DAMAGE AND RESIDUAL STRENGTH ASSESSMENT OF STRUCTURES USING MEASURED STATIC DEFLECTIONS

[1]B.Subhan Ramji and [2]P.Venkata Muthyalu
Department of Civil Engineering, Pragati Engineering College, Kakinada. A.P, India

ABSTRACT

Damage occurrence in Civil Engineering Structures such as bridges and buildings is quite inevitable either during its construction or during its life time. Whole structure cannot be discarded just because part of the structure is damaged. Hence, the damaged structure has to be restored by repairing it. Robust schemes are developed and used for damage assessment by many a researcher all over the world. In this investigation, it is attempted to provide an empirical model for damage assessment using simple deflections due to static loading based on fracture mechanics principles.

KEY WORDS: *Crack length, deflection, curvature, Damage, Damage parameter, and residual strength.*

I. INTRODUCTION

Rehabilitation of damaged structure demands determination of location and degree of damage in the structure. Damage assessment is an inverse problem which is non-linear. This is also known as system identification. Robust schemes are developed and used for damage assessment by many a researcher all over the world. A sizeable numbers of efforts have been sought to instrument bridges for structural monitoring and damage assessment, which are expensive and cumbersome. Therefore, as an alternative, it is aimed to investigate and provide a simple damage model with following objectives

1. To determine if a structure is damaged form it's measured static deflections.
2. To determine location of the damage, if damaged, from its static deflections.
3. To quantify the damage from measured static deflections of the structure.
4. To determine the residual strength of the structure.

In this investigation, damage in a structure is modeled as a discrete crack. The present study is carried out in three stages. Finite Element Analysis (Computational work) is done using Fracture Analysis Code for 2 Dimensional problems (FRANC 2D) and damage model; from measured deflections at regular intervals along the beam span; is proposed in the first stage. In the second stage, an empirical model for residual strength assessment is proposed based on fracture mechanics principles. In the third stage, an experimental work is carried out on a simply supported beam with known damage and validated the proposed "Damage Assessment Model".

Finite Element Analysis

In the first stage, a simply supported beam with a concentric point load is modeled using finite element analysis package (FRANC 2D). Deflections in the beam at regular intervals along the span are noted. Deflections, slopes and curvatures along the span are plotted from the noted deflections as shown in the Figure 1.

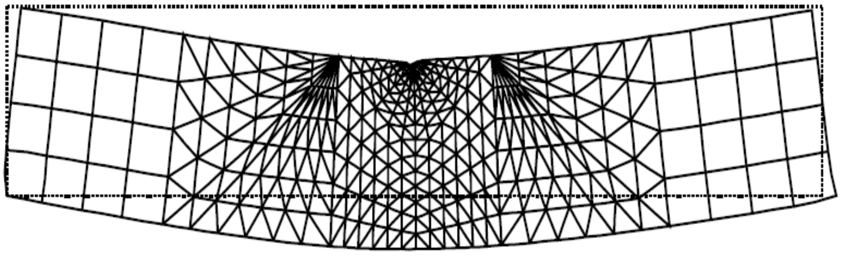

Figure 1: Deflected profile of the undamaged beam

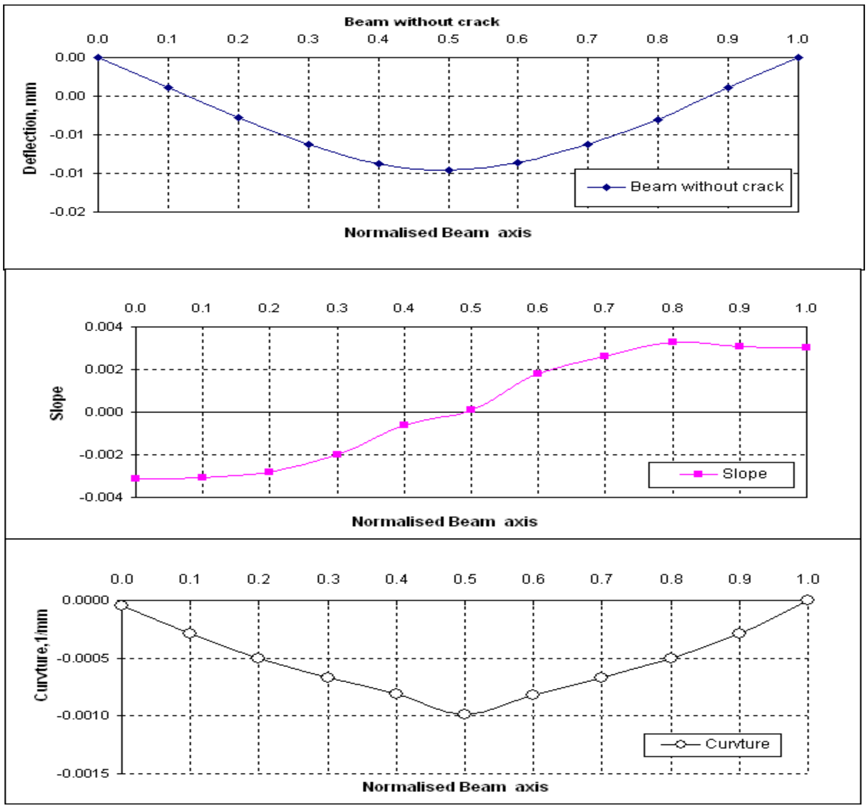

Figure 2: a) Deflected b) Slope and c) Curvature profile of an intact beam

F E Analysis of Beam with Damage (Discrete Crack)

Then a single concentric crack of length a_1 is introduced in the tension zone of the beam at its mid span. Deflections in the cracked beam are noted at the same points where deflections are measured in the beam and tabulated as shown in the Table 1.

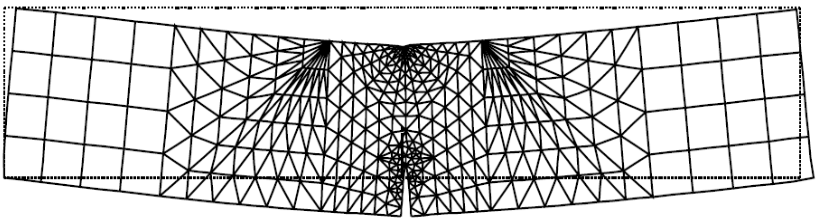

Figure 3: Deflected profile of a damaged beam

Deflections, slopes and curvatures of the cracked beam are plotted. FE Analysis is carried out with increments of crack length in the beam and deflections are noted. Using the noted vertical deflection; slopes and curvatures at the points where deflections are noted; are computed using finite difference method. The slopes and the curvatures along the span are plotted as shown in Figure 1.

Table 1: Deflections in a Damaged Beam with crack at centre

Deflection in a Damaged Beam with crack at centre									
Span/L	**a0**	**a1**	**a2**	**a3**	**a4**	**a5**	**a6**	**a7**	**a8**
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.10	-0.003	-0.003	-0.004	-0.004	-0.005	-0.007	-0.010	-0.017	-0.036
0.20	-0.006	-0.006	-0.007	-0.008	-0.010	-0.014	-0.020	-0.034	-0.072
0.30	-0.009	-0.009	-0.010	-0.012	-0.015	-0.020	-0.030	-0.050	-0.107
0.40	-0.011	-0.012	-0.013	-0.015	-0.019	-0.026	-0.039	-0.066	-0.142
0.48	-0.012	-0.013	-0.015	-0.018	-0.023	-0.031	-0.047	-0.080	-0.175
0.50	-0.012	-0.013	-0.014	-0.017	-0.022	-0.031	-0.046	-0.081	-0.173
0.52	-0.012	-0.013	-0.015	-0.018	-0.023	-0.031	-0.047	-0.080	-0.175
0.60	-0.011	-0.011	-0.013	-0.015	-0.019	-0.026	-0.039	-0.066	-0.141
0.70	-0.009	-0.009	-0.010	-0.012	-0.015	-0.020	-0.030	-0.050	-0.107
0.80	-0.007	-0.007	-0.008	-0.009	-0.011	-0.014	-0.021	-0.034	-0.072
0.90	-0.004	-0.004	-0.004	-0.005	-0.006	-0.008	-0.011	-0.018	-0.037
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

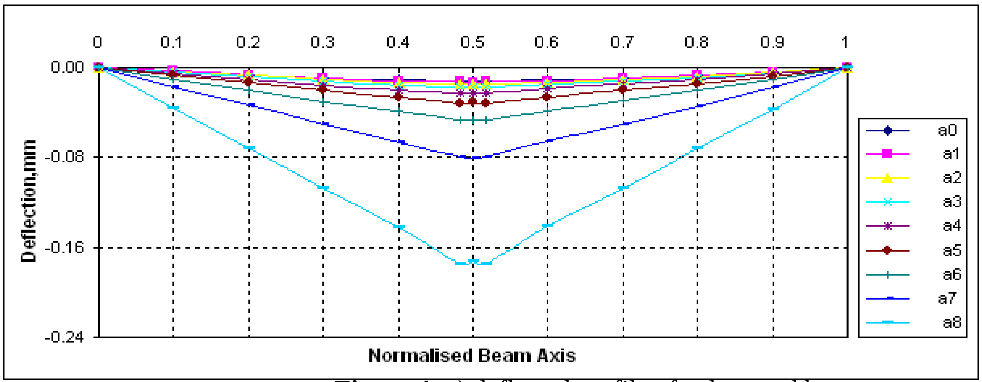

Figure 4: a) deflected profile of a damaged beam

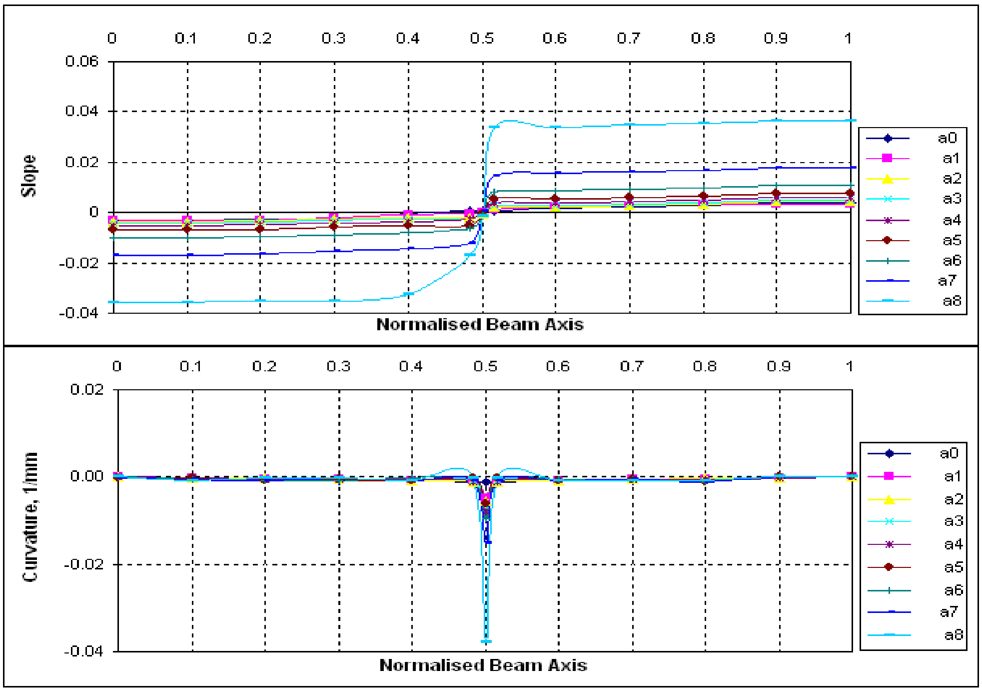

Figure 4: b) Slope and c) Curvature profile of a damaged beam

II. IDENTIFICATION AND QUANTIFICATION OF DAMAGE

The curvature plots as shown in Figure 4(c) reveal that an abrupt change in curvature in the beam is an indication of damage and location of damage as well. Further, it can be understood that magnitude of change in curvature is in proportion of degree of damage. Here a term, curvature ratio, β is introduced which is defined as ratio of curvature of damaged beam to that of undamaged beam. The curvature ratio β is called as "Damage parameter". The damage parameter β is plotted along the span. From the plot as shown in Figure 5, it is obvious that curvatures of undamaged and damage beams remain coincide although out the span except at damage location; and an abrupt increase in β at the damage location.

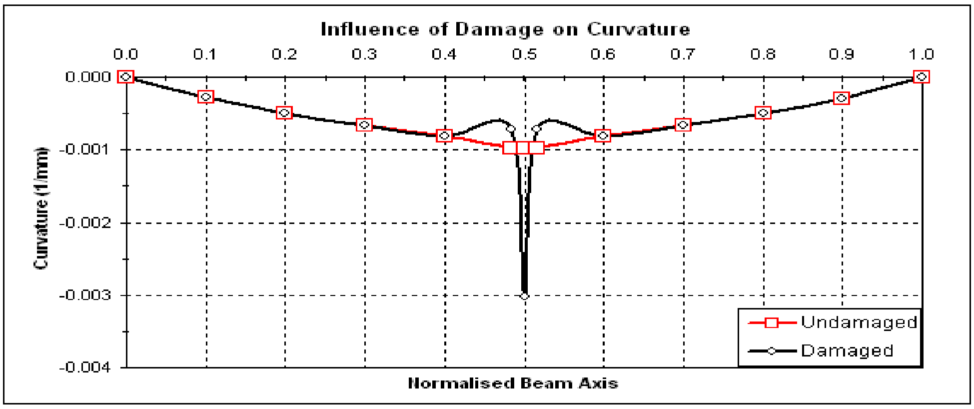

Figure 5: Damage parameter, β plot.

III. DISCUSSION ON THE RESULTS

Study of the plots reveals that presence of damage (modeled as crack) influences the deflections and slopes and curvatures in the beam, overall behavior of the beam. There is an abrupt increase in curvature at crack position. The curvature remains unchanged in between the cracks. Magnitude of curvature at crack position is found to be in proportion to crack length (damage).The curvature ratio, β (Damage parameter) is found to be remained constant for a given crack length irrespective of crack position.

Therefore it is learnt that location and quantity of damage can be assessed from simple measured static deflections of damaged structure from the proposed damage assessment model through curvature ratio β. Limitations of the method employed here are that the damage is modeled as a single discrete crack and combination of discrete cracks. Based on the above observations, a simple damage model is proposed using curvature ratio β.

Emperical Damage Assessment Model

The underlying principles for damage assessment models are explained through an example as follows here. Suppose; a simply supported beam is loaded with concentric point load. Since the beam material is elastic, the beam got deflected. Deflections at various points in the beam along its span are as shown in Fig. 15

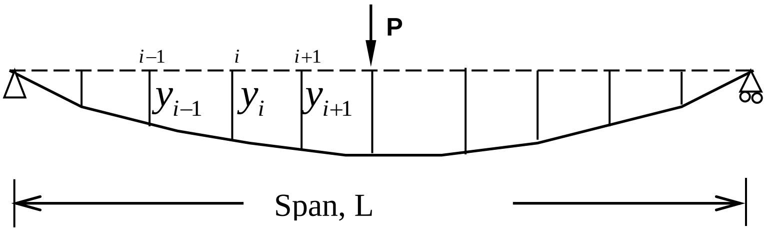

Figure 6 : Deflected profile of a beam with concentric point load

Let y_{i-1}, y_i *and* y_{i+1} be the deflections at points $i-1, i$ *and* $i+1$ respectively in the beam at a regular interval, Δx. Small deflection theory is invoked keeping in view the realistic situation. According to Euler-Bernoulli's beam theory for small deflections, the curvature at any section in a beam is

$$\psi_x = \frac{1}{R_x} = \left(\frac{M_x}{EI_x}\right) = \left(\frac{d^2y}{dx^2}\right) \quad \text{......................(1)}$$

It is obvious from the Eqn. (1) that curvature at any section in the beam is function of bending moment and flexural rigidity. More explicitly, bending moment M_x is directly related to curvature; and flexural rigidity EI_x is inversely related to curvature as indicated in the eqn (1).

If damage occurs in the beam in the form of discrete crack at any section, curvature shoots up at the section due to the obvious fact that moment of inertia decreases all of a sudden at that section. Bending moment M_x at any section in the beam depends on given loading on the beam. Let I_{ox} be the moment of inertia of undamaged beam at any section. Therefore, curvature at any section is

$$\psi_{ox} = \frac{1}{R_{ox}} = \left(\frac{M_x}{EI_{ox}}\right) = \left(\frac{d^2y}{dx^2}\right)_{ox} \quad \text{......................(6)}$$

Where, ψ_{ox} is curvature in undamaged beam at any section in the beam. Suppose the beam is damaged in the form of a discrete crack. Let I_{dx} be the moment of inertia of the beam at cracked section. It is to note that moment of inertia decreases all of a sudden. Therefore, the curvature at any section of the damaged beam,

$$\psi_{dx} = \frac{1}{R_{dx}} = \left(\frac{M_x}{EI_{dx}}\right) = \left(\frac{d^2y}{dx^2}\right)_{dx} \quad \text{......................(7)}$$

Where, I_{ox} is moment of inertia of original (undamaged) beam and I_{dx} is moment of inertia of damaged beam. From Eqns- (6) and (7), ratio of the curvatures of the damage beam and the undamaged beam at any section, called as curvature ratio is

$$\beta = \frac{\psi_{ox}}{\psi_{dx}} = \left(\frac{I_{dx}}{I_{ox}}\right) \quad \text{.....................(8)}$$

The Eqn. 8 reveals that the curvature ratio is independent of loading/ bending moment; but depends on moment of inertia. Let the slope and curvature at any point i in undamaged beam are

$$\text{Slope at } i, \quad \theta_i = \frac{y_{i+1} - y_i}{\Delta x} \quad \text{..................(9)}$$

$$\text{Curvatuere at } i, \quad \psi_{oi} = \frac{\theta_{i+1} - \theta_i}{\Delta x} = \frac{y_{i-1} - 2y_i + y_{i+1}}{(\Delta x)^2} \quad \text{..........(10)}$$

As it has been concluded in the previous chapters; deflections and slopes increase due to the presence of damage. The influence of the damage can be understood from Fig. 4.2

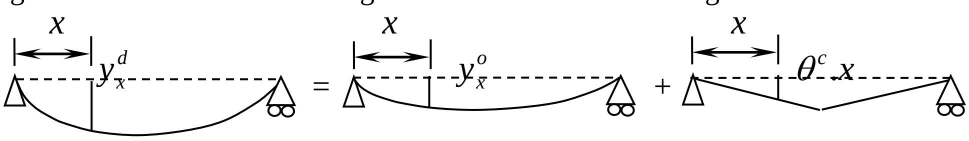

Fig. 7(a) Fig. 7(b) Fig. 7(c)

Figure 7: a) Deflections in damaged beam b) Undamaged beam c) Due to crack.

$$y_x^t = y_x + \theta^c.x \quad \text{..........................(11)}$$

Where θ^c is increase in slope due to presence of crack. y_i^t is total deflection at i in a damaged beam. Therefore total deflection and curvature in a damaged beam, over undamaged portion, can be expressed as in Eqns. (12) and (13).

$$\therefore \quad y_i^t = y_i + y_i^c = \left(y_i + \theta_{i-1}^c \Delta_i\right) \quad \text{.....................................(12)}$$

$$\psi_d = \frac{\left(y_{i+2}^t - 2y_{i+1}^t + y_i^t\right)}{\Delta^2} \quad \text{.....................................(13)}$$

$$= \frac{\left(y_{i+2} + \theta_{i+1}^c \Delta - 2\left(y_{i+1} + \theta_i^c \Delta\right) + y_i + \theta_{i-1}^c \Delta\right)}{\Delta^2} \quad \text{.......(13a)}$$

$$= \frac{\left(y_{i+2} - 2y_{i+1} + y_i\right) + \left(\theta_{i+1}^c - 2\theta_i^c + \theta_{i-1}^c\right)\Delta}{\Delta^2} \quad \text{......................(13b)}$$

It is concluded in previous chapter that θ^c remains constant over undamaged beam. Therefore,

$$\theta_{i-1}^c = \theta_i^c = \theta_{i+1}^c \quad \text{.................................(14)}$$

Hence, from Eqns. (13b) and (14), the curvature in damaged beam over the undamaged portion is

$$\psi_d = \frac{\left(y_{i+2} - 2y_{i+1} + y_i\right)}{\Delta^2}. \quad \text{............................(15)}$$

Eqns (10) and (15) enable to conclude that the curvature in damaged beam over undated portion remains unaffected. But the curvature at the damage location shoots up abruptly as evidenced from Eqn. (8). The slopes (due to damage alone) in undamaged portions are not equal, as a result $\left(\theta_{i+1}^c - 2\theta_i^c + \theta_{i-1}^c\right)$ cannot be zero. Hence curvature at the damage location shoots up.

Quantification Of Damage From Damage Parameter β

Suppose width and depth of damaged beam cross-section is "b" and "D" respectively. Let the crack length in the beam be "a" at the damage location. Therefore moment of inertia of the undamaged beam section is in undamaged beam portion,

$$I_o = \frac{bD^3}{12} \quad \text{..............(16)}$$

And moment of inertia of the damaged beam at the damage location is, where $D_d = (D - a)$,

$$I_d = \frac{bD_d^3}{12} \quad \text{.............(17)}$$

Let the degree of damage, $\alpha = \left(\dfrac{a}{D}\right) \Rightarrow a = \alpha D$ ....(18)

And $D_d = (1-\alpha)D$ ....(19)

According to definition of damage parameter [Eqn. (8)], curvature ratio, is

$$\beta = \frac{\psi_{ox}}{\psi_{dx}} = \left(\frac{I_{dx}}{I_{ox}}\right) = \frac{1}{(1-\alpha)^3} \quad \text{.....................(20)}$$

$$\alpha = \left(1 - \sqrt[3]{\frac{1}{\beta}}\right) \quad \text{....................(21)}$$

Hence the damage assessment model, [eqn. (21)] can be used to predict the damage quantity

Procedure For Damage Identification And quantification From Damage Parameter β

Damage can be quantified from measured static deflection in the damaged structure. In order to measure response of the damaged beam in terms of deflections, the structure has to be loaded. The loading should be as simple as possible. Concentric point load is preferable as the related information is simple and straight forward. That is why; simple concentric point load is used in this investigation. The magnitude of the applied load on the damaged structure should be such that measurable deflection can be made and the structure should not be further damaged. Damage can be assessed using the proposed damage model in this investigation. Step by step systematic procedure for damage quantification is as detailed below.
Considerable number of deflections at regular intervals along the span has to be measured.
Deflections at closer intervals ensure better results in damage assessment.

1. The measured deflections have to be plotted along the span; and checked from the deflection plot whether beam is damaged.
2. Slopes have to be calculated from the measured deflections and plotted, and checked from slope plot, whether beam is damaged.
3. Then curvatures ψ_d of the damaged beam have to be calculated from the measured deflections and plotted. The curvature plot surely enables to detect whether beam is damaged. If damaged, location of the damage and degree of the damage can easily be worked out from curvature ratio β.
4. Curvatures of the undamaged (original) beam (*at the location of damage*), ψ_0 can be determined by interpolating the curvature values near by the damage location in the damaged beam
5. It is proved earlier that curvatures in the damaged beam remains unchanged over the undamaged portions and shoots up at the locations of damage. i.e., curvature ratio β is almost equal to ONE over undamaged portion.
6. Plot the curvature ratios $\left(\beta = \dfrac{\psi_d}{\psi_0}\right)$ of the beam. The plot gives directly the location and the degree of damage in the beam.
7. Then calculate the damage parameter $\left(\beta = \dfrac{\psi_d}{\psi_0}\right)$ at the damage location.
8. Then calculate the degree of damage, $\alpha = \left(1 - \sqrt[3]{\dfrac{1}{\beta}}\right)$
9. Then length of the crack in the beam, $a = \alpha D$

Validation of the Damage Model

The second stage of the present study incorporates experimental work on damage assessment of structures using measured static deflections, as shown in Figure 6. Experiment is conducted on a simply supported beam with known crack length and deflections at regular internal in the beam are noted. Then curvature ratio, β is determined form the measured static deflections. Damage, in the form of crack length, is estimated using proposed damage assessment model and found it to be about 90% accurate.

Determination of Damage Parameter β from Measured Static Deflections

From the Table 9, deflections of the simply supported wooden beams [Fig. 17, 18] are plotted as shown in Fig. 19a. Slopes and curvatures of the beam are also calculated using Eqns. (2) & (3) and plotted as shown in Figures 19b & c respectively. Curvature shoots up in the damage beam at location of crack, as shown in Fig. 19c. The damage parameters β of the beams with 4mm crack length and

8mm crack length are found to be 2.28 and 5.92 respectively [Table 10] Then curvature ratios the damaged beams are calculated and plotted as shown in Fig. 20a & b.

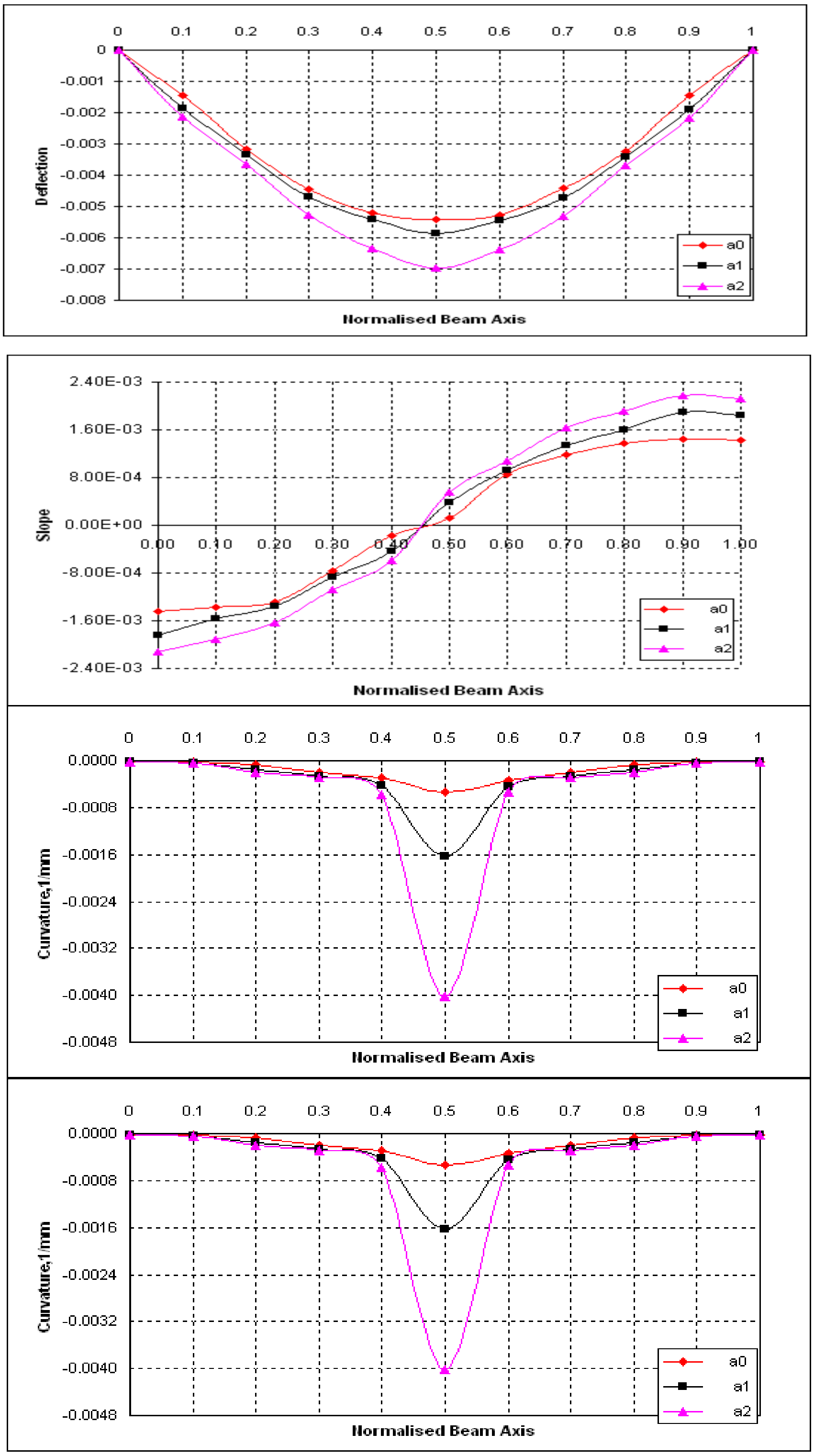

Figure 8 (a), (b) and (c): Deflections, slopes and curvatures of simply supported Beam with Centre crack

Table 2: Experimental Results from wooden beam (Deflections)

Deflection of wooden Beam (Experimental)			
Span/L	**a0**	**a1**	**a2**
0.00	0.000	0.000	0.000
0.10	-0.001	-0.002	-0.002
0.20	-0.003	-0.003	-0.004
0.30	-0.004	-0.005	-0.005
0.40	-0.005	-0.005	-0.006
0.50	-0.005	-0.006	-0.007
0.60	-0.005	-0.005	-0.006
0.70	-0.004	-0.005	-0.005
0.80	-0.003	-0.003	-0.004
0.90	-0.001	-0.002	-0.002
1.00	0.000	0.000	0.000

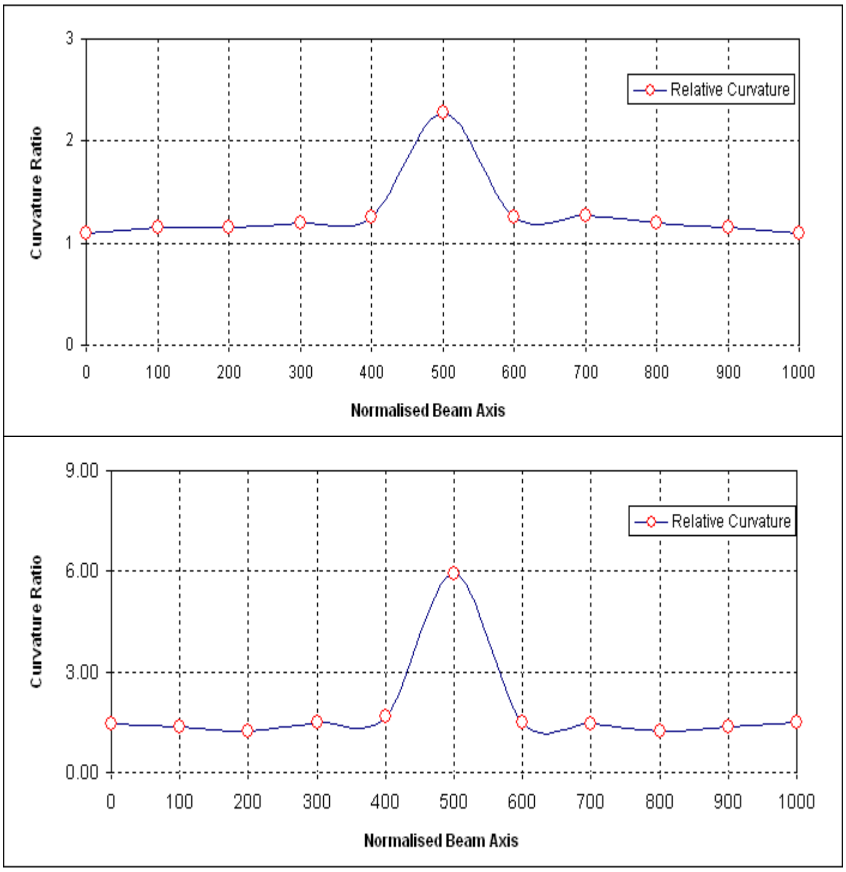

Figure 9: (a) & (b) Curvature ratio plots of the beam with 4mm & 8mm crack lengths.

Damage Assessment Using Damage Parameter β

The (Validation of the damage assessment model) damage parameters β are reckoned from the curvature ratio plots [Fig. 20a & b] and tabulated in the Table 10. Using the proposed damage assessment model [Eqn. 21], the degree of damages α in the beams is

$$\alpha = \left(1 - \sqrt[3]{\frac{1}{\beta}}\right) \quad \text{...(21)}$$

computed, and furnished in the Table 10. The crack lengths are estimated from Eqn. (18) and tabulated in the Table 10. The estimated crack lengths are compared with the originally introduced crack lengths and % error is also calculated as indicated in the Table 3.

$$\text{Degree of damage, } \alpha = \left(\frac{a}{D}\right) \Rightarrow a = \alpha D..........(18)$$

Table 3. Damage parameters, β degree of damage, α and error estimation

Undamaged Beam / Damaged Beam	Actual crack Length, mm	Curvature Ψ	Damage Parameter β	Degree of Damage Α	Estimated crack length from proposed model, mm	% Error
(Undamaged) a0	0	7.10E-04	1.0000	0.00	0.0	0
(Damaged) a1	4	1.62E-03	2.2817	0.24	4.3	-8
(Damaged) a2	8	4.21E-03	5.9296	0.45	8.1	-1

Residual Strength Assessment from Damage Parameter β

Knowing the degree of damage, in the beam, in terms of crack length; residual strength of the beam can be reckoned from the proposed "Residual Strength Assessment Model" [Eqn.22] as given below. The model is based on Fracture Mechanics Principles, as explained in preceding Chapters. Here, fracture toughness (K_{Ic}) of the beam material should be known for assessment of residual strength. Fracture toughness is a material property which has to be determined experimentally. Geometric factor, $g(\alpha)$ corresponding to the relative crack length α, is also mandatory in this model. The residual strength of the beam is expressed in terms of Uniformly Distributed Load (UDL) $w_r\ /unit\ \ length$

$$w_r = \left[\frac{bD^{3/2}}{3c(1-c)l^2\sqrt{\pi\left\{1-\sqrt[3]{\frac{1}{\beta}}\right\}}g(\alpha)}\right]K_{IC}..............(22)$$

Here, the proposed "Residual Strength Assessment Model" is not validated in this investigation owing to time and other constraints. However, it is sure that this model ensures acceptable accuracy in predicting residual strength of the damaged structure. From the above discussions, the following conclusions are drawn.

1. Degree of damage in the damaged beam has been assessed. The error in damage assessment is found to be around 8%.
2. Residual strength of the damaged structure is corresponding to a simply supported beam with uniformly distributed load w_r.
3. Intentionally induced damage in the experiment beam could not be identified from deflection and slope plots, as the deflection are small.
4. Residual strength of the damaged beam is not estimated here for want of fracture toughness of wood material.

IV. CONCLUSIONS

Based on the above results and discussions the following conclusions can be drawn.

1. Curvature changes abruptly at damage (crack) position.
2. The curvature remains unchanged in between the cracks.
3. Magnitude of curvature at crack position is found to be in proportion to crack length (damage).
4. The curvature ratio, β (Damage parameter) is found to be remained constant for a given crack length irrespective of crack position.

5. Location and quantity of damage can be assessed from simple measured static deflections of damaged structure from the proposed damage assessment model through curvature ratio β. Limitations of the method employed here are that the damage is modeled as a single discrete crack.
6. Damage, in the form of crack length, is estimated using proposed damage assessment model and found it to be about 90% accurate.

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AUTHORS DETAILS

Billa. Subhan Ramji was born in Kakinada, A.P, India, in Year 1983. He received the Bachelor degree in Civil Engineering from the University of JNTU, Hyderabad, in 2006 and the Masters in Structural Engineering from the University of JNTU, Kakinada, in Year 2009, both in Civil Engineering. Presently he is working as Assistant Professor in Civil Engineering Department in Pragati Engineering College, Kakinada, AP

P. Venkata Muthyalu was born in Avinigadda, A.P, India, in Year 1988. He received the Bachelor degree in Civil Engineering from the University of JNTU, Kakinada, in 2009 and the Masters in Geo-Technical Engineering from the University of JNTU, Kakinada, in Year 2011, both in Civil Engineering. Presently he is working as Assistant Professor & HOD in Civil Engineering Department in Pragati Engineering College, Kakinada, AP