MHD BOUNDARY LAYER FLOW INDUCED BY A PERMEABLE STRETCHING SURFACE

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ABSTRACT

The self similar steady boundary layer flow induced by a permeable continuous surface stretching with velocity $U_w\,(X) = A.x^{-1/2}$ in a quiescent incompressible fluid in the presence of a transverse magnetic field of uniform strength Bo, with suction velocity $V_w(x) = -\left(\frac{\upsilon A}{4}\right)^{1/2}.\ x^{-3/4} f_w$ is considered. Numerical solution of the resulting similarity momentum equation using Runge-Kutta-Fehlberg Forth-Fifth order method is obtained. The influence of various parameters is presented. It is observed that the velocity boundary thickness decreases with the increasing values of the suction parameter (f_w) and the magnetic parameter(M).

KEYWORDS: *boundary layers flow, MHD, stretching surface, numerical study.*

NOMENCLATURE:

A	Strength of stretching velocity
B_0	Constant applied magnetic field
f	Dimensionless stream function
f_w	Dimensionless suction velocity
m	Stretching exponent
u	Downstream velocity
U_w	Stretching velocity
v	Transversal velocity
V_w	Suction velocity
x	Coordinate in direction of surface motion
y	Coordinate in direction normal to surface motion
M	Dimensionless magnetic field parameter
Greek symbols	
η	Dimensionless similarity variable
ρ	Density of fluid
σ_e	Current density
ν	Kinematic viscosity
Superscript	
$'$	Derivative with respect to η
Subscript	
w	Condition at the wall

I. INTRODUCTION

The flow problems with obvious relevance to polymer extension are an interesting area of present day research. In a melt-spinning process, the extradite from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching imports a unidirectional orientation to the extradite, thereby improving its mechanical properties and the quality of the final product greatly depends on the rate of cooling. The classical problem was introduced by Blasius [4] where he considered the boundary layer flow on a fixed plate. The behavior of boundary layer flow due to a moving flat surface immersed in a quiescent fluid was first studied by Sakiadis [5], who investigated it theoretically by both exact and approximate methods. Crane [6] presented a closed from exponential solution for the planar viscous flow of linear stretching case, this problem was then extended by Afzal and Varshney [3] to a general power law of stretching velocity $u_w \sim x^m$, where, x is the distance from the issuing slit and m is a contact. The development of the boundary layer due to stretching permeable sheet was studied by Gupta and Gupta [20], who reported an exact solution for the flow field and a solution in incomplete gamma, functions for the thermal field Ali [15] studied the general case. When the sheet is stretched with stretching velocity of the form x^m . the stretching surface is either considered as an impermeable by Magyari and Keller [8].

As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by number of researchers [3,8,14,15,21]. Miklavcic and Wang [16] obtained an analytical solution for steady viscous hydrodynamic flow over a permeable shrinking sheet. Then, Hayat et al. [11] derived both exact and series solution describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. Grubka and Bodha [14] analyzed heat transfer studies by considering the power law variation of surface temperature. Costell [21] studied the magneto hydrodynamics flow of a power-law fluid over a stretching sheet. Chen [6] analyzed mixed convection of a power law fluid past a stretching surface ion the presence of thermal radiation and magnetic field. For that reason Cortell [22] studied the effects of viscous dissipation and work done by defamation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Abel et al. [17,18] extended the work and studied the viscoelastic MHD flow and heat transfer over stretching sheet with viscous and ohmic dissipation, non-uniform heat source and radiation. Pal and Talukdar [7] studied the unsteady MHD heat and mass transfer along with heat source past a vertical permeable plane using a perturbation analyzed.

The aim of the present paper is to investigate the self similar steady boundary layer flow induced by a permeable continuous surface stretching with velocity $U_w(x) = Ax^{-1/2}$ in a quiescent incompressible fluid in the presence of a transverse magnetic field, with, suction velocity

$V_w(x) = -\left(\frac{\upsilon A}{4}\right)^{\frac{1}{2}} . x^{-\frac{3}{4}} f_w$ is considered, using numerical approach.

II. MATHEMATICAL FORMULATION

Consider the steady boundary layer on a permeable plane wall, stretching with velocity

$u_w = U_w\,(x)$ in a quiescent incompressible fluid in the presence of a transverse magnetic field of uniform strength B_0 fixed to the wall.

We consider the case of a short circuit problem in which the applied electric field $E = 0$, and also assure that the induced magnetic field is small compared to the external magnetic field B_0. This implies a small magnetic Reynolds number. The governing boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_0^2}{\rho}u \tag{2}$$

Along with the boundary conditions for the problem are given by :

$$u(x,0)=U_w(x), \qquad v(x,0)=V_w(x)$$

$$u(x,\infty)=0. \tag{3}$$

The x-axis is directed along the continuous stretching surface and points in the direction of motion. The y-axis is perpendicular to x and to the direction of the slot (z-axis) where the continuous stretching plane issues. u and v are the x and y components of the velocity field of the steady plane boundary flow, respectively. υ denotes the kinematic viscosity of the ambient fluid and will be assumed constant, we mention here that Prandtl's equations (1)-(2), being of parabolic type, require for a complete specification of the problem also and initial condition $u\ (x_0, y>0)=u_0(y)$ at some station $x=x_0$. In the theory of self similar stretching induced flows it is usual to specify this condition by placing the origin of the coordinate system on the slot (which plays the role of a leading edge). In this way the effect of the initial condition $u(0,y>0)=u_0(y)$ on the dimensionless stream function$f(\eta)$, which depends on a similarity variable of the form $\eta \propto x^{-\alpha}\ .y$, is just the same as that of the boundary condition $u(x,\infty)=u_\infty(x)$. in the present case both of them require$f'(\infty)=0$. In other words, in such cases it is usual to ignore the initial condition by absorbing it tacitly in the asymptotic condition of the flow.

For $m=-1/2, i.e.\ U_w(x)=A.x^{-1/2}$. To convert the governing equations into a set of similarity equations, we introduce the following transformation [15]:

$$u(x,y)=A.x^{-\frac{1}{2}}f'(\eta), \qquad v(x,y)=-\frac{1}{2}(A.\upsilon)^{\frac{1}{2}}.x^{-\frac{3}{4}}\left[f(\eta)-3\eta f'(\eta)\right],$$

$$\eta=\frac{1}{2}\left(\frac{A}{\upsilon}\right)^{\frac{1}{2}}.x^{-3/4}.y \tag{4}$$

Which identically satisfies (1), and substituting (4) into (2), we obtain the following non-liner ordinary differential equation :

$$f'''(\eta)+f(\eta)f''(\eta)+2f'^2(\eta)-Mf'(\eta)=0 \tag{5}$$

Where, $M=\frac{4\sigma_e B_0^2}{\rho}.\frac{x}{U_\infty}$ (non dimensional magnetic parameter) (6)

The boundary condition defined as in (3) will take the form,

$$f(0)=f_w, \quad f'(0)=1,$$

$$f'(\infty)=1 \tag{7}$$

Here f_w denotes the dimensionless suction velocity

$$f_w=f(0)=-2(\upsilon A)^{-\frac{1}{2}}x^{\frac{3}{4}}.v(x,0)>0.$$

III. Numerical Solutions

The non- liner ordinary differential equation (5) subject to boundary condition (7) are solve numerically using Runge-Kutta-Fehlberg forth-fifth order method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well-known Runge-Kutta-Fehlberg Fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The boundary condition $\eta = \infty$ were replaced by those at $\eta = 5$ in accordance with standard practice in the boundary layer analysis. The effects of the f_w and M on the velocity distribution and skin-friction are shown in figures 1 to 7.

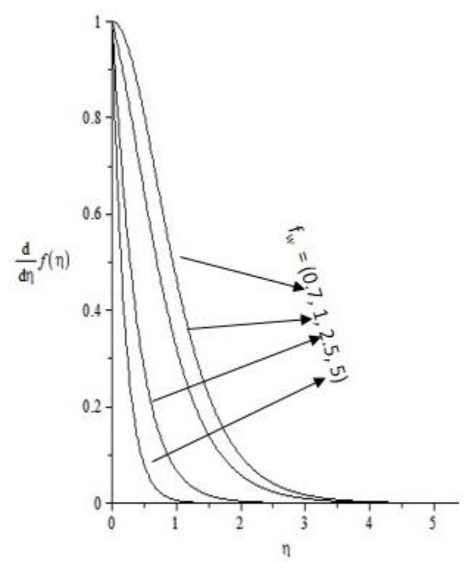

Figure 1. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M=0

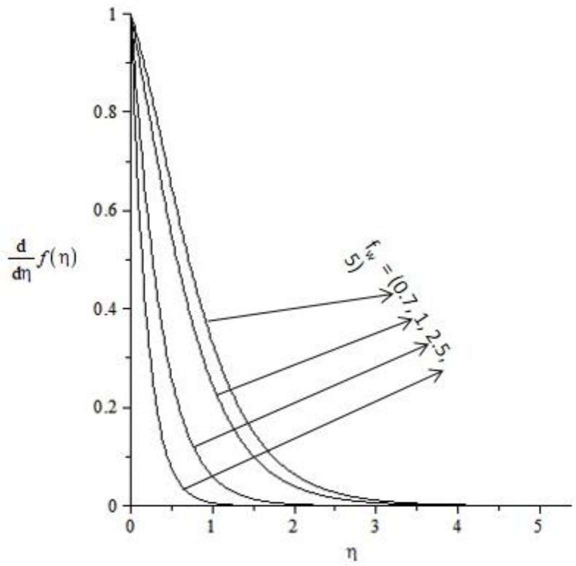

Figure 2. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M=0.1

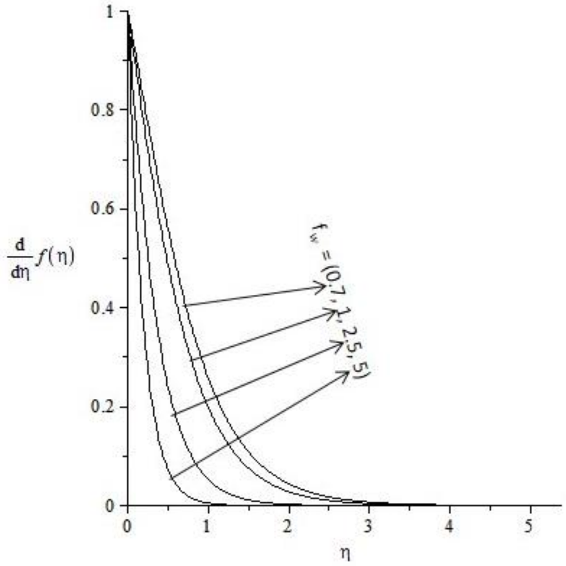

Figure 3. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M=0.2

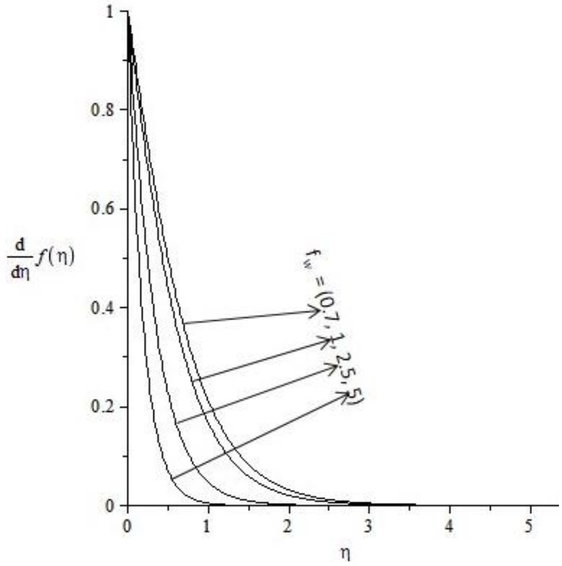

Figure 4. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M=0.3

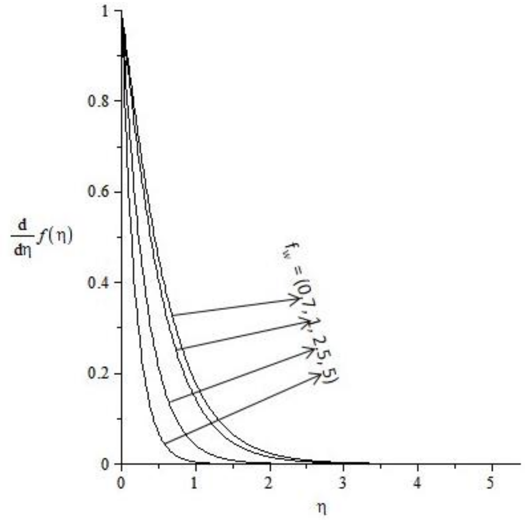

Figure 5. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M=0.4

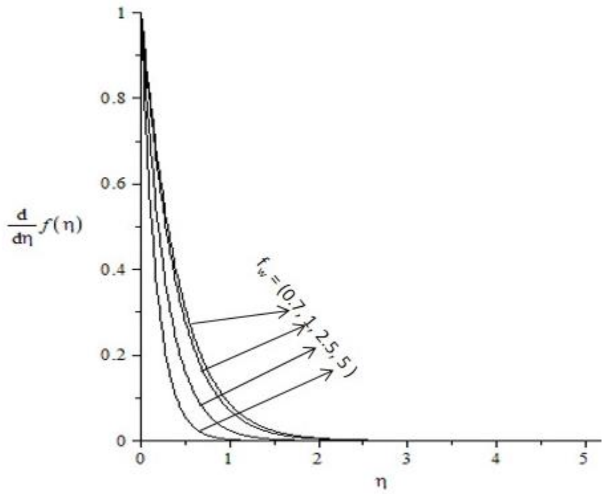

Figure 6. Velocity profile $f'(\eta)$ for various values of suction parameter (f_w), when M= 1

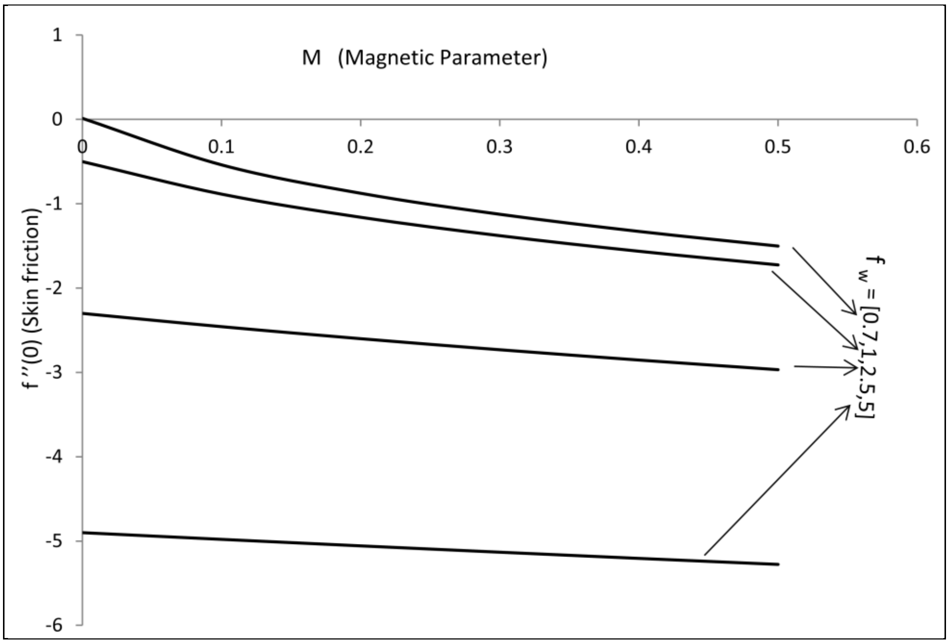

Figure 7. Skin friction $f''(0)$ against magnetic parameter M for various Values of suction parameter (f_w).

IV. CONCLUSION

A mathematical model has been presented for the steady boundary layer flow induced by a permeable continuous surface stretching with velocity $U_w(x) = Ax^{-\frac{1}{2}}$, we notice from figures 1 to 6 if suction parameter (f_w) increases we can find the decrease in the fluid phase velocity i.e. velocity boundary layer thickness decreases. Similar results accrue for increasing value of magnetic parameter(M).
The skin friction against magnetic parameter (M) are shown in figure 7 for different values of suction parameter (f_w). It is noted that for increasing value of f_w , the skin friction increases but it decreases with the increasing values of$M$.
Thus we conclude that we can control the velocity field by suction parameter and by introducing magnetic field.

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PROFILE

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