis of systems. A system can be modeled in the state space form [1]. In this state space form, stability can be determined by computing the eigenvalues of the system matrix A. There exist several methods in the literature for the computation of eigenvalues [2, 3]. Moreover, in the engineering applications, some structural matrices have been used and their eigenvalues computations are also important. In the mathematical literature, we found that there exists Gerschgorin theorem [4-6], which gives the bounds under which, all eigenvalues lie. Now a day's eigenvalues can be calculated easily using MATLAB. But, we found that the proposed method computes eigenvalues without involving iterative numerical technique. In this paper a simple formulae has been derived that helps in the computation of the eigenvalues, which is faster than the MATLAB for the class of structural matrices.

II. GERSCHGORIN THEOREM [4-6]

For a given matrix A of order (nxn), let P_k be the sum of the moduli of the elements along the kth row excluding the diagonal elements a_{kk}. Then every eigenvalues of A lies inside the boundary of at least one of the circles.

$$\left|\lambda - a_{kk}\right| = P_k \tag{1}$$

III. DETERMINATION OF EIGENVALUES OF THE STRUCTURAL MATRICES

For the given matrix [A] of the form

$$A = \begin{bmatrix} a & -b & -b & \cdots & -b \\ -b & a & -b & \cdots & -b \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -b & -b & -b & \cdots & a \end{bmatrix} \quad (2)$$

where, $a > 0, b > 0, a = |(n-1)b|$ (3)

The Gerschgorin's circle of the above matrix are given below

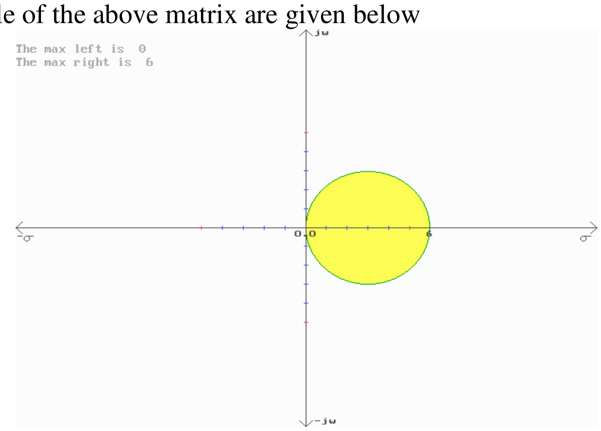

FIG (1):- GERSCHGORIN'S BOUND $[0,\ (n-1)\,a]$

Eigenvalues of the above matrix are

$$\lambda = 0,\ \lambda = \underbrace{(a+b),\ (a+b),\ \ldots,\ (a+b)}_{(n-1)\ \text{times}} \quad (4)$$

Step (1):-In the above matrix [A] if all the principle diagonal elements are changed by ε we obtain the following matrix [B] as

$$[B] = \begin{bmatrix} (a+\varepsilon) & -b & -b & \cdots & -b \\ -b & (a+\varepsilon) & -b & \cdots & -b \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -b & -b & -b & \cdots & (a+\varepsilon) \end{bmatrix} \quad (5)$$

such that

$$(a+\varepsilon) > b,\ a > 0,\ b > 0 \text{ and } a = |(n-1)\,b| \quad (6)$$

Gerschgorin's circle of the above matrix is

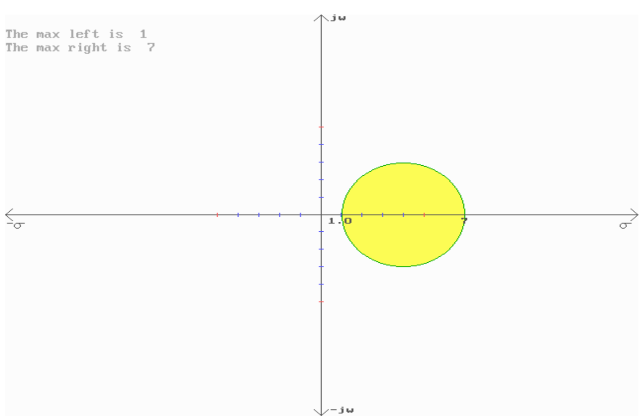

Fig (2):- Gerschgorin bound $[\varepsilon,\ (n-1)\,a - \varepsilon]$

Applying above Gerschgorin theorem to the above matrix [B] , we get

$$\left|\lambda - a_{ii}\right| \le \sum_{\substack{i=1 \\ i\ne j}}^{n} \left|a_{ij}\right| = r_j \quad (7)$$

In the matrix [A] replace

$$\text{aii} = (a+\varepsilon) \quad r_j = |\,(n-1)\ b| \quad (8)$$

From eq. (4) and eq. (5) we get

$$|\,\lambda - (a+\varepsilon) \ \le |\,(n-1)b| \quad (9)$$

$$|\,\lambda - (a+\varepsilon)\ | \le a \quad (10)$$

By removing modulus of the above equation, we get

$$\pm\ (\lambda - (a+\varepsilon)) \le a \quad (11)$$

So $(\lambda - (a+\varepsilon)) \le a$ (12)

or $-(\lambda - (a+\varepsilon)) \le a$ (13)

Now consider the equation (13)

$$-\ (\lambda - (a+\varepsilon)) \le a \quad (14)$$

$$-\ \lambda - \le \varepsilon \quad (15)$$

$$\lambda \ \le \varepsilon$$

Here $\lambda < \varepsilon$ is rejected since Gerschgorin's bound are positive. So, from eq.(15) , ε is one of the eigenvalues of the structural matrix . Thus for the above matrix [B] one of the eigenvalues is at ε . Let us consider one of the eigenvalues $\lambda_n = \varepsilon$. Now, we calculate remaining eigenvalues which is given below in the following steps

Applying Gerschgorin theorem to the matrix [B], we get

$$|\,\lambda_1 - (a+\varepsilon) \quad | \le |\ (n-1)\ b| \quad (16)$$

$$|\,\lambda_2 - (a+\varepsilon) \quad | \le |\ (n-1)\ b| \quad (17)$$

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$$|\,\lambda_{n-1} - (a+\varepsilon) \quad | \le |\ (n-1)\ b| \quad (18)$$

By subtracting eq,(16) from eq,(18) we get ,

$$|\,\lambda_1 - \lambda_2| \le \ 0 \quad (19)$$

In the above eq. (18), $|\,\lambda_1 - \lambda_2 \ | < 0$ is rejected, since the absolute value of any number is always positive. Thus, we get, $|\,\lambda_1 - \lambda_2 \ | = 0$ i.e., $\lambda_1 = \lambda_2$ (20)

Similarly from subtracting remaining equations we can show that

$\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_{n-1} = k$ where, k is the repeated eigenvalues

. **Step 2:** Using definitions of trace of the matrix, i.e.,

$$Trace(A) = \sum_{i=1}^{n} a_{ii} \quad (21)$$

$$Trace(A) = \sum_{i=1}^{n} \lambda_i \quad (22)$$

We calculate

$$Trace(A) = na \quad (23)$$

$$n(a+\varepsilon) = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{n-1} + \lambda_n \quad (24)$$

Since, $\lambda_n = \varepsilon$ we get (25)

$$n(a+\varepsilon) = \varepsilon + k(n-1) \quad (26)$$

$$(n(a+\varepsilon) - \varepsilon)/(n-1) = k \quad (27)$$

Thus, the eigenvalues of the system matrix [B] are1 $\varepsilon, k_1, k_2, ..., k_{n-1}$, where $k = k_1 = k_2 = k_{n-1}$

Remark: In matrix $[A]$, if we replace $(a+\varepsilon)$ by $-(a+\varepsilon)$ and $-b$ by b, then eigenvalues for the matrix $[A]$ are $\varepsilon, k_1, k_2, ..., k_{n-1}$ where $k = k_1 = k_2 = ...k_{n-1}$ and

$$-(n(a+\varepsilon)-\varepsilon)/(n-1) = k \tag{28}$$

4. Examples and figures

Consider the matrix [A] as

$$[A] = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}$$

Gerschgorin's circle of the above matrix is

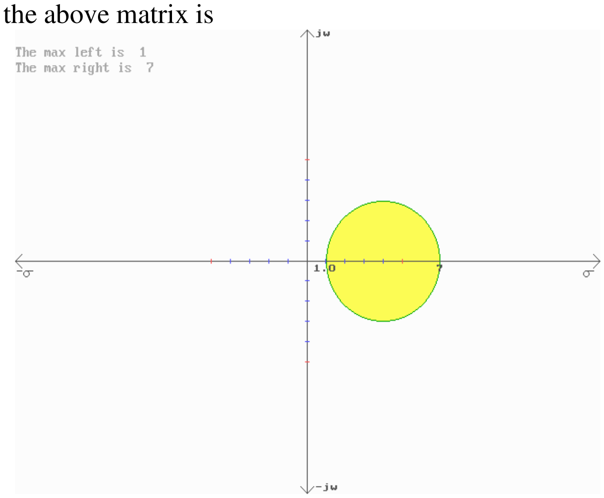

Fig (3): - Gerschgorin bound is [1, 7]

The above structural matrix is similar to matrix as shown in eq (5). In this above matrix, $n = 4$ $(a+\varepsilon) = 4$ $a = 3$ $\varepsilon = 1$. So we have directly determined its eigenvalues. From eq. (25) it has one eigenvalue at ε and the remaining eigenvalues can be calculated using eq.(26) as follows

$k = (4(3+1)-1)/3 = 5$

Thus the eigenvalues of the matrix B are 1 , 5 , 5 , 5.

IV. CONCLUSIONS

In this paper, we have proposed a simple technique for calculating eigenvalues of the structural matrices. It has been observed that as the Gerschgorin's circle move ε distances on the real axis, then one of the eigenvalues will ε for the structural matrix and the other eigenvalues are repeated. The proposed method needs on iterative methods and instead of that a simple formula is derived using Gerschgorin's theorem to calculate the repeated eigenvalues. Computations of eigenvalues have many application in Computer Engineering and the control systems.

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