# GENERALIZATION OF NEAREST NEIGHBOUR TREATMENTS USING EUCLIDEAN GEOMETRY

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***ABSTRACT***
*Neighbour design is important to know that the competition effects between neighbouring plots that is the response on a given plot is affected by the treatment applied on neighbouring plots as well as by the treatments applied to that plot. In this paper Neighbour design is constructed for OS1 series using Euclidean Geometry with parameters v=s2, b=s(s+1), r=s+1, k=s, λ=1, whether s is a prime number or power of a prime number. Here two-sided (left & right) neighbours for every treatment upto q=(s-1)-th order for neighbour design is obtained. And it is observed that for these neighbour treatments property of circularity holds for the complete design as well as within the each set of s series. It is further observed that neighbour treatments of qth - order follow the property of circularity of the same order.*

***KEY WORDS:*** *Galois field, BIBD, Euclidean geometry, Neighbour Design, left neighbours, Right neighbours, Circularity.*

## I. INTRODUCTION

Neighbour balanced designs are more useful to remove the neighbour effects in experiments because these designs are a tool for local control in biometrics, agriculture, horticulture and forestry. These designs are, therefore, useful for the cases where the performance of a treatment is affected by the treatments applied to its neighboring plots. Rees (1967) introduced neighbour designs in serology and constructed these designs in complete blocks as well as incomplete blocks. Some suitable designs have also been given by Lawless (1971), Hwang (1973), Dey and Chakravarty (1977), Azais *et al*. (1993), Iqbal *et al*. (2009). Laxmi and Rani (2009) obtained the patterns of neighbour treatments of first - order neighbours for every treatment of neighbour designs of the OS1 series considering two sided (left &right) border plots. Laxmi and Parmita (2010) suggested a method of finding left neighbours of a treatment in a neighbour design for OS2 series without constructing the actual design. Laxmi and Parmita (2011) further suggested a method of finding right neighbours for the OS2 series. Rani and Laxmi observed left and right (two-sided) first - order neighbours of every treatment for OS1 series using Euclidean Geometry and has shown that these neighbours follow the property of circularity of first-order. Rani et al. (2013) observed left and right (two-sided) second-order neighbours of every treatment for OS1 series and has shown that these neighbours follow the property of second-order circularity.

The objective of this paper is given a systematic procedure for finding out two-sided neighbours for neighbour design with parameters v= $s^2$, b= s(s+1), r= s+1, k=s, λ=1 for different values of s even without constructed the actual design. The designs of OS1 series can be obtained by first forming a finite 2-dimensional EG(2, s), where s is either a prime number or power of a prime number, by using the elements of G.F.(s), treating the points as treatments, all possible lines as blocks, and then the points on a line as the contents of the block corresponding to the line. This method is considered for s being a prime number or power of a prime number by taking s=3 and s=4. With the resulted BIBD neighbour design is constructed by using the border plots. In bordered designs only the neighbours of treatments in interior plots are of interest. Thus a border plot has no neighbours but can be a neighbour of an interior plot and all interior plots have two nearest neighbours.

## II. NEIGHBOUR DESIGN OF OS1 SERIES USING EUCLIDEAN GEOMETRY

Orthogonal series for Balanced Incomplete Block Design (B.I.B.D.) with parameters $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$ and $v = b = s^2+s+1$, $r = k = s+1$, $\lambda=1$ were introduced by Yates (1936). The first series was named as OS1 series and the second series was named as OS2 series. Construction of neighbour design of OS1 series using Euclidean geometry one may refer to Rani and Laxmi ( ). The first order left and right (two-sided) neighbours for every treatment of OS1 series have been discussed by Rani and Laxmi ( ). Further, second-order left and right (two-sided) neighbours for every treatment of OS1 series have been discussed by Rani and Meena (2013). This paper interests to find out left and right (two-sided) neighbours upto $q^{th}$ order for neighbour design of OS1 series where q=1, 2, … , s-1. Let us consider the neighbours of a treatment for the neighbour design for different value of s.

### (2.1) Neighbours of a treatment When s=3

The parameters of OS1 series when s=3 will becomes: v=9, b=12, r=4, k=3, $\lambda=1$ and neighbours of first and second- order for a treatment can be easily obtained using the method discussed by Rani and Laxmi ( ) and Rani and Meena (2013). Authors considered the third- ordered neighbours of every treatment for the neighbour design and found that these third-ordered neighbours are the treatments themselves. Further, higher ordered neighbours that is fourth -ordered, fifth- ordered and so on is the repetition of first – order and second- order neighbour treatments respectively. It implies that neighbours for a treatment upto second-order [(s-1)-th] can be obtained after that the repetition of treatments is there. Rani and Laxmi ( ) has found and given a list of neighbours of first-order for every treatment of neighbour design for OS1 series when s=3. Rani and Meena (2013) has given a list of neighbours of second- order neighbours for every treatment for the same design for s=3. Here these left and right (two-sided) neighbours (nbhrs) of first- order, second-order are further summarized in Table 2.1.1.

**Table 2.1.1**Two-sided neighbours upto q-th order when s=3

| Treatment no. 'i' | Series in which 'i' lies | $1^{st}$-order left nbhrs | $1^{st}$-order right nbhrs | Other nbhrs | $2^{nd}$-order left nbhrs | $2^{nd}$- order right nbhrs | other nbhrs |
|---|---|---|---|---|---|---|---|
| **1** | 1≤i≤s | 7,8,9 | 4,5,6 | 3,2 | 4,5,6 | 7,8,9 | 2,3 |
| **2** | | | | 1,3 | | | 3,1 |
| **3** | | | | 2,1 | | | 1,2 |
| **4** | s+1≤i≤2s | 1,2,3 | 7,8,9 | 6,5 | 7,8,9 | 1,2,3 | 5,6 |
| **5** | | | | 4,6 | | | 6,4 |
| **6** | | | | 5,6 | | | 6,5 |
| **7** | $2s+1\leq i\leq s^2$ | 4,5,6 | 1,2,3 | 9,8 | 1,2,3 | 4,5,6 | 8,9 |
| **8** | | | | 7,9 | | | 9,7 |
| **9** | | | | 8,9 | | | 9,8 |

It is further observed that these neighbours have a systematic pattern which is summarized in the form of series in Table 2.1.2

**Table 2.1.2** Two- sided neighbours in form of series upto q-th order when s=3

| Treatment | Series in | $1^{st}$-order | $1^{st}$-order | Other | $2^{nd}$-order | $2^{nd}$- order | other |
|---|---|---|---|---|---|---|---|

| no. 'i' | which 'i' lies | common left nbhr series | common right nbhr series | nbhrs | common left nbhr series | Common right nbhr series | nbhrs |
|---|---|---|---|---|---|---|---|
| **1**<br>**2**<br>**S** | 1≤i≤s | 2s+1≤i≤$s^2$ | s+1≤i≤2s | i-1,i+1 | s+1≤i≤2s | 2s+1≤i≤$s^2$ | i-2,i+2 |
| **s+1**<br>**s+2**<br>**2s** | s+1≤i≤2s | 1≤i≤s | 2s+1≤i≤$s^2$ | i-1,i+1 | 2s+1≤i≤$s^2$ | 1≤i≤s | i-2,i+2 |
| **2s+1**<br>**2s+2**<br>**$s^2$** | 2s+1≤i≤$s^2$ | s+1≤i≤2s | 1≤i≤s | i-1,i+1 | 1≤i≤s | s+1≤i≤2s | i-2,i+2 |

## (2.2) Neighbours of a treatment When s=4

The parameters of the OS1 series thus becomes: v=16, b=20, r=5, k=4, λ=1. Neighbours of first- order and second -order for a treatment can be easily obtained using the method discussed by Rani and Laxmi ( ) and Rani and Meena (2013). Authors considered the third- ordered and fourth- ordered neighbours of treatments for the neighbour design and found that the fourth- ordered neighbours are the treatments themselves. Further, higher ordered neighbours i.e. fifth- ordered and so on is the repetition of first- order, second- order and third- ordered neighbour treatments respectively. It implies that neighbours for a treatment of third- order (s-1) can be obtained after that the repetition of neighbour treatments takes place. Rani and Laxmi has found and given a list of neighbours of first order for every treatment for neighbour design of OS1 series when s=4. Rani and Meena (2013) has given a list of neighbours of second- order for every treatment for the same design for s=4. Neighbours of third- order for a treatment can be easily obtained by observing the treatment as two plot away in left direction and two plot away in right direction for left and right neighbors respectively i.e. using circularity property as suggested by Rani and Laxmi ( )and Rani and Meena (2013). It is further observed that these neighbours (nbhrs) have a systematic pattern which is summarized in the Table 2.2.1:

**Table 2.2.1** Two- sided neighbours in form of series upto q-th order when s=4

| oln. no.→1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| **treatment no. 'i'** | **Series in which 'i' lies** | **1st-order common left nbhr series** | **1st-order common right nbhr series** | **1st - order other nbhrs** |
| **1**<br>**2**<br>**3**<br>**S** | 1≤i≤s | 3s+1≤i≤$s^2$ | s+1≤i≤2s | i-1,i+1 |
| **s+1**<br>**s+2**<br>**s+3**<br>**2s** | s+1≤i≤2s | 1≤i≤s | 2s+1≤i≤3s | i-1,i+1 |
| **2s+1**<br>**2s+2**<br>**2s+3**<br>**3s** | 2s+1≤i≤3s | s+1≤i≤2s | 3s+1≤i≤$s^2$ | i-1,i+1 |
| **3s+1**<br>**3s+2**<br>**3s+3**<br>**$s^2$** | 3s+1≤i≤$s^2$ | 2s+1≤i≤3s | 1≤i≤s | i-1,i+1 |

**Table contd.**

| 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| **2nd-order common left** | **2nd-order common right** | **2nd –order other nbhrs** | **3rd-order common left** | **3rd-order common right nbhr series** | **3rd-order Other** |

| nbhr series | nbhr series | | nbhr series | | nbhrs |
|---|---|---|---|---|---|
| 2s+1≤i≤3s | 2s+1≤i≤3s | i-2,i+2 | s+1≤i≤2s | 3s+1≤i≤$s^2$ | i-3,i+3 |
| 3s+1≤i≤$s^2$ | 3s+1≤i≤$s^2$ | i-2,i+2 | 2s+1≤i≤3s | 1≤i≤s | i-3,i+3 |
| 1≤i≤s | 1≤i≤s | i-2,i+2 | 3s+1≤i≤$s^2$ | s+1≤i≤2s | i-3,i+3 |
| s+1≤i≤2s | s+1≤i≤2s | i-2,i+2 | 1≤i≤s | 2s+1≤i≤3s | i-3,i+3 |

## (2.3) Neighbours of a treatment When s=5

The parameters of the OS1 series thus becomes: v=25, b=30, r=6, k=5, λ=1. Here authors considered the neighbours of first- order, second- order and higher-ordered for the neighbour design and found that the fifth- ordered neighbours are the treatments themselves. Further, higher ordered neighbours that are sixth- ordered and so on, is the repetition of first- order, second- order upto fourth- ordered neighbour treatments respectively. It implies that neighbours for a treatment upto fourth- order (s-1) can be obtained. Left and right neighbours of fourth- order for a treatment can be further obtained by observing the treatments as three-plot away in left and right direction of that treatment i.e. using property of circularity of fourth order. Further neighbours (nbhrs) have a systematic pattern which is summarized in the Table 2.3.1:

**Table 2.3.1** Two- sided neighbours in form of series upto q-th order when s=5

| Coln. no. 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| **Treatment no. 'i'** | **Series in which 'i' lies** | **1st-order common left nbhr series** | **1st-order common right nbhr series** | **1st-order other nbhrs** | **2nd-order common left nbhr series** | **2nd-order common right nbhr series** |
| **1<br>2<br>3<br>4<br>s** | 1≤i≤s | 4s+1≤i≤$s^2$ | s+1≤i≤2s | i-1,i+1 | 3s+1≤i≤4s | 2s+1≤i≤3s |
| **s+1<br>s+2<br>s+3<br>s+4<br>2s** | s+1≤i≤2s | 1≤i≤s | 2s+1≤i≤3s | i-1,i+1 | 4s+1≤i≤$s^2$ | 3s+1≤i≤4s |
| **2s+1<br>2s+2<br>2s+3<br>2s+4<br>3s** | 2s+1≤i≤3s | s+1≤i≤2s | 3s+1≤i≤4s | i-1,i+1 | 1≤i≤s | 4s+1≤i≤$s^2$ |
| **3s+1<br>3s+2<br>3s+3<br>3s+4<br>4s** | 3s+1≤i≤4s | 2s+1≤i≤3s | 4s+1≤i≤$s^2$ | i-1,i+1 | s+1≤i≤2s | 1≤i≤s |
| **4s+1<br>4s+2<br>4s+3<br>4s+4<br>$s^2$** | 4s+1≤i≤$s^2$ | 3s+1≤i≤4s | 1≤i≤s | i-1,i+1 | 2s+1≤i≤3s | s+1≤i≤2s |

**Table contd.**

| 8 | 9 | 10 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| **2nd-order other nbhrs** | **3rd-order common left nbhr series** | **3rd-order common right nbhr series** | **3rd-order other nbhrs** | **4th-order common left nbhr series** | **4th-order common right nbhr series** | **4th-order other** |

| | | | | | | nbhrs |
|---|---|---|---|---|---|---|
| i-2,i+2 | 2s+1≤i≤3s | 3s+1≤i≤4s | i-3,i+3 | s+1≤i≤2s | 4s+1≤i≤$s^2$ | i-4,i+4 |
| i-2,i+2 | 3s+1≤i≤4s | 4s+1≤i≤$s^2$ | i-3,i+3 | 2s+1≤i≤3s | 1≤i≤s | i-4,i+4 |
| i-2,i+2 | 4s+1≤i≤$s^2$ | 1≤i≤s | i-3,i+3 | 3s+1≤i≤4s | s+1≤i≤2s | i-4,i+4 |
| i-2,i+2 | 1≤i≤s | s+1≤i≤2s | i-3,i+3 | 4s+1≤i≤$s^2$ | 2s+1≤i≤3s | i-4,i+4 |
| i-2,i+2 | s+1≤i≤2s | 2s+1≤i≤3s | i-3,i+3 | 1≤i≤s | 3s+1≤i≤4s | i-4,i+4 |

### (2.4) Neighbours of OS1 series for any value of s

The parameters of the OS1 series are : v=$s^2$, b=s(s+1), r=s+1, k=s, λ=1 for which both - sided neighbours are to be found whether s is either a prime number or power of a prime number.

**(i)**Consider the treatment number 'i' where i=1,2,…,$s^{2.}$

**(ii)** Then find the series in which the treatment number 'i' lies

The series is defined in such a way that the sequence of first's' treatments i.e. (1 to s ) of the design form the first series, the sequence of next 's' treatments i.e. (s+1 to 2s) form the second series and the sequence of next 's' treatments i.e. (2s+1 to3s) form the third series and so on so the last series consists of the extreme last treatment from s(s-1)-th to $s^2$-th treatment. Thus, there are 's' series upto the treatment number $s^2$. The (s+1)-th series of treatment numbers $s^2$+1 to $s^2$+s reduces to 1 to s with modv. So the (s+1)-th series is again the first series of the design. It again holds true for the next (s+2)-th and so on series which proves that the design is circular.

**(iii)** Then find out the common left neighbour series and common right neighbour series for any treatment number:

Let the treatment number 'i' lies in the $j^{th}$ series where (j=1,2,…,s) then (j-1)-th series i.e. the previous series is the first- order common left neighbour series and (j+1)-th series i.e. the next series is the first-order common right neighbour series. (j-2)-th series is the second-order left neighbour series and (j+2)-th series is the second-order right neighbour series. For example: let the treatment number 'i' lies in the j-th series where (j=1,…,s) then (j-2)-th series is the one –series away in left direction is the second-order common left neighbour series and (j+2)-th series is the one series away in right direction is the second order common right neighbour series. In the similar fashion, q-th left and right neighbour series can be observed where q=1,…,s-1. As there are s+1 replications of each treatment there will be s+1 left neighbors and s+1 right neighbors making 2s+2 in total number of neighbours of any order for a treatment.

**(iv)** Two more neighbours other than these two common series can be find by the concept of neighbour means successor or predecessor:

These should be (i-q)-th and (i+q)-th neighbour treatments for the q-th order neighbours of treatment number i, where q=1,…,s-1. These two neighbour treatments follow the property of circularity of q-th order within the series in which series the treatment number 'i' lies. Hence the systematic way of finding left neighbours and right neighbours (nbhrs) of q-th order of any treatment is summarized in the Table 3.1.

**Table 3.1**Two- sided neighbours in form of series upto q-th order for any value of s

| **Coln. no. 1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|
| **treatm-ent no. 'i'** | **Series in which 'i' lies** | **1st-order common left nbhr series** | **1st-order common right nbhr series** | **1st-order other nbhrs** | **2nd-order common left nbhr series** |
| **1<br>2<br>.<br>.<br>.<br>s** | 1≤i≤s | s(s-1)+1≤i≤$s^2$ | s+1≤i≤2s | i-1,i+1 | s(s-2)+1≤i≤s(s-1) |

| | | | | | |
|---|---|---|---|---|---|
| **s+1<br>s+2<br>.<br>.<br>.<br>2s** | s+1≤i≤2s | 1≤i≤s | 2s+1≤i≤3s | i-1,i+1 | s(s-1)+1≤i≤$s^2$ |
| **2s+1<br>2s+2<br>.<br>.<br>.<br>3s** | 2s+1≤i≤3s | s+1≤i≤2s | 3s+1≤i≤4s | i-1,i+1 | 1≤i≤s |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. |
| **s(s-2)+1<br>s(s-2)+2<br>.<br>.<br>.<br>s(s-1)** | s(s-2)+1≤i≤s(s-1) | s(s-3)+1≤i≤s(s-2) | s(s-1)+1≤i≤$s^2$ | i-1,i+1 | s(s-4)+1≤i≤s(s-3) |
| **s(s-1)+1<br>s(s-1)+2<br>.<br>.<br>.<br>$s^2$** | s(s-1)+1≤i≤$s^2$ | s(s-2)+1≤i≤s(s-1) | 1≤i≤s | i-1,i+1 | s(s-3)+1≤i≤s(s-2) |

**Table contd.**

| **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|
| **$2^{nd}$-order common right nbhr series** | **$2^{nd}$-order other nbhrs** | . . . | **$q^{th}$-order common left nbhrs series** | **$q^{th}$-order common right nbhr series** | **$q^{th}$-order other nbhrs** |
| 2s+1≤i≤3s | i-2,i+2 | . . . | s+1≤i≤2s | s(s-1)+1≤i≤$s^2$ | i-q, i+q |
| 3s+1≤i≤4s | i-2,i+2 | . . . | 2s+1≤i≤3s | 1≤i≤s | i-q, i+q |
| 4s+1≤i≤5s | i-2,i+2 | . . . | 3s+1≤i≤4s | s+1≤i≤2s | i-q, i+q |
| .<br>.<br>. | .<br>.<br>. | . . . | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. |
| 1≤i≤s | i-2,i+2 | . . . | s(s-1)+1≤i≤$s^2$ | s(s-3)+1≤i≤s(s-2) | i-q, i+q |
| s+1≤i≤2s | i-2,i+2 | . . . | 1≤i≤s | s(s-2)+1≤i≤s(s-1) | i-q, i+q |

After observing the s-th order two-sided neighbours it was found that these s-th ordered neighbours are the treatments themselves. Further, higher-ordered neighbours that is (s+1)-th, (s+2)-th and so on, is the repetition of first-order, second-order upto the (s-1)-th ordered neighbour treatments respectively. It implies that neighbours for a treatment upto (s-1)-th i.e. highest - order can be obtained after that the repetition takes place.

It was noted from the above table that the second-order left neighbours and right neighbours of every treatment are same when s=4. This may be due to the circularity of second-order as shown by Rani and Meena (2013). It was further observed that for s=8, fourth-ordered left and right neighbours are same, for every treatment. It was found true for each even value of s (for which the incomplete block neighbour design can be constructed) of OS1 series. So it is inferred that whenever s is an even number there (s/2)-th order two-sided (left & right) neighbours of a treatment shall be same due to circularity of the design.

## III. CONCLUSION

In case of OS1 series where $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$ either s is a prime number or power of a prime number neighbour treatments are same either using the method of MOLS discussed by Laxmi et al ( ) and Laxmi et al ( ) or Euclidean Geometry. It is further observed that for a given value of s, the neighbour treatments can be find out from 1-th order to (s-1)-th order for treatment number from 1 to $s^2$ and these neighbour treatments follows the property of circularity of the same order for the neighbour treatments between the complete design as well as within each set of s series.

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