REVIEW OF OPTIMALITY CRITERION APPROACH SCOPE, LIMITATION AND DEVELOPMENT IN TOPOLOGY OPTIMIZATION

Avinash Shukla, Anadi Misra
Department of Mechanical Engineering,
G. B. Pant University of Agriculture and Technology Pantnagar, Uttarakhand, India

ABSTRACT
Out of many available optimization algorithms, classical approach to the numerical solution of a discretized structural optimization problem is the optimality criteria (OC) method. OC method is very efficient for solving the topology optimization method. OC method is used in various fields of engineering application as a very strong method of optimization. This paper gives an insight into the OC method. Here we evaluate scope and limitation of OC method and discuss the recent development in OC method to expand its application form the optimization of a simple energy functional with a single constraint on material resource to the case of malty constraint.

KEYWORDS: *Topology optimization, optimality criterion method, finite element method, discretized structural optimization.*

I. INTRODUCTION

A classical approach to the numerical solution of a discretized structural optimization problem is the optimality criteria (OC) method. OC method has turned out to be very efficient for solving the topology optimization problems. We can see in the layout of structural optimization that SIMP is one of the gradient based density distribution methods and optimality criterion is one of the mathematical model which works on SIMP.

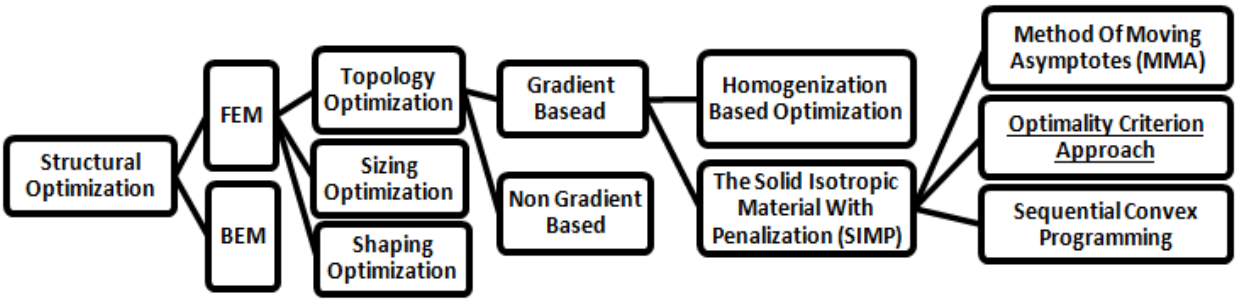

Figure 1: Layout of structural optimization

In contrast to the mathematical programming methods, the optimum criteria methods take advantage of the knowledge on the physics and mechanics of the respective problem set. A well-known and ascertained physical law relating to structural mechanics is for instance the Fully Stressed Design which can actually only are applied to statically determined structures. Regarding the optimum criteria methods, these criteria and the response behavior of modifications of the physical model are implemented into the algorithm. With suitable redesign rules, a convergence behavior is achieved which cannot be attained with mathematical optimizers. Applying this particular physical and mechanical knowledge, the optimum criteria methods remain limited to the certain application areas.

Applying this knowledge makes the individual optimization steps comprehensible. The optimum criteria are particularly well proven for shape and topology optimization where a large number of design variables are required. The convergence speed is independent of the number of design variables there are commercial programs to solve only simple topology optimization problem.

II. THE OPTIMALITY CRITERION

The optimality criterion is a simple method frequently used for updating the design variables. It is a heauristic method based on the Lagrangian function. The Lagrangin multipliers are found through an iterative process.

The Lagrangian *L* for the optimization problem is given by

$$L = c + \lambda(V - fV_0) + \lambda_1^T(Ku - f) + \sum_{e=1}^{N} \lambda_{2e}(x_{min} - x_e) + \sum_{e=1}^{N} \lambda_{3e}(x_e - x_{max}), \quad 2.1$$

Where λ and λ_1 are the global Lagrangian multipliers and λ_{2e} and λ_{3e} are Lagrangian multipliers for the lower and upper side constraints.

Optimality is found when the derivatives of Lagrangian function with respect to the design variable are zero:

$$\frac{\partial L}{\partial x_e} = 0, \text{ for } e = 1, N. \quad 2.2$$

$$\text{Here } \frac{\partial L}{\partial x_e} = \frac{\partial c}{\partial x_e} + \lambda \frac{\partial V}{\partial x_e} + \lambda_1^T \frac{\partial Ku}{\partial x_e} - \lambda_{2e} + \lambda_{3e} \quad 2.4$$

Assuming that the lower and upper bound constraints are not active ($\lambda_{2e} = \lambda_{3e} = 0$) and that the loads are design independent ($\frac{\partial f}{\partial x_e} = 0$), we obtain

$$\frac{\partial L}{\partial x_e} = \frac{\partial u^t}{\partial x_e} Ku + u^T \frac{\partial K}{\partial x_e} u + u^T K \frac{\partial u}{\partial x_e} + \lambda v_e + \lambda_1^T \left(\frac{\partial K}{\partial x_e} u + K \frac{\partial u}{\partial x_e}\right) \quad 2.5$$

This reduces to:

$$\frac{\partial L}{\partial x_e} = u^T \frac{\partial K}{\partial x_e} u + \lambda_1^T \frac{\partial K}{\partial x_e} u + \frac{\partial u}{\partial x_e}(2u^T K + \lambda_1^T K) + \lambda v_e \quad 2.6$$

Since $\lambda_1^T$is arbitrary, it is chosen in such a way as to eliminate the derivatives$\frac{\partial u}{\partial x_e}$. λ_1^T is set equal to , to set equal to ---$2u^T$, to set$(2u^T K + \lambda_1^T K)$ equal to zero.

Hence,
$$\frac{\partial L}{\partial x_e} = -u^T \frac{\partial K}{\partial x_e} u + \lambda v_e$$
$$= -p(x_e)^{p-1} u_e^T K_0 u_e + \lambda v_e = 0 \quad 2.7$$

Where $u_e^T K_0 u_e$ is the energy of a solid element with density 1. Since the strain density remain constant throughout the domain, the design domain, the design variable can be updated based on

$$\frac{p(x_e)^{p-1} u_e^T K_0 u_e}{\lambda v_e} = \frac{\frac{\partial L}{\partial x_e}}{\lambda v_e} = B_e^k = 1 \quad 2.8$$

The heuristic scheme or updating the design variable is then

$$x_e^{k+1} = x_e^k \left(\frac{p(x_e)^{p-1} u_e^T K_0 u_e}{\lambda v_e}\right)^{\varsigma} = x_e^k (B_e^k)^{\varsigma} \quad 2.9$$

Where ς is the damping, which can from zero to one. A positive move limit m, which can also vary from zero to one, is introduced from zero to one, is introduced to stabilize the iteration, that is, to ensure that no big change in relative density is allowed between two successive iterations, so that an element does not go from void to solid or vice versa in one iteration. Thus, the heuristic scheme for updating the design variable then becomes

$$x_e^{k+1} = \begin{cases} \max(x_{min}, x_e^k - m) \ if \ x_e^k (B_e^k)^{\varsigma} \le \max(x_{min}, x_e^k - m) \\ x_e^k (B_e^k)^{\varsigma} \ if \ \max(x_{min}, x_e^k - m) < x_e^k (B_e^k)^{\varsigma} < \min(1, x_e^k + m) \\ \min(1, x_e^k + m) \ \ if x_e^k (B_e^k)^{\varsigma} \ge \min(1, x_e^k + m) \end{cases}$$

The damping is normally set to 0.5 and its purpose is also to stabilize the iteration. The Lagrangian multiplier is updated iteraratively using bisection, such that the Lagrangian also satisfies the volume constraint.

2.1 Scope of Optimality Criterion Method

The type of algorithm described above has been used to great effect in a large number of structural topology design studies and is well established as an effective (albeit heuristic) method for solving

large scale problems. The effectiveness of the algorithm comes from the fact that each design variable is updated independently of the update of the other design variables; except for the resealing that has to take place for satisfying the volume constraint. The algorithm can be generalized to quite a number of structural optimization settings (see for example [**Rozvany (1989), Rozvany (1992)**]) but it is not always straightforward. For cases where for example constraints of a non-structural nature should be considered (e.g., representing geometry considerations), when non-self-adjoint problems are considered or where physical intuition is limited, the use of a mathematical programming method can be a more direct way to obtain results. Typically, this will be computationally more costly, but a careful choice of algorithm can make this approach as efficient as the optimality criteria method.
The OC does not require any great programming efforts in order to solve the compliance topology design problem. When access to a FEM code is provided, only a few lines of extra code is required for the update scheme and for the computation of the energies involved. The optimality criteria method is closely related to the concept of fully stressed design. However, it is important to note that the specific strain energy is constant in areas of intermediate density, while it is lower in regions with a density $\rho = \rho_{min}$ and higher in regions with a density equal to 1.

2.2 Limitation of Optimality Criterion Method

Like other optimization methods OC also suffers from the problem of checkerboards, mesh dependence and local minima. There come some problems for which the OC method does not converge. One of the major limitation of OC approach is applicable to problems with only volume as the constraint (or "volume as the only constraint") i.e. existing framework of OC method is limited to optimization of simple energy functional (compliance or eigen frequencies) with a single constraint on material resource.

III. RECENT DEVELOPMENT IN OPTIMALITY CRITERION METHOD

The optimality criteria method is typically used in situation when the number of the global constraints is much less than the number of the design variables. Accordingly, the number of Lagrange multipliers is very small, and the cost for the procedure of computing Lagrange multipliers becomes negligible. Viewed from this perspective, the extra computational cost on the criteria optimality method procedure form constraint to multiple constraints is rather minor. From the design variable updating procedure, we observe that computer time for the updating procedure is virtually independent of the present method over various method of the updating procedure of the latter, including the method of sequential line programming, depends on the number of design variables, which is unfortunately very large in a topology design problem.
The gradient based Taylor series expansion is employed to present the relationship between constraints and design variables in as explicit form **[4].** The computational cost for updating the Lagrangian multipliers is proved to be rather minor when the number of displacement constraints is small. The computational burden mainly comes from the analysis of various adjoin structures.

IV. CONCLUSIONS

The OC method is very useful for the topology optimization, its results are more accurate and convergence is independent of the number of design variables. It has the potential to be used as malty constraint method in place of single constraint function if gradient based Taylor series expansion is employed. Also as number of global constraints is much less than the number of design variable, so the number of Lagrange multipliers is very small, and the cost for the procedure of computing Lagrange multipliers becomes negligible. So modified version of OC can we used to improve the software based optimization tools to save time and energy. Using this concept other algorithms can also be modified to expand their area of applications. Use of improved version of OC method will definitely bring the revolution in the field of Topology Optimization as it will reduce our dependency on other less efficient methods which are used in malty constraint cases.

REFERENCES

[1]. Nima Bakhtiar. (1996) "A New Approach for Sizing, Shape and Topology Optimization" SAE International Congress and Exposition, Detroit, Michigan USA, February 26-29.
[2]. Dineo Khoza. (2006) "Topology optimization of plate-like structure" University of Pretoria etd.
[3]. Peter W. Christensen, Anders Klarbring. (2009) "An Introduction to Structural Optimization" Springer, Solid Mechanics and Its Applications, Vol 153.
[4]. Luzhong Yin, Wei Yang. (2001) "Optimality criterion method for topology optimization under multiple constrains" Pergamon, Vol 79: 1839-1850.
[5]. Ashish Das. (2002) "An Introduction to Optimality Criterion and Some Results on Optimal Block design" Design Workshop Lecture Notes, ISI, Kolkata, November, 25-29, 2002, pp. 1-21.
[6]. D. Tcherniak and O. Sigmund, "A web-based topology optimization program" Struct. Multidisc. Optim. Springer-Verlag 2001, Vol 22: 179-187.
[7]. M. P. Bendsøe, and N. Kikuchi, (1988) "Generating optimal topologies in structural design using a homogenization method" Comput. Meth. Appl. Mech. Eng., Vol 71: 197-224.
[8]. C. C. Swan and I. Kosaka, (1997) "Voigt-Reuss topology optimization for structures with linear elastic material behaviors", Int. J. Numer. Meth. In Eng. Vol 40: 3033-3057.
[9]. Cheng, C.S. (1978). "Optimality of certain asymmetrical experimental designs" Ann. Statist, Vol 6:1239-1261.
[10]. Bapat, R.B. and Dey, A. (1991) "Optimal block designs with minimal number of observations" Statist. Probab. Letters, Vol 11: 399-402.
[11]. D. Tcherniak and O. Sigmund, "A web-based topology optimization program" Struct. Multidisc. Optim. Springer-Verlag 2001, Vol 22: 179-187.
[12]. G. Allaire, F. Jouve and A. M. Toader, (2002) "A level set method for shape optimization" C. R. Acad. Sci. Paris.
[13]. S. F. Rahmatalla and C. C. Swan, (2004) "A Q4/Q4 continuum structural topology optimization implementation", Struct. Multidisc. Optim. Springer-Verlag, Vol 27: 130-135.
[14]. J. Du and N. Olhoff, (2005) Topology optimization of continuum structures with respect to simple and multiple Eigen-frequencies. 6th World Congr. Struct. Multidisc. Optim. Brazil.
[15]. O. Sigmund and P. M. Clausen, (2007) "Topology optimization using a mixed formulation: An alternative way to solve pressure load problems" Comput. Meth. Appl. Mech. Eng., Vol 196: 1874-1889.
[16]. G. I. N. Rozvany, (2008) "A critical review of established methods of structural topology optimization." Struct. Multidisc. Optim. Springer-Verlag.
[17]. Hassani B, Hinton E. (1998) "A review of homogenization and topology optimization III: Topology optimization using optimality criteria" Comput Struct, Vol 69:739-56.
[18]. Sigmund O. (1997) "On the design of compliant mechanisms using topology optimization" Mech Struct, Vol 25:493-524.

AUTHORS

Avinash Shukla obtained his bachelor's degree (B. Tech.) in Mechanical Engineering from Ajay Kumar Garg Engineering College, Ghaziabad, U. P., in the year 2009 and M. Tech. in Design and Production Engineering from G. B. Pant University of Agriculture and Technology, Pantnagar, Uttarakhand in the year 2013. He is currently working as Assistant Professor in the Mechanical Engineering department of IFTM University, Moradabad, U. P. His areas of interest are optimization and finite element analysis.

Anadi Misra obtained his Bachelor's, Master's and doctoral degrees in Mechanical Engineering from G. B. Pant University of Agriculture and Technology, Pantnagar, Uttarakhand, with a specialization in Design and Production Engineering. He has a total research and teaching experience of 26 years. He is currently working as professor in the Mechanical Engineering department of College of Technology, G. B. Pant University of Agriculture and Technology, Pantnagar and has a vast experience of guiding M. Tech. and Ph. D. students.