

FUZZY FRACTIONAL ORDER SLIDING MODE CONTROLLER FOR DC MOTOR

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ABSTRACT

This paper gives fractional surface Sliding Mode Control for DC Motor. The fractional PID controller relying on the sliding-mode theory is used to improve the dynamical characteristics of the drive system. Sliding mode control method is studied for controlling DC motor because of its robustness against model uncertainties and external disturbances. A little chattering phenomenon & fast reaching velocity into the switching hyper plane in the hitting phase is desired in a Sliding Mode Control. To reduce the chattering phenomenon in Sliding Mode Control (SMC) a Fuzzy Logic Controller (FLC) is used to replace discontinuity in the Signum function. For this purpose, we have used a Fractional PID outer loop in the control law then the gains of the sliding term and Fractional PID term are tuned on-line by a fuzzy system, so the chattering is avoided and response of the system is improved here. Initially a sliding PID surface is designed and then, a fractional form of this network $PI^\lambda D^\mu$ is proposed. Then the performance of the controlled system for DC Motor is investigated. The simulation results signify performance of Fuzzy Fractional Order Sliding Mode Controller. Presented implementation results on a DC motor confirm the above claims and demonstrate the performance improvement in this case.

KEYWORDS: PID Controller, Fractional Controller, Sliding Mode Control, Fuzzy Logic, DC Motor.

I. INTRODUCTION

The PID controller is by far the most dominating form of feedback in use. Due to its functional simplicity and performance robustness, the proportional-integral-derivative controller has been widely used in the process industries. The reason for their wide popularity lies in the simplicity of design and good performance including low percentage overshoot and small settling time for slow process plants. The most appreciated feature of the PID controllers is their relative easiness of use, because the three involved parameters have a clear physical meaning. This makes their tuning possible for the operators also by trial-and error and in any case a large number of tuning rules have been developed. Although all the existing techniques for the PID controller parameter tuning perform well, a continuous and an intensive research work is still underway towards system control quality enhancement and performance improvements.

On the other hand, in recent years, it is remarkable to note the increasing number of studies related with the application of fractional controllers in many areas of science and engineering. This fact is due to a better understanding of the fractional calculus potentialities [1]. In the field of automatic control, the fractional order controllers which are the generalization of classical integer order controllers would lead to more precise and robust control performances. Although it is reasonably true that the fractional order models require the fractional order controllers to achieve the best performance, in most cases the researchers consider the fractional order controllers applied to regular linear or non-linear dynamics to enhance the system control performances. The $PI^\lambda D^\mu$ controller, involving an integrator of order λ and a differentiator of order μ has the better response in comparison with the classical PID controller [2]. The form of fractional PID followed by:

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (1)$$

Sliding Mode Control guarantees the stability and robustness of the resultant control system, which can be systematically achieved but at the cost of chattering effect. Unfortunately an ideal Sliding Mode Controller has a discontinuous switching function. Due to imperfect switching, the issue of chattering will be arisen in the practice. Chattering can be made negligible if the width of the boundary layer is chosen large enough. Due to discontinuity in the sign function at the reaching phase a chattering phenomenon is occurred [3-5].

Sliding Mode Control for DC Motor is studied. To suppress the chattering several techniques have been investigated [9]. In this paper, initially PID surface Sliding Mode Controller is proposed. Thereafter a fractional surface Sliding Mode Controller is applied. To reduce the chattering phenomenon Fuzzy Logic Control is used to replace the discontinuous function in the Sliding Mode Control [8-9]. Conventional PID Controllers has many limits due to dead line, noise, etc. Fractional order PID Control is the development of integer order PID Control [11-12].

II. FUNDAMENTALS OF FRACTIONAL CALCULUS

Fractional calculus has gained wide acceptance in last couple of years. J Liouville made the first major study of fractional calculus in 1832. In 1867 A. K. Grunwald worked on the fractional operations. G. F. B. Reimann developed the theory of fractional integration in 1892. Fractional order mathematical phenomena allow us to describe and model real object more accurately than the classical integer methods.

The past decade has seen an increase in research efforts related to fractional calculus and use of fractional calculus in control system. For a control loop perspective there are four situations like (i) integer order plant with integer order controller. (ii) Integer order plant with fractional order controller. (iii) Fractional order plant with integer order controller. (iv) Fractional order plant with fractional order controller. Fractional order control enhances the dynamic system control performance.

The fractional-order differentiator can be denoted by a general fundamental operator as a generalization of the differential and integral operators, which is defined as follows

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (dt)^{-\alpha} & \alpha < 0 \end{cases} \quad (2)$$

Here α is the fractional order, a and t are the limits. The three definitions used for the general fractional differ integral are the Grunwald–Letnikov (GL) definition, the Riemann–Liouville (RL) and the Caputo definition. There are two commonly used definitions for the general fractional differentiation and integration, i.e., the Grunwald–Letnikov (GL) and the Riemann–Liouville (RL). The GL definition is as

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\lfloor \frac{t-a}{h} \rfloor} (-1)^j \binom{\alpha}{j} f(t - jh) \quad (3)$$

where $\lfloor . \rfloor$ is a flooring-operator.

Here

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \quad (4)$$

while the RL definition is given by:

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(T)}{(t-T)^{\alpha-n+1}} dT \quad (5)$$

The condition for above equation is $n-1 < \alpha < n$. $\Gamma(.)$ is called the gamma function.

III. MODEL OF DC MOTOR

DC motors are widely used in industrial and domestic equipment. The control of the position of a motor with high accuracy is required. The electric circuit of the armature and the free body diagram of a motor are shown in fig. 1

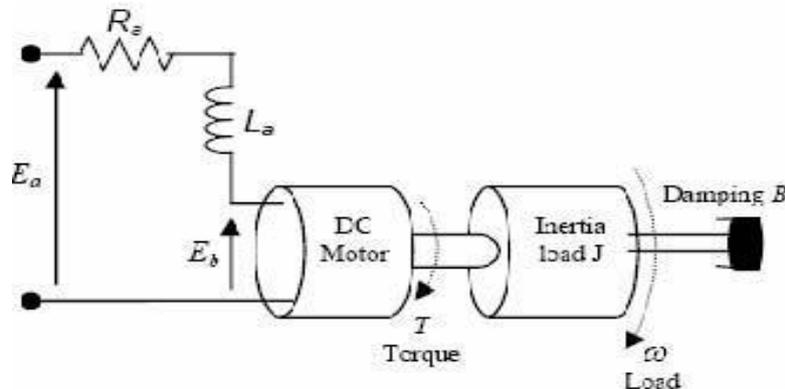


Fig.1 The Structure Of a DC Motor

A desired speed may be tracked when a desired shaft position is also required. In fact, a single controller may be required to control both the position and the speed. The reference signal determines the desired position and/or speed. The controller is selected so that the error between the system output and reference signal eventually tends to its minimum value, ideally zero. There are various DC motor types. Depending on type, a DC motor may be controlled by varying the input voltage whilst another motor only by changing the current input. In this paper a DC motor is controlled via the input voltage. The control design and theory for controlling a DC motor via current is nearly the same. For simplicity, a constant value as a reference signal is injected to the system to obtain a desired position. However, the method works successfully for any reference signal, particularly for any stepwise time-continuous function. This signal may be a periodic signal or any signal to get a desired shaft position, i.e. a desired angle between 0 and 360 degrees from a virtual horizontal line. The dynamics of a DC motor may be expressed as:

$$\begin{aligned}
 E_a &= R_a I_a + L_a \left(\frac{dI_a}{dt} \right) + E_b \\
 T &= J \left(\frac{d\omega}{dt} \right) + B\omega - T_l \\
 T &= K_T I_a ; E_b = K_b \omega \\
 \left(\frac{d\omega}{dt} \right) &= \Theta
 \end{aligned}
 \tag{6}$$

With the following physical parameters:

E_a : The input terminal voltage (source), (v);

E_b : The back emf, (v); R_a : The armature resistance, (ohm);

I_a : The armature current (Amp);

L_a : The armature inductance, (H);

J : The moment inertial of the motor rotor and load, (Kg.m²/s²);

T : The motor torque, (Nm)

ω : The speed of the shaft and the load (angular velocity), (Rad/s);

Θ : The shaft position, (rad);

B : The damping ratio of the mechanical system, (Nms);

K_T : The torque factor constant, (Nm/Amp);

K_b : The motor constant (v-s/ rad). Block diagram of a DC motor is shown in fig.2.

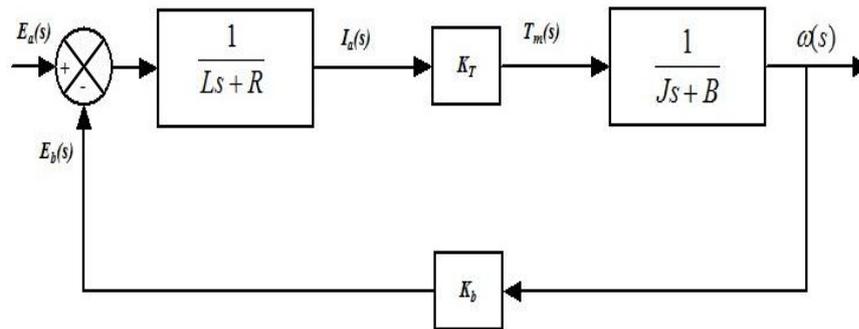


Fig.2 Block diagram Of a DC Motor

So the final transfer function will be

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T}{[(R+Ls)(Js+B)+K_TK_b]} \quad (7)$$

After applying the values of DC motor parameters as mentioned in appendix A, final transfer function can be represent as

$$G(s) = \frac{1.5}{s^2 + 14s + 40.02} \quad (8)$$

IV. SLIDING MODE CONTROLLER

A Sliding Mode Controller is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that can map plant state to a control. Surface and the switching among different functions are determined by plant state that is represented by a switching function. Without loss of generality, consider the design of a sliding mode controller for the following second order system: Here $u(t)$ is the input to the system:

$$u = u_s + u_{eq} \quad (9)$$

Where $u = -k \text{sat}(s/\varphi)$ and constant factor φ defines thickness of the boundary layer.

$\text{Sat}(s/\varphi)$ is a saturation function that is defined as:

$$\text{sat}\left(\frac{s}{\varphi}\right) = \begin{cases} \frac{s}{\varphi}, & \text{if } \left|\frac{s}{\varphi}\right| \leq 1 \\ \text{sgn}\frac{s}{\varphi}, & \text{if } \left|\frac{s}{\varphi}\right| > 1 \end{cases} \quad (10)$$

The function between u_s and s/φ is shown in fig.3.

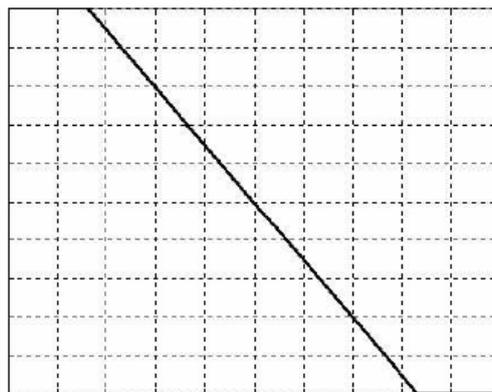


Fig 3. Switching surface in the phase plane

The control strategy adopted here will guarantee the system trajectories move toward and stay on the sliding surface $s=0$ from any initial condition if the following condition meets

$$ss \leq \eta |s| \quad (11)$$

Where η is a positive constant that guarantees the system trajectories hit the sliding surface infinite time. Using a sign function often causes chattering in practice. One solution is to introduce a boundary layer around the switch surface. This controller is actually a continuous approximation of the ideal relay control. The consequence of this control scheme is that invariance of sliding mode control is lost. The system robustness is a function of the width of the boundary layer. The principle of designing sliding mode control law for arbitrary-order plants is to make the error and derivative of error of a variable is forced to zero. In the DC motor system the position error and its derivative are the selected coordinate variables those are forced to zero. Switching surface design consists of the construction of the switching function. The transient response of the system is determined by this switching surface if the sliding mode exists. First, the position error is introduced

$$e(k) = \Theta_{ref}(k) - \Theta(k) \tag{12}$$

Where $\Theta_{ref}(k)$, $\Theta(k)$ are the respective responses of the desired reference track and actual rotor position, at the K the sampling interval and $e(k)$ is the position error. The sliding surface (s) is defined with the tracking error (e) and its integral ($\int e dt$) and rate of change (\dot{e})

$$s = \dot{e} + \lambda_1 e + \lambda_2 \int e dt \tag{13}$$

Where $\lambda_1, \lambda_2 > 0$ are strictly positive real constant. The basic control law of Sliding Mode Controller is given by

$$u = -k \operatorname{sgn}(s) \tag{14}$$

Where k is a constant parameter, $\operatorname{sgn}(\cdot)$ is a sign function and s is the switching function.

V. DESIGN OF FUZZY FRACTIONAL PID SLIDING MODE CONTROL

In this section, a fuzzy sliding surface is introduced to develop a sliding mode controller. Which the expression $\operatorname{sat}(s/\varphi)$ is replaced by an inference fuzzy system for eliminate the chattering phenomenon. In addition, to improve the response of system against external load torque, the sliding mode controller designs with a Fractional PID out loop. The designed fuzzy logic controller has two inputs and an output. The inputs are sliding surface (s) and the change of the sliding surface in a sample time, and output is the fuzzy gain (k_{fuzz}). The fuzzy controller consists of three stages: Fuzzyfication, inference engine and Defuzzyfication. Then, a 3*3 rule base was defined (Table) to develop the inference system. Both Fuzzyfication and inference system were tuned experimentally. The membership function of inputs variables and control variable are depicted in Fig. 4, 5 resp.

Table 1: Fuzzy Sliding Mode Control Rule Table

s	N	Z	P
\dot{s}			
N	NB	NM	Z
Z	NM	Z	PM
P	Z	PM	PB

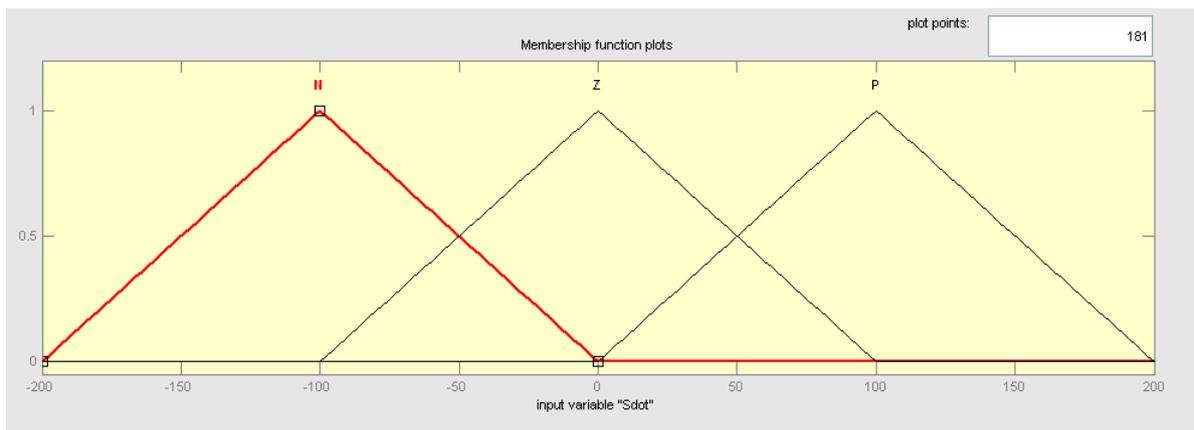
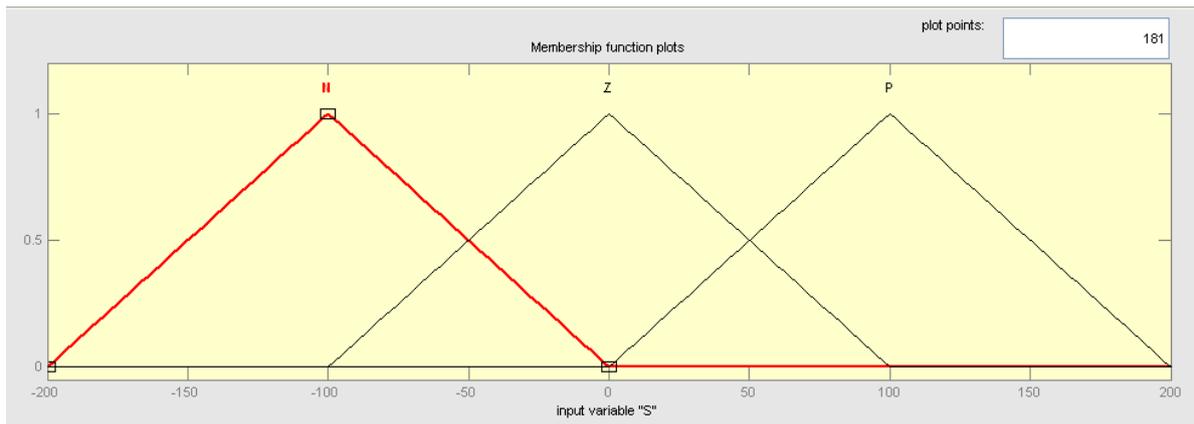


Fig 4. Membership function for Input variables

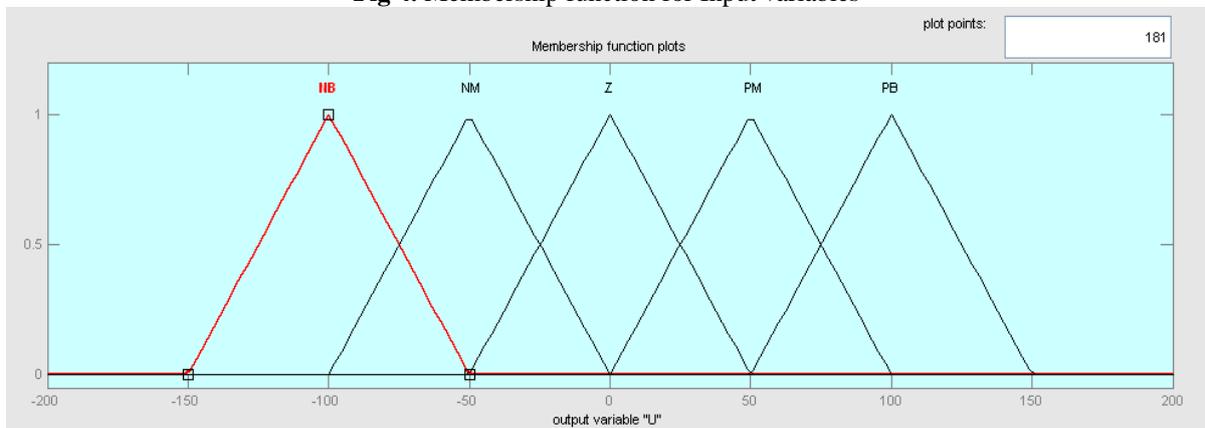


Fig 5. Membership function for Control variable

Fig.6 shows the Simulink model for the DC Motor using Fuzzy PID sliding mode control. For the robust control of DC Motor the fractional order PID Controller & PID Controller are designed and simulated using Simulink model

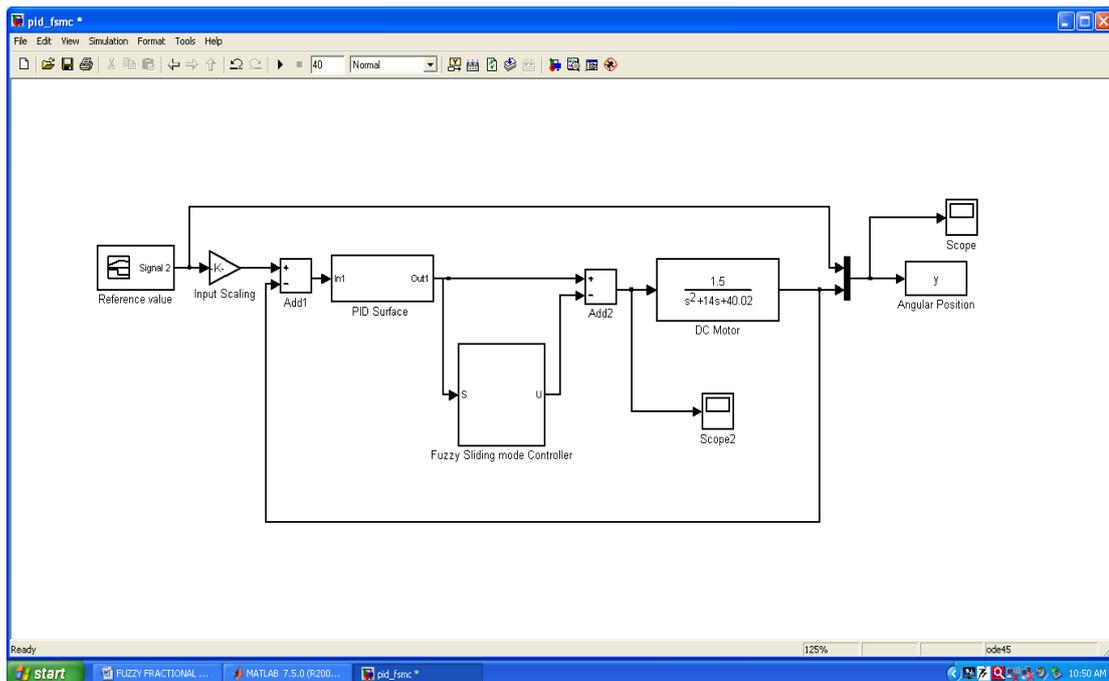


Fig.6 Simulink Model of DC Motor using Fuzzy PID sliding Mode Control

Fig.7 shows the Simulink Model of DC Motor using fuzzy Fractional order PID Sliding Mode Control and Fuzzy Sliding Mode Controller is shown in fig.8.

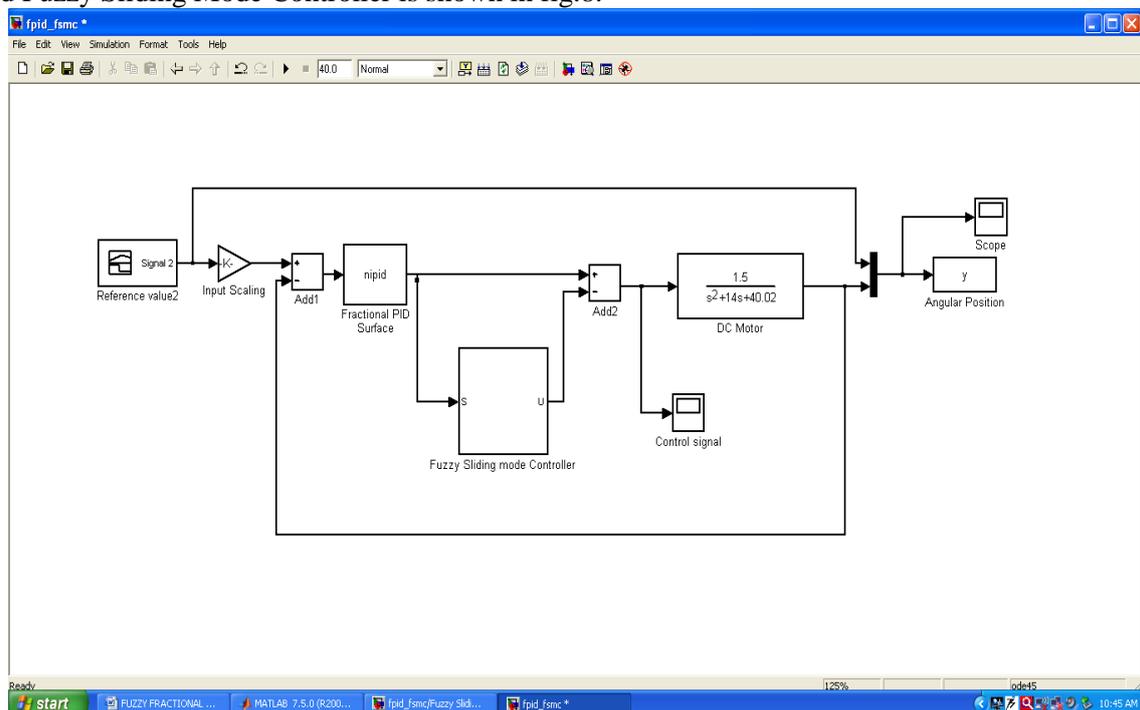


Fig.7. Simulink Model of DC Motor using Fuzzy Fractional order PID Sliding Mode Control.

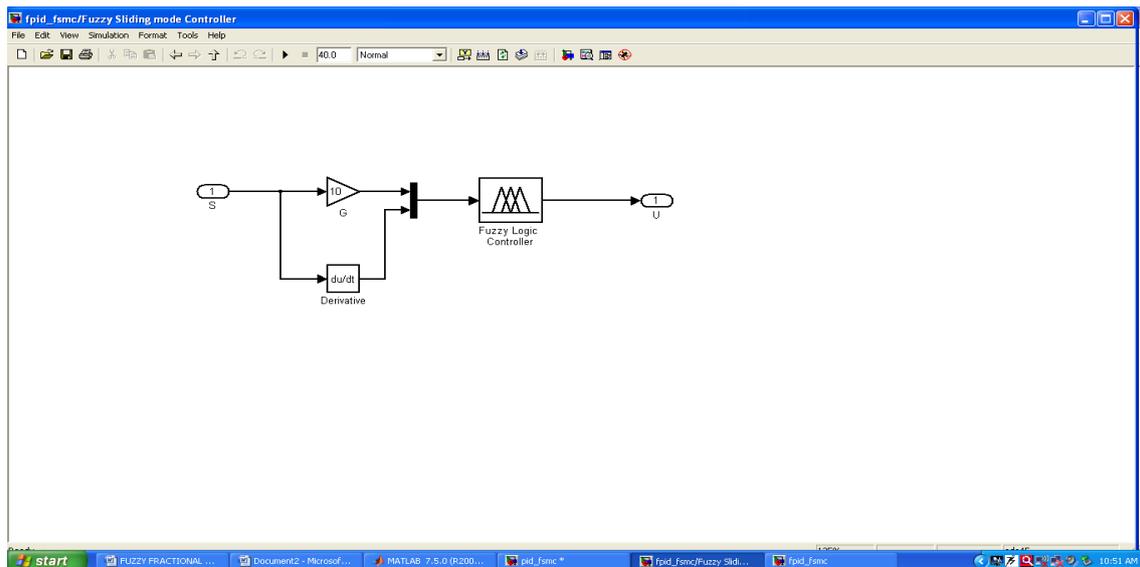


Fig.8. Simulink Model of Fuzzy Sliding Mode Controller.

VI. SIMULATION RESULTS

In this section, the overall model of DC motor with sliding mode controller and fuzzy logic and Fractional PID is implemented in MATLAB/Simulink. The simulink model of the PID with Fuzzy SMC is shown in Fig. 6. And the Simulink Model of FOPID with Fuzzy SMC is shown in Fig.7. The system is tuned for the following parameter for the PID Controller.

$$\begin{aligned} K_p &= 12 \\ K_i &= 0.01 \\ K_d &= 0.01 \end{aligned}$$

PID controller works well for the system with fixed parameters. However, in the presence of large parameter variations or major external disturbances, the PID controllers usually face trade-off between:

- i. Fast response with significant overshoot.
- ii. Smooth but slow response.

For a DC Motor system a controller with more number of tuning parameters and which works well for the complex non-linear systems is to be used. Fractional PID controller is one such controller which is to be design for the DC Motor control. A fractional PID controller is designed for the system by experimental method with following parameters:

$$\begin{aligned} K_p &= 12 \\ K_i &= 0.01 \\ \lambda &= 0.2 \\ K_d &= 0.01 \\ \mu &= 0.5 \end{aligned}$$

Fig.9 shows of the response of the system using conventional PID with FSMC and Fig.10 shows the response of the system using Fractional PID Controller with FSMC.

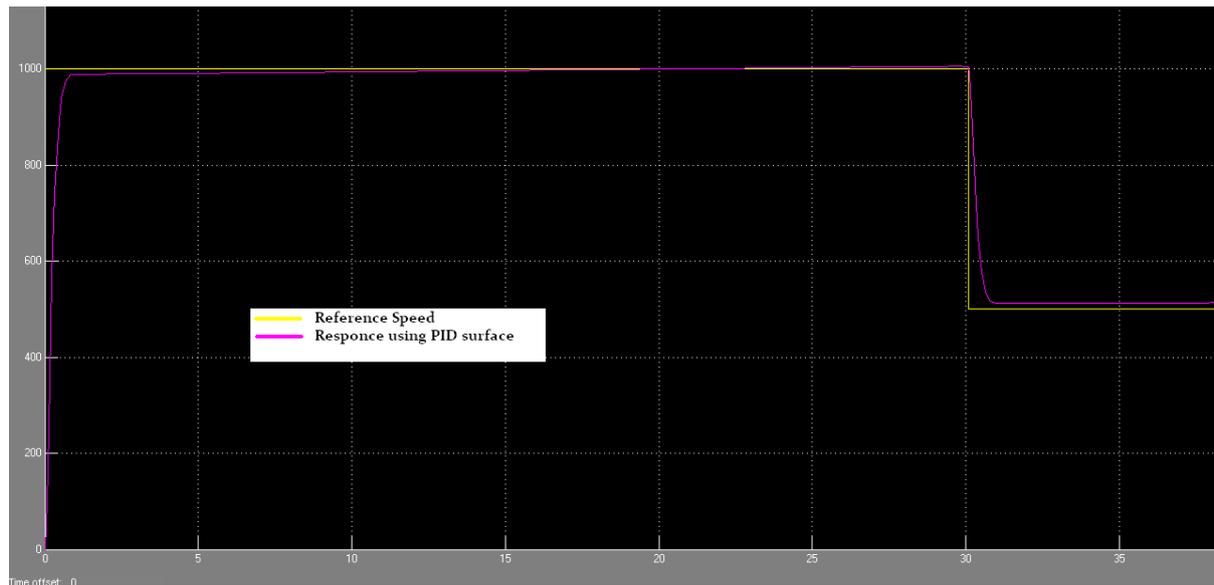


Fig.9 Response of DC Motor using Fuzzy PID surface Sliding Mode Control

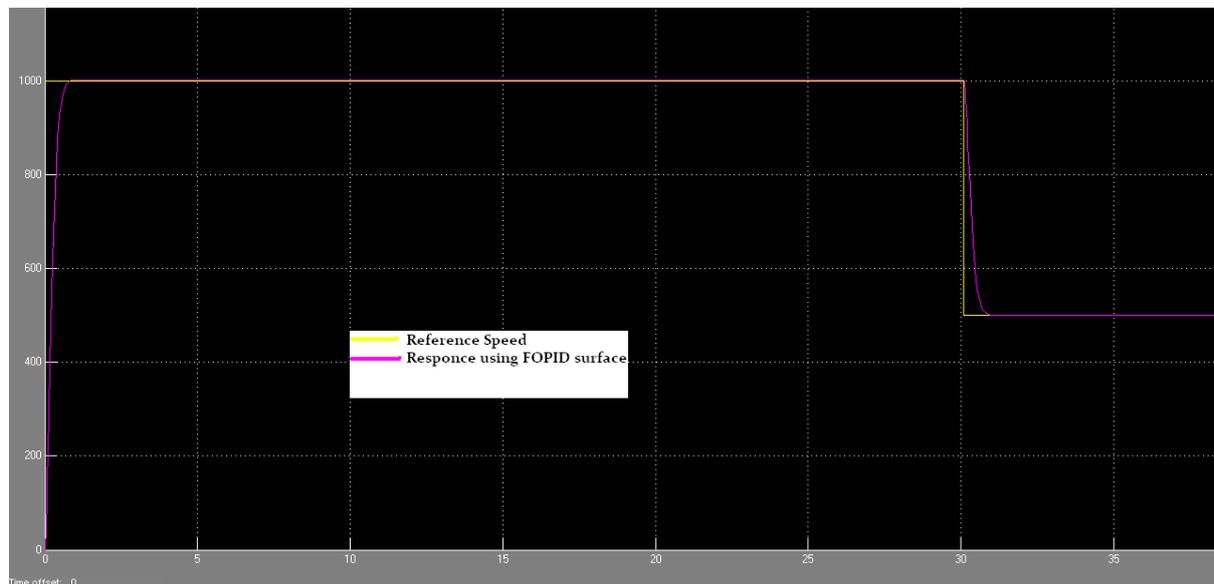


Fig.10 Response of DC Motor using Fuzzy Fractional PID surface Sliding Mode Control

VII. CONCLUSION

In present work, performance comparison of PID controller with that of fractional order PID controller is presented. Firstly, a simulation model of DC Motor is constructed with the help of MatLab/Simulink module. Then, performance comparison of PID controller with that of fractional order PID controller are simulated and studied. According to the simulation results the Fuzzy Fractional PID sliding mode controller can provide the properties of insensitivity and external disturbances, and response of DC Motor for this controller is very smoother and better than the response obtained with Fuzzy PID sliding mode controller. Fractional order PID controller for integer order plants offer better flexibility in adjusting gain and phase characteristics than the PID controllers, owing to the two extra tuning parameters i.e. order of integration and order of derivative in addition to proportional gain, integral time and derivative time.

APPENDIX A

Rated Power (P)	5 Hp
Rated Armature voltage	240 V
Armature Resistance R	2Ω
Armature Inductance L	0.5H
Back EMF constant K_b	0.015
Motor constant K_t	0.015
Friction coefficient of motor B	0.2Nms/rad
Moment of Inertia J	0.02kg-m ²

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