INVENTIVE BURNING SHIP

Shafali Agarwal[1] and Ashish Negi[2]
[1]Research Scholar, Singhania University, Rajasthan, India
[2]Dept. of Computer Science, G.B. Pant Engg. College,
Pauri Garwal, Uttarakhand, India

ABSTRACT
Michelitsch and Rossler in 1992 have applied seemingly small changes in complex analytic Mandelbrot set function and it makes a remarkable difference in the appearance of obtained fractal images. They named it burning ship as the image looks like a ship is going up in flame. The purpose of this paper is to explore the unidentified geometrical structure of various fractal images after applying iteration methods such as Mann iteration and Ishikawa iteration methods. Some of the obtained images characterize a strong connection between Mandelbrot set and burning ship fractal.

KEYWORDS: *Mann Iteration, Ishikawa Iteration, M-Burning Ship, I-Burning Ship, Quasi Julia Set.*

I. INTRODUCTION

Mandelbrot set is a well known fractal for many mathematicians. A lot of researches about the various format of Mandelbrot set have been done by researchers. In 1992, Michelitsch and Rossler have applied a small change in standard Mandelbrot set function in terms of its imaginary part. Initially Michelitsch had done experiment in 1992 with the absolute part of the function and got a completely new fractal images see Michelitsch [3]. Appearances of obtained images are incredibly beautiful and completely different from previous fractal images known as Burning ship. The Burning ship fractal is generated by iterating the function:

$$z_{n+1} = (|\operatorname{Re}(z_n)| + i|\operatorname{Im}(z_n)|)^2 + c \qquad \text{(Eq.1)}$$

Here the real and imaginary components of the complex quadratic equation are:

$$x_{n+1} = x_n^2 - y_n^2 - c_x \qquad \text{(Eq. 2)}$$

And

$$y_{n+1} = 2x_n y_n - c_y \qquad \text{(Eq. 3)}$$

For $x_0 = 0$ and $y_0 = 0$, the above equations (2 & 3) yield the fractal images in the c-plane (parameter space) which will either escape or remain bounded. See Mandelbrot & Peitgen et al.[1, 6].

Now we are considering absolute value of x_n and y_n which helps to shown a lot of curviness and turn it into angles and lines. It makes the difference between Mandelbrot set and burning ship fractal as in case of Burning ship the values to be considered in its absolute form of its real and imaginary components before squaring, whereas in Mandelbrot set no absolute part is considered.

Based on these differences, the Mandelbrot set contains images of classical beauty, organic forms and ornate scrollwork and the burning ship contains cartoonist forms and patterns that look like war print, paw print, tokens and towers.

Earlier Ivo et al. have used the geometrical configuration such as iterations, range, average speed of generating etc of burning ship and other fractals for encrypting the information [4]. In the similar way, the various fractals are used to simulate the real world objects so that the analysis can be made easy for all those objects like structure of Eiffel tower, coastline of Britain etc.

The outline of this study is organized as follows: it is started with a detailed structural description of burning ship. The core of this study is to apply Mann iteration and Ishikawa iteration methods and generate beautiful fractals named as M-burning ship and I-burning ship respectively. Later we discuss about the process involved to create Julia set images followed by conclusion.

II. STRUCTURE OF BURNING SHIP

Burning ship is a region of chaos that contains wild and noisy images. Most fractals look like oil or watercolours as they were spray painted on a brick wall. The intensity of colour assigned to each pixel is determined by how fast a pixel tends to infinity. E.g. a pixel shown in black colour remains in the centre of fractal and never diverges to infinity. It will either oscillate or converges to single point. Change the colour of pixel from black to light colour, it shown the divergence of pixel very quickly to infinity. Even the Mandelbrot set creation used the same technique.

The main figure shows diversified images like on top border you will be surprised to see a lady as passenger on the ship. This image shows a lady is waving her hands, probably calling for help. She is standing on the deck where the lady structures take a deep dip down. On the upper side of burning ship, there is a large dusty pattern in contrast to other corresponding parts. Within the dust dumb-bell like voids of all sizes are randomly distributed with some mini burning ships.

A tower like structure is strangely ordered and as well magnify that we are astounded to find endlessly varying kaleidoscopic images. These patterns are weird, wonderful, and undeniably beautiful and still show that twisted edginess we have come to expect from burning ship. This structure is somewhat related to image reported in Rossler [10] for the simplest non-analytic Julia set.

On zooming the sail of burning ship, there are infinite incredibly intricate small mini ships are shown the presence of midgets on the external ray. Midgets are small images of the same fractal found in the scattered surrounding of the superior set [5]. Burning ship image can also be extended with different power values. *For eg.* A burning ship equation with power *3* is known as bird of pray.

III. RESULT AND ANALYSIS

Initially we have described the basic geometrical structure of burning ship function, which by default used Picard iteration method. *i. e.*

$$x_{n+1} = f(x_n)$$

Now we define burning ship with respect to Mann iterates. We named it M-Burning ship.

3.1 M-Burning Ship

The method is given as below. See e.g. Negi A & Rani M. [4] and Rani M. & Kumar V.[8, 9].

$$z_d = sf(z_{d-1}) + (1-s)z_{d-1}, \qquad \text{(Eq. 4)}$$

where *f(z)* is the function on which we applied the given iteration method, *z* is a complex number and $0<s<1$ and *s* is convergent to a non-zero number. Mann iteration is based on one step feedback machine.

In case of cubic polynomial, for $s=0.3$, a smooth sword like fractal was shown. As we changed the value of *s* from *0.3 to 0.7* with rotation angle 270°, a beautiful image of Eiffel tower with sea waves was obtained. Now for $s=1$, an identical mini burning ship shapes on its external rays was publicized. As we know the burning ship function is a form of Mandelbrot set function, accordingly for $s=0.5$, there is a Mandelbrot set image was identified for cubic polynomial. It proved that both functions were related to each other. For bi-quadratic function at $s=0.7$, another stunning sparkling crackers like image was obtained which is diagonally symmetrical.

After applying Mann iteration, we got some strange but self explanatory images which can be useful for further research work.

3.2 I-Burning Ship

Next we have applied another iteration method i.e. Ishikawa iteration. This iteration method works on two step feedback process.
Ishikawa Iteration method: The method is given as below. See e.g. Rana R. et al. [7]

$$y_n = (1-s')x_n + s'Tx_n, \qquad n \geq 0 \qquad \text{(Eq. 5)}$$

$$x_{n+1} = (1-s)x_n + sTy_n, \qquad n \geq 0 \qquad \text{(Eq. 6)}$$

where z is a complex number, *0<s<1 and 0<s'<1, where s and s'* is convergent to a non-zero number.
After applying the above iteration process, obtained images were termed as I-Burning ship.
In our experiment we have considered higher powers of the burning ship function to get the momentous images. Let's start with the power value *5* at *s=0.5* and *s'=0.8*, we have shown three images for the same values but with different magnification factor.
There is one image on magnification value *0.45* which was diagonally symmetrical. On zooming there was sparkling Mandelbrot set bulb is attached with the original image. Again magnifying its external ray with value *30*, we got a beautiful image look like earthen lamp (diya) with decorative rays in all direction.
Further increase the power value to *8* and at *s=0.3 and s'=0.5*, an appearance of two elephants are hand shaking with each other is shown. You will be surprised to see the Julia set image in the burning ship fractal function without switching to Julia set function for *n=10, s=0.8 and s'=0.5* with magnification factor *2732*.

3.3. Quasi Julia Set

Michelitsch and Rossler entitled to Julia set as Quasi Julia Set because it refers to non analytic images which don't obey Cauchy-Riemann conditions (Eq. 7): See Michelitsch M. and Rossler O.E. [2]

$$\begin{aligned} \partial f_1(x,y)/\partial x &= \partial f_2(x,y)/\partial y \\ \partial f_1(x,y)/\partial y &= -\partial f_2(x,y)/\partial x \end{aligned} \qquad \text{(Eq. 7)}$$

The creation of Quasi Julia Set along real axis is identical to Julia set taken from Mandelbrot set because at $y=0$ implies $z^2 = x^2$. Otherwise for $y!=0$ not a single Julia set point will be common between Mandelbrot set and burning ship.
To consider different Julia set from Mandelbrot set we have to consider the orbit of all points that will take a point somewhere in second or fourth quadrants i.e. $(-x,y)$ or $(x,-y)$. The y value of next iteration from within one of these quadrants will be:

$$z = (-x)^2 + (y)^2 + i*2*(-x)*(y) + c \qquad \text{(Eq. 8)}$$

Or

$$z = (x)^2 + (-y)^2 + i*2*(x)*(-y) + c \qquad \text{(Eq. 9)}$$

If a point is already in one of these quadrants, condition is satisfied. Otherwise we have three conditions:
(1) If $x^2 = y^2$, then the series will be

$$(x^2 - y^2, 2*|x|*|y|) = (0, 2*|x|*|y|)$$

Next iteration will be

$$(0-(2*|x|*|y|)^2, 2*0*2*|x|*|y|) = (-(2*|x|*|y|)^2, 0) = (-x, 0)$$

(2) If $x^2 < y^2$, then the series will be

$$(x^2 - y^2, 2*|x|*|y|) = (-x, y)$$

(3) If $x^2 > y^2$, then the series will be

$$(x^4 - 6*x^2*y^2 + y^4, 4*|x|^3*|y| - 4*|y|^3*|x|)\ (x^2 - y^2, 2*|x|*|y|) = (x, y)$$

$$((x^2-y^2)^2 - 4*(x^2-y^2, 2*|x|*|y|) =$$

$$(-x, y)*x^2*y^2, 2*2*|x|*|y|*(x^2-y^2))$$

After solving it gives

$$(x^4 - 6*x^2*y^2 + y^4, 4*|x|^3*|y| - 4*|y|^3*|x|)$$

If we keep iterating it and ignore the smaller chunk of fractal, we will get different Julia set than Mandelbrot set's See e.g. 10.

We have obtained some Quasi Julia sets for different seed values. There were lots of surprising images of Julia sets which maps different points in burning ship. According to mapping some times we got an image having spiritual feeling see figures *16* & *17*, which shows the south-western paw print motif. Another image shows a frog like structure for power *3*.

IV. CONCLUSION

Burning Ship is a kind of escape time fractal. It can be analyzed on the basis of escape points set and prisoner points set after applying various Iteration methods. Burning ship has its own unique images like lady on deck, dumbbell like shape, bird of pray etc but sometimes images resembled to Mandelbrot set fractal also see fig. 9. In burning ship fractal midgets are also found on the external ray in terms of mini burning ship. A completely different and religious Julia set images are shown in the form of paw print which mapped to different values in burning ship see fig. 16.

All the fractals form a link with realistic images which can be further useful for approaching researchers. We can also calculate the fixed points for the given images to explain the number of iterations needed to converge.

V. FUTURE WORK

This is just a mild beginning of study the various fractal images of iterated burning ship. One can calculate the fixed point of obtained images, represent the number of iterations used to convergence. These images with its constant *c*-value can be used in fractal cryptography.

REFERENCES

[1]. Mandelbrot B. B., "Fractal Aspects of the Iteration of z->λz (λ-z) for Complex 2", z. *Ann. NY Acad. Sci.* 357, 249(1980).

[2]. Michelitsch M. and Rossler O.E., "The Burning Ship and Its Quasi Julia Sets", Chaos and Fractals: Computer & Graphics, Vol.16, No.4, pp. 435-438 (1992).

[3]. Michelitsch M. and Rossler O. E., "Spiral structures in Julia sets and related sets", In: *Spiral Symmetry*, I. Hargittai and C. A. Pickover (Eds.), World Scientific, Singapore 123-134(1992).

[4]. Motyl I., Jasek R. and Varacha P., "Analysis of Fractal Structure for the Information Encrypting Process", International Journal of Computers, Issue 4, Volume 6, 2012, 224-231.

[5]. Negi A. & Rani M. "Midgets of Superior Mandelbrot set", Chaos, Solitons, and Fractals 36, 237– 245 (2008).

[6]. Peitgen, Heinz-Otto and Richter, Peter. "The Beauty of Fractals: Images of Complex Dynamical Systems", Heidelberg: Springer-Verlag, 1986

[7]. Rana R., Dimri R. C. and Tomar A., "Remarks on Convergence among Picard, Mann and Ishikawa iteration for Complex Space", International Journal of Computer Applications (0975-8887), Vol. 21, No. 9, May 2011, 20-29.

[8]. Rani M. and Kumar V. "Superior Julia set", J Korea Soc Math Educ Ser D Res Math Educations. 8(4):261–77, (2004).

[9]. Rani M. and Kumar V. "Superior Mandelbrot sets", J. Korea Soc. Math. Educ. Ser. D, Res. Math. Educations, 279- 291,(2004).

[10]. Rossler O. E., Kahlert C., Parisi J., Peinke J., and Rohricht B., "Hyperchaos and Julia sets", Z *Naturforsch.* 41a, 819-822 (1986).

[11]. http://theory.org/fracdyn/burningship/julias.html

ITERATED IMAGES OF BURNING SHIP

Figure 1: $f(z)= (|Re(z_n)| + i|Im(z_n)|)^2 + c$

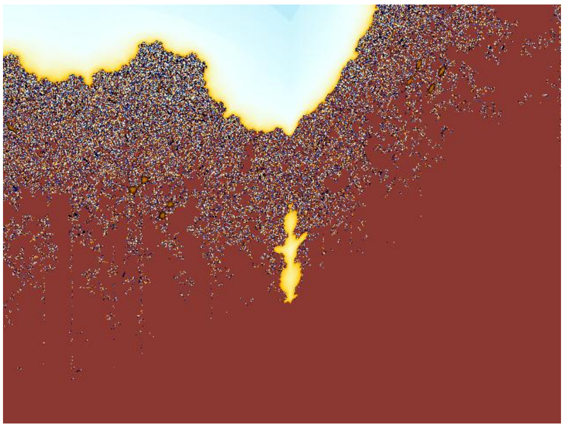

Figure 2: Burning Ship Lady

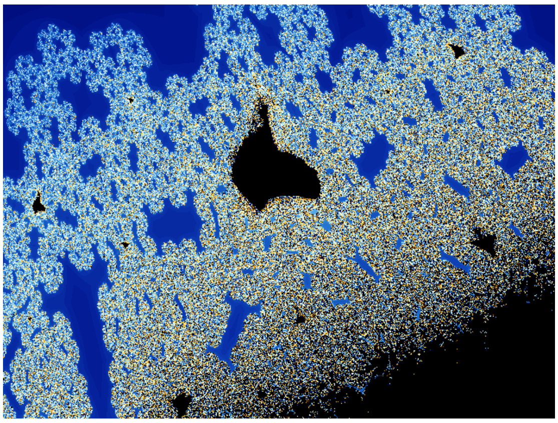

Figure 3: Dusty area showing dumbbell like voids with mini-burning ships

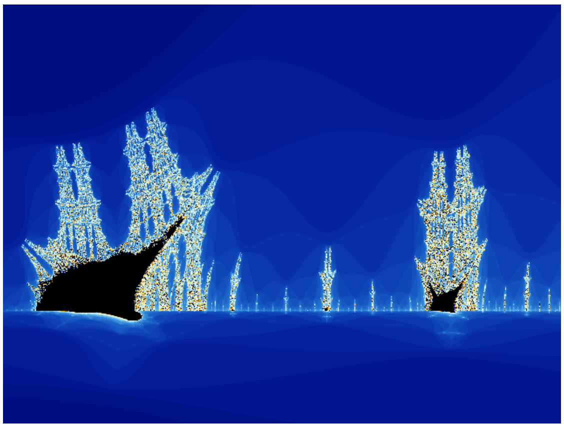

Figure 4: Mini burning ships on external ray with Eiffel Tower structure

Figure 5: Bird of pray with n = 3

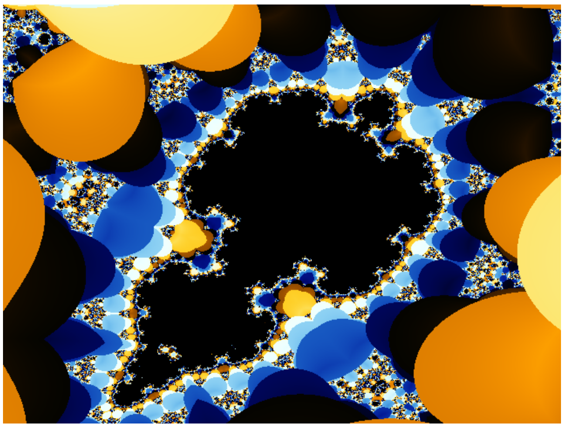

Figure 6: Sword like structure

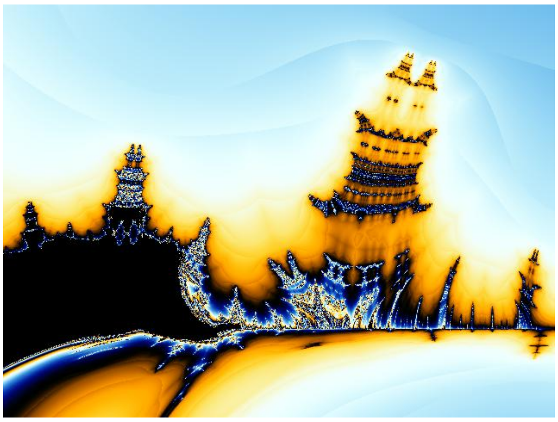

Figure 7: Eiffel Tower structure in sea waves

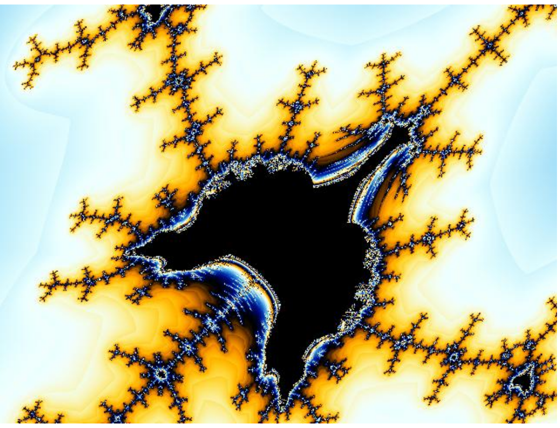

Figure 8: Bird of pray with Midgets (mini bird of pray) on external rays

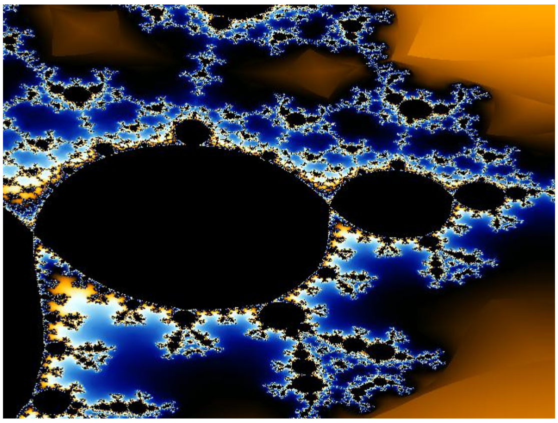

Figure 9: Mandelbrot set image in burning ship fractal

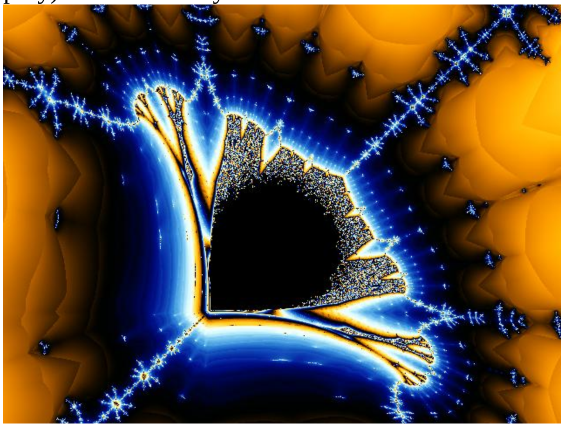

Figure 10: Diagonally symmetrical Sparkling cracker image

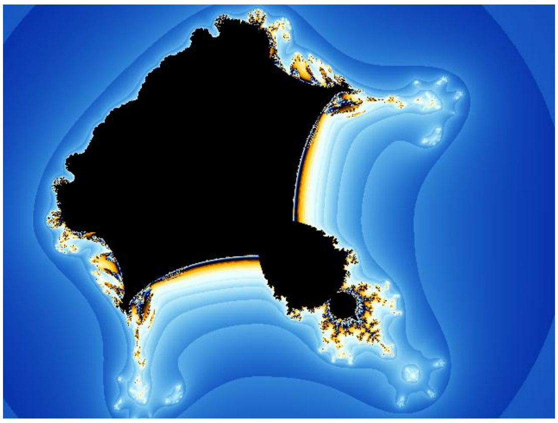

Figure 11: Burning ship function with n=5, magnification=0.45

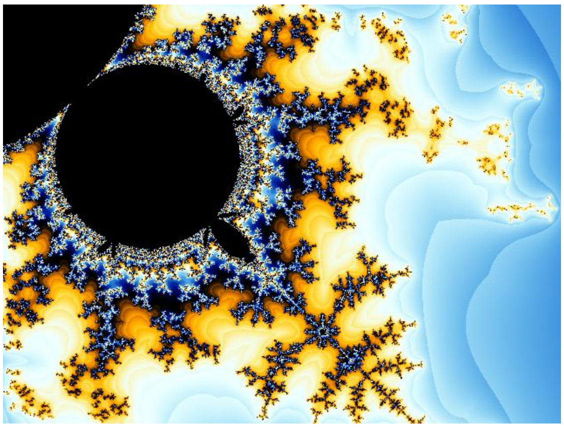

Figure 12: Burning ship function with n=5, magnification=3

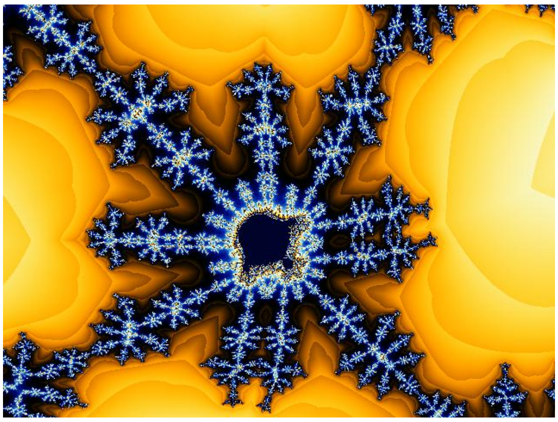

Figure 13: Burning ship function with n=5, magnification=30

Figure 14: Two elephants are hand shaking with each other

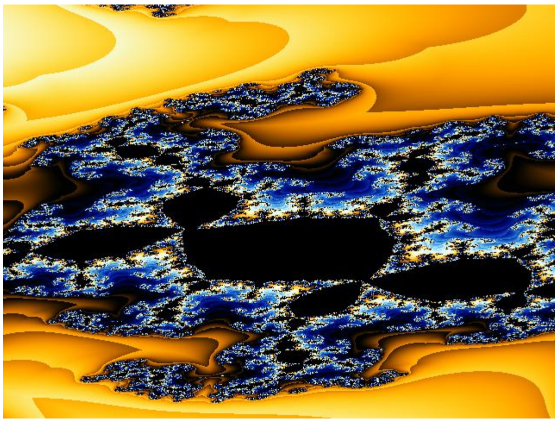

Figure 15: Julia set image in burning ship fractal

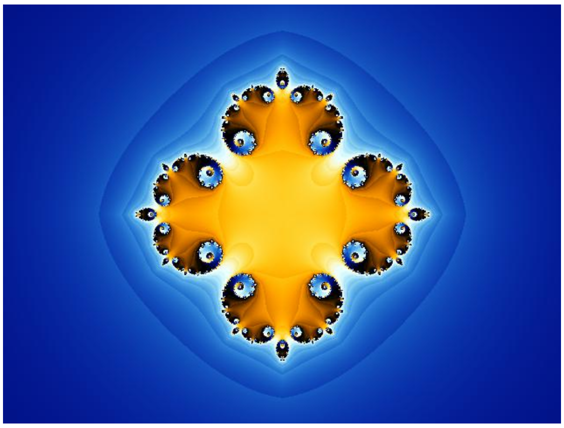

Figure 16: Paw Print Motif 1

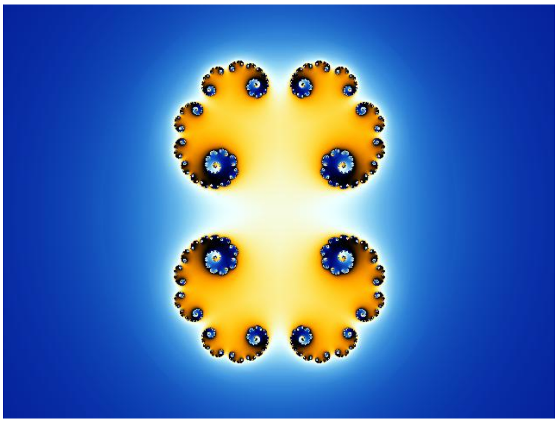

Figure 17: Paw Print Motif 2

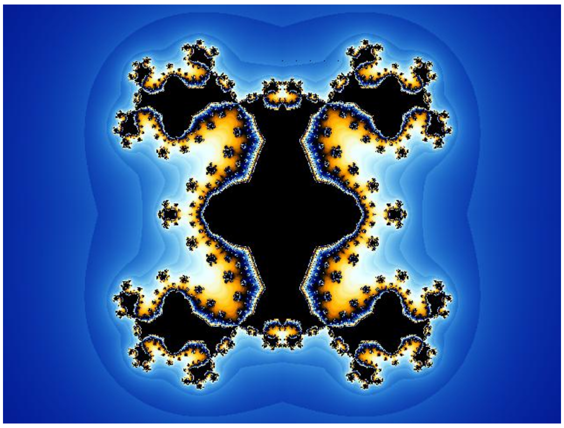

Figure 18: Quasi Julia set with n=3

AUTHORS

Shafali Agarwal is associated as an assistant professor with JSSATE, Noida, formerly I worked with NIET, Greater Noida. I am pursuing Ph.D. from Singhania University, Rajasthan. My research area is fractal analysis which is a part of Computer Graphics. I got published a book titled “Data Structure using C” for engineering students. I have published seven papers in national conference, International conference and International journal which are indexed by ACM, springer, Citeseer, ProQuest, Index Copernicus, EBSCO, Scribd and many more. I have completed graduation in 2001, master in computer applications in 2004 and after that MPhil in 2013.

Ashish Negi is working as an associate professor in G. B. Pant Engineering College, Pauri Garwal, Uttarakhand. He has done B.Sc., M.Sc., P.G.D.C.A., M.C.A. and Ph.D. Now he is pursing MTech from Karnataka State Open University. He is a very eminent person for his organization. Hs had published more than twenty five papers in national conference, International conference and International journal including Elsevier, World journal of Science & Technology etc. He is an active member of "Computer Society of India".