SEALS IN HIGH PRESSURE PLUNGER PUMPS A MATHEMATICAL APPROACH TO LEAK PROOF SEAL DESIGN

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ABSTRACT
High pressure plunger pumps generally use fixed contour metal seals for sealing the inlet and the outlet areas during its working cycle. Pressures to be sealed are high and pulsating in nature. Assembly and in situ service requirements necessitate a radial expanding seal design; which is preloaded to generate enough radial pressure so that there is no leakage and at the same time control its expansion so as not deform into plastic range and thus hinder seal extraction in service. The objective of this paper is to put forward, a mathematical relationship between clamp load and radial / axial bearing pressure which allows design engineer to assess effect of radial play, friction, seal wedge angle and the preload requirement. A design engineer will be able to decide on dimensions and the taper angle along with the bolt system dimensions at a very early stage of design and be sure of his design concept. Seal size determines the pump throughput and the system maximum pressure and thus the pump rating. Hence importance of this simple and effective evaluation methodology.

KEYWORDS: *Radial seal expansion, Plunger metal seal, Plunger Pump, Sealing*

I. INTRODUCTION

The requirements for a seal in a given application are multifaceted. Seals could be Contact, Non-contact; Fixed Contour solids or Flexible Elastomers. They could be operating in Static, Dynamic or Elasto Hydrodynamic conditions. However, what is demanded from all sealing systems, is, that under the given operating conditions it fulfils its objective of providing system safety at commercially competitive costing for an optimum sealing solution.
High pressure hydraulic plunger pumps (+500 bar) use solid metal seals for ensuring sealing between the pressure cylinder and inlet / outlet valve systems. They seal both in axial and radial direction under a known preload *Fig. 1* If external clamp or preload is high and the seal undergoes localized plastic deformation, it becomes a maintenance nightmare to remove the seal without damaging the polished cylindrical ID. On the other hand if preload relaxes or is not adequate, the possibility of seal failing, by allowing leakage past the seal, is obvious. To meet this dual objective of controlled expansion and leak proof sealing, a hypothesis, using simplified form of ring element under internal and external load, is proposed so as to:

- **a)** Bring out mathematical relationship between the external preload and radial displacement and bearing pressure, of radial expanding seals and
- **b)** Use the finite element analysis technique to correlate the variances in the hypotheses

1.1. Current Published Information

As most of relayed applications lie in the realm of patents and copy rights [1], little published information was available on the behaviour of radial expanding seals, which took into account, assembly play, friction and radial bearing pressure generation. Most published work related to the area of seals in high pressure tubing and fittings. They primarily dealt with seal performance under varying conditions and effect of use of different materials. It has been found that perfect sealing is

achieved after complete plastic deformation of the seal at the joint interface takes place. Sealing then is independent of the surface finish quality and the fluid medium to be sealed. Lehmann [2] also differentiates between critical internal pressure (when the medium just starts to seep) and the blow-by pressure at which then the seal fails. Siebel and Raible [3] analysed the effect of da/di (outer to inner diameter ratio) of various seals on their performance behaviour. They concluded that the sealing behaviour did not change as long as the ratio da /di remained same.
From experimental evaluation [4] it has been concluded that, in general, for a seal to be effective, the clamp load i.e. the external load, must be three times greater than force resulting because of the internal pressure to be sealed. The results from [4] however must be considered, taking into account the seal design and the application.

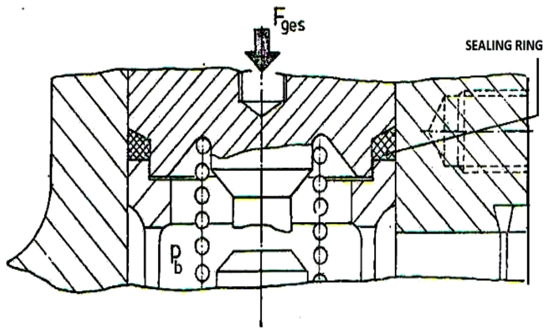


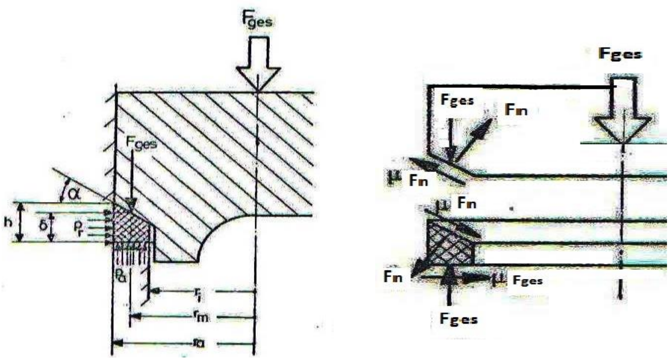
Fig 1 Assembly of a typical seal

Author assumes that a similar result for radial bearing pressure is expected. A typical plunger seal is generally sandwiched between the valve seat and the taper body. The preload F_{ges} is exerted by the bolt arrangement. Fig 1 shows assembly of a typical seal used in such arrangements.

II. MATHEMATICAL MODEL DEVELOPMENT

2.1. Force Polygon

The axial Load F_{ges} is directed to the seal through the valve body. For ease of understanding it is considered to be acting at the seal mid as a uniformly distributed line load along the circumference
The resulting components in the seal plane are shown in ***Fig** 2 & 3,* wherein *Fn* is the normal force perpendicular to the parting plane, *μ.Fn* and *μ. Fges* frictional forces opposing the radial displacement of the sealing ring.



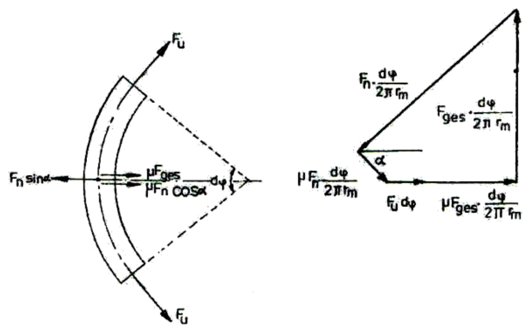


Fig 2 Forces acting on the sealing ring. **Fig 3** Force Polygon of forces acting on ring element

The Force polygon in vertical direction gives

$$\mu F_n \frac{d_\varphi}{2\pi r_m}\cdot \sin\alpha + F_n \frac{d_\varphi}{2\pi r_m}\cos\alpha - F_{ges}\frac{d_\varphi}{2\pi r_m} = 0 \quad (1)$$

And solving for Radial and vertical direction forces

$$F_r = F_{ges} \cdot \frac{(\sin\alpha - \mu\cos\alpha)}{(\mu\sin\alpha + \cos\alpha)} \qquad (2)$$

and, $$F_z = F_{ges} \qquad (3)$$

2.2 Basic Equations

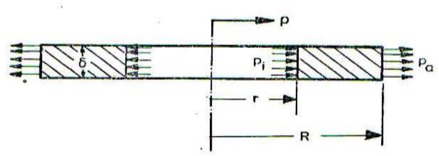


Fig 4 Simplified seal section

If on a uniform thickness ring, bearing pressures *pa* and *pi* act on the inner and the outer surface ref ***Fig** 4,* then the radial displacement, based on Kantorowich [**5**], is given by the equation:

$$u_\rho = \frac{1\text{-}v}{E}\frac{P_a R^2 + P_i\ r^2}{R^2\text{-}r^2} \cdot \rho + \frac{1+v}{E} \cdot \frac{(P_a+P_i)R^2 r^2}{R^2\text{-}r^2} \cdot \frac{1}{\rho} \qquad (4)$$

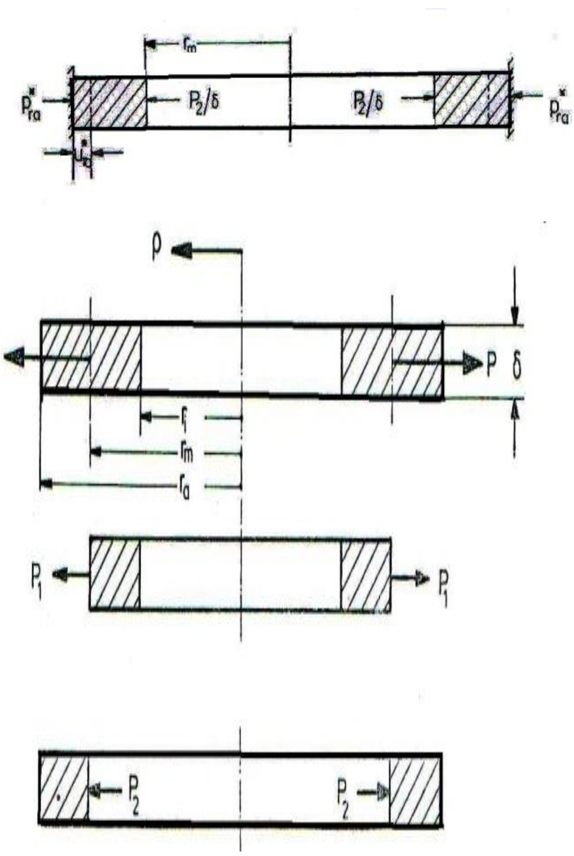


Fig 6 & 7 Ring under radial loads

2.2.1 Case I : Hypothesis neglecting friction; µ = 0

Considering *µ = 0* we get from *Eqn (2)*

$$F_{r\uparrow\mu=0} = F_{ges} .\tan\alpha \qquad (5)$$

and therefore $$P = \frac{F_r}{2\pi r_m} = F_{ges} \cdot \frac{\tan\alpha}{2\pi r_m} \qquad (6)$$

This outwardly directed radial Line load ***P***, we propose, is acting at mean radius r_m ($r_i < r_m < r_a$) Using *Eqn* (4) we consider a case where the basic seal ring cut in two parts, whose radii are r_i / r_m for the inner ring and r_m / r_a for the outer ring, with δ being the ring thickness at the mean radius r_m. We consider that a part of total line load *P*, say P_1 acts on the inner ring and the balance $P2 = P - P_1$ on the outer ring as shown in ***Fig*** *6*

This gives us $\mathrm{P} = \mathrm{P}_1 + \mathrm{P}_2$ (7)

The radial displacement u_1 of the outer circumference of the inner ring must be same as the displacement u_2 of the inner circumference of the outer ring.

Thus: $u_1 = u_2 = u_m$ (8)

Based on above conditions (*Eqns 7 & 8*) P_1 and P_2 can now be calculated. Under these conditions outer ring with internal bearing pressure P_2 / δ (= p_i) can be examined . Now, should an external radial pressure p_{ra}^* (in our case as a result of prevention of expansion = sealing) act on this outer ring, can be equated to radial play u_{ra}^* between sealing ring and cylinder, as depicted in ***Fig*** *7*

ie $u_{ra}^* = f(p, p_{ra}^*)$ (9)

- *Basic Equation - Inner Ring Displacement*

The expansion u_1 of the outer diameter of the inner ring ***Fig*** *6* because of the external pressure $p_a = P_1/\delta$ can be calculated with:

$$p_a = \frac{P1}{\delta}, \quad p_i = 0, \quad R = r_m, \quad r = r_i, \quad \rho = r_m \tag{10}$$

Substituting and simplifying the values in *Eqn* ***(4)***, we get :

$$u_1 = \frac{(P_1 / \delta)\, r_m^2}{(r_m^2 - r_i^2)} \cdot r_m \cdot [(1-\nu) + (1+\nu)\, r_i^2] \tag{11}$$

- *Basic Equation - Outer Ring Displacement*

Similarly expansion u_2 (= u_m) of outer circumference of inner ring based on external stress pi = P_2/δ and $p_a = -p_{ra*}$ can be calculated with:

$$p_i = P_2/\delta, \quad p_a = -p_{ra}^*, \quad R = r_a, \quad r = r_m, \quad \rho = r_m. \tag{12}$$

Substituting the values in *Eqn (4)*, we get u2 and similarly expression for $u_{ra}^* \ / \ u_{ra}$

$$u_2 = \frac{1-\nu}{E} \cdot \frac{\left(-p_{ra}^* \cdot r_a^2 + \left(\frac{P_2}{\delta}\right) r_m^2\right)}{r_a^2 - r_m^2} \cdot r_m + \frac{1+\nu}{E} \cdot \frac{\left(-p_{ra}^* + \frac{P_2}{\delta}\right) r_a^2 \cdot r_m^2}{r_a^2 - r_m^2} \cdot \frac{1}{r_m} \tag{13}$$

$$u_{ra}^* = \frac{1-\nu}{E} \cdot \frac{\left(-p_{ra}^* \cdot r_a^2 + \left(\frac{P_2}{\delta}\right) r_m^2\right)}{r_a^2 - r_m^2} \cdot r_a + \frac{1+\nu}{E} \cdot \frac{\left(-p_{ra}^* + \frac{P_2}{\delta}\right) r_a^2 \cdot r_m^2}{r_a^2 - r_m^2} \cdot \frac{1}{r_a} \tag{14}$$

- *Basic Equations – Radial Bearing Pressure*

From *Eqn (8)* we get the expression

$$\frac{(P_1/\delta)\, r_m}{E\,(r_m^2 - r_i^2)} \cdot [(1-\nu)\, r_m^2 + (1+\nu)\, r_i^2]$$

$$= \frac{1-\nu}{E} \cdot \frac{(-p_{ra}^* \cdot r_a^2 + (P_2/\delta)\, r_m^2)}{r_a^2 - r_m^2} \cdot r_m + \frac{1+\nu}{E} \cdot \frac{\left(-p_{ra}^* + \frac{P_2}{\delta}\right) r_a^2 \cdot r_m^2}{r_a^2 - r_m^2} \cdot \frac{1}{r_m} \tag{15}$$

The three equations *Eqn (7)*, *(14)* and *(15)* contain three unknowns , namely P1, P2 and p_{ra}^* and can be easily resolved for p_{ra}^*. Substituting suitably and simplifying we get:

$$\frac{r_a}{E} \cdot \frac{P}{\delta} \cdot \frac{(1-\nu)\, r_m^2 + (1+\nu)\, r_i^2}{(r_a^2 - r_i^2)} - u_{ra}^* = \frac{p_{ra}^*}{E} \cdot \frac{[(1-\nu)\, r_a^2 + (1+\nu) r_i^2]}{(r_a^2 - r_i^2)} \cdot r_a \tag{16}$$

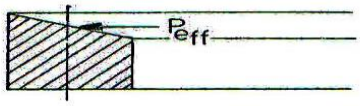


Fig 8 Considering frictional force

2.2.2 Case II: Hypothesis Considering frictional resistance during radial displacement

In *Eqn (16)* calculated bearing pressure and displacement of ring outer profile are under the conditions when friction $\mu = 0$. Should the friction be considered then radial acting force Fr, gets reduced by an amount *Fr'* , where *Fr'* is given by:

$Fr' = (-)\ \mu \cdot F_{ges}$

Thus, taking μ into account , the effective radial line load, acting on median circumference, P_{eff} is :

$$P_{eff} = \frac{F_r}{2\pi r_m} - \frac{F'_r}{2\pi r_m} \tag{17}$$

$$P_{eff} = P \cdot \frac{1-\mu 2 \ -2\,\mu \cot\alpha}{1+\ \mu \tan\alpha} \tag{18}$$

By meeting the condition :

$$1-\ \mu 2 \ \ -2\,\mu \cot\alpha \ \geq \ 0 \tag{19}$$

ie $\alpha \geq \ \arctan(\ 2\mu/1-\ \mu^2\ )$

it is now possible to evaluate the radial bearing pressure p^*_{ra} under the influence of frictional resistance where now in *Eqn (16)* P is simply to be replaced by P_{eff}.

From *Eqn (19)* it is also clear that for small values of α the radial displacement does not occur ie. in such a case only the lateral expansion is to be considered.

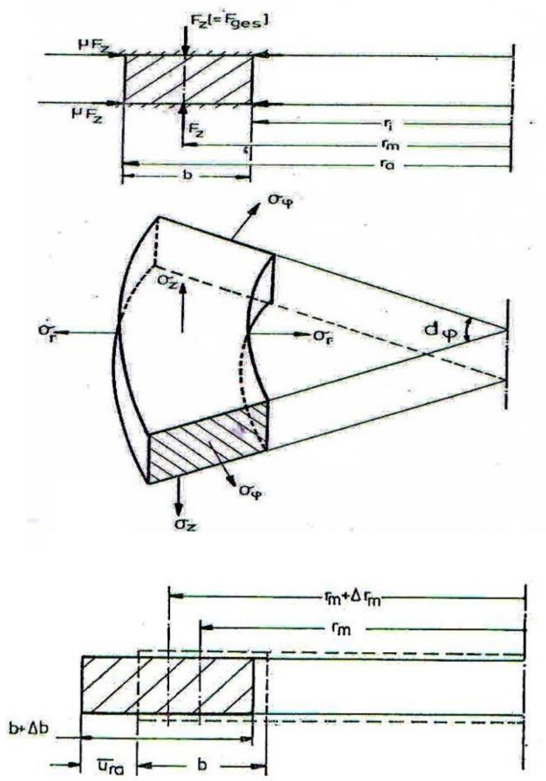


Fig9 Lateral deformation; ring deforms,

2.2.3 Case III: Hypotheses Considering effect of lateral deformation

The effect of lateral elongation as a result of the vertical load Fz *(=Fges; Eqn 2)* can be approximated by a conditional consideration: ie considering a unidirectional stress state ie elongation through deformation of the seal as a whole neglecting any effect of friction.

The actual expansion as a result of lateral deformation will be smaller because of the presence of friction. Thus the actual value of radial expansion will lie between the results calculated considering and not considering lateral deformation.

Further, is the ring free to expand without resistance, we can consider it as a protracted case of unidirectional state of stress.
Thus, referring ***Fig** 9*; With $\sigma_\varphi = \sigma_r = 0$ we get

In radial direction, $\epsilon_r = (-)\frac{\sigma_z}{E} \cdot \upsilon$ (20)

And in circumference direction $\epsilon_\varphi = (-)\frac{\sigma_z}{E} \cdot \upsilon$ (21)

$\Delta r_m = \epsilon_\varphi \cdot r_m \qquad \Delta b = \epsilon_r \cdot b$ (22)

The lateral deformation $\vec{u}_{ra}$ thus equals

$$\vec{u}_{ra} = \nu \cdot \frac{r_a}{\pi E} \cdot \frac{F_{ges}}{r_a^2 - r_i^2} \qquad (23)$$

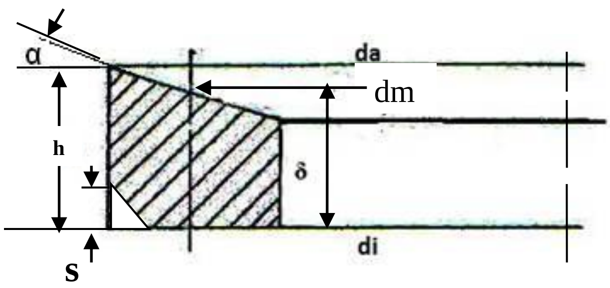
2.3 Mathematical Model Hypothesis - Summary

Thus incorporating the lateral deformation $\vec{u}_{ra}$ *Eqn* (23) and frictional resistance **μ** through P_{eff} *Eqn (18)* in the basic *Eqn (16)* and simplifying we get expression *Eqn (24)*. This allows us to evaluate u_{ra} (u_{ra}^*) or p_{ra}^*
ie From Hypotheses I + II + III, we get the mathematical model as :

$$\frac{r_a}{E} \cdot \frac{P_{eff}}{\delta} \cdot \frac{(1-\nu)\, r_m^2 + (1+\nu)\, r_i^2}{(r_a^2 - r_i^2)} + \vec{u}_{ra} - u_{ra}^* = \frac{p_{ra}^*}{E} \cdot \frac{[(1-\nu) r_a^2 + (1+\nu) r_i^2]}{(r_a^2 - r_i^2)} \cdot r_a \qquad (24)$$

III. FEM SIMMULATION RESULTS

A simulation [1] with finite element method using a *TRIAX* element was conducted with different ring shapes FEM results (rated comparison) is summarised and tabulated below in *Fig 10*. Also given there in is the result using the computational mathematical model as summarised in the hypotheses Eqn (24)



Common Dimensions / Data

da = 65 mm **E** = 20.6×10^4 N/mm2

d i = 53 mm **μ** = 0.1

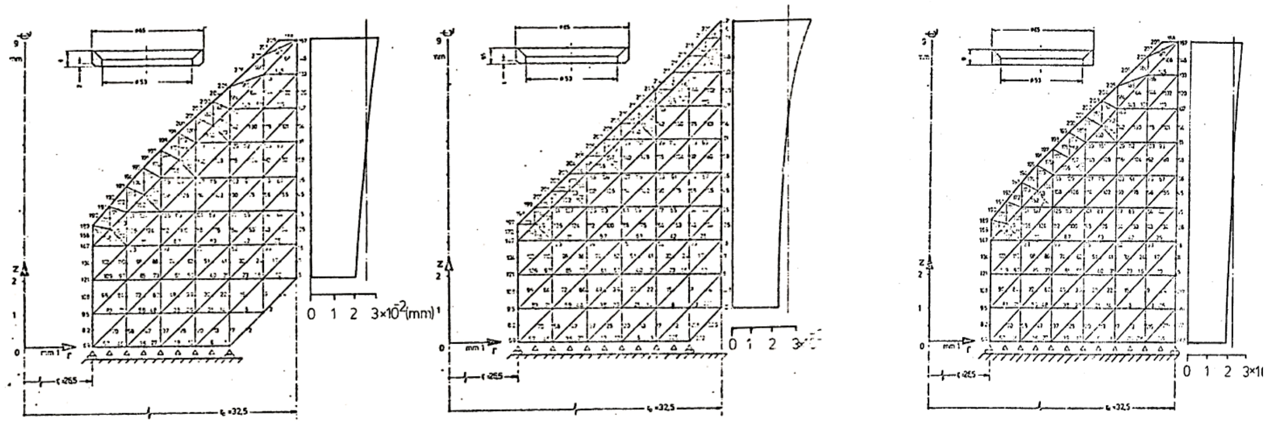
dm = 59 mm **ν** = 0.3

1 micron = 10^{-3} mm ; 1 bar = 10^{+5} N/m^2

Ring No	Material	h	S x 45^0	α	δ	u_0 = Expansion / Ton load (Fges)		p_0 = Radial Pressure / Ton load (Fges)	
						FEM	EQN 24	FEM	EQN 24
		mm	deg	-	mm	micron/Ton	micron/Ton	bar/Ton	bar/Ton
1	Mild Steel	9	2	45	6.5	5.2	4.3	70.2	60.5
2	Mild Steel	9.5	1	45	6.7	5.3	4.5	58.2	58.5
3	Mild Steel	9	0.2	45	6.5	4.7	4.3	50.3	60.5

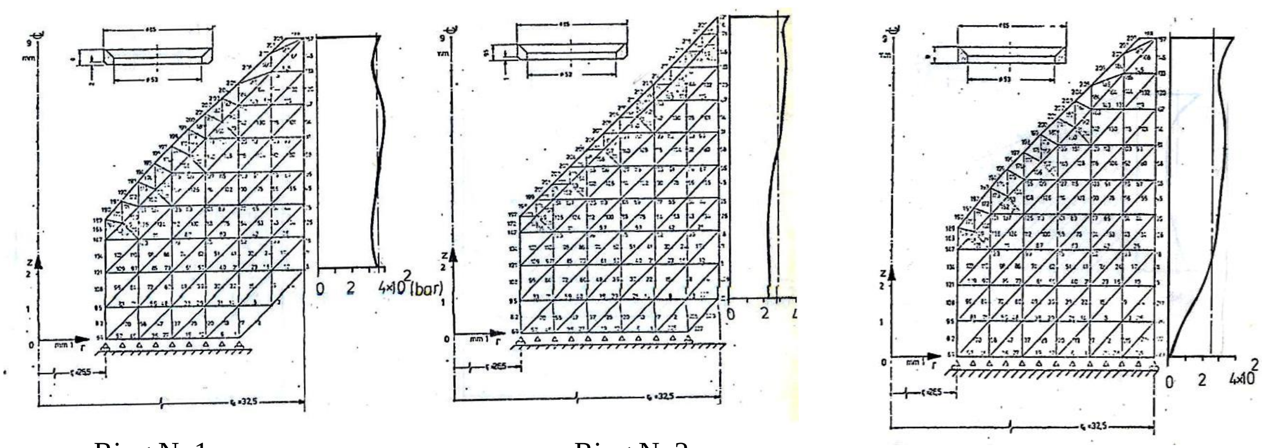
Fig 10 FEM and Computational results for rated Radial expansion u_0 and Radial pressure p_0 Tabulated

The actual displacement profile and the profile of radial pressure generated under known load conditions are shown in *Figs 11 and 12* .



Ring No1 Ring No2 Ring No3

Fig 11 u_{ra} ie Distribution of Radial Expansion (@ Fges= 4905 daN)



Ring No1 Ring No2
Ring No3

Fig 12 p_{ra*} ie Distribution of Radial Pressure (@ Fges = 4905 daN)

3.1 FEM and Mathematical Model Result Discussion

A quick FEM comparison shows a fairly good overall relationship as far as radial expansion u_0 and its distribution is concerned. The radial pressure mean value is in consonance with the calculated value but its distribution does shows its dependence on the leading edge chamfer. It is to be assumed that the edge chamfer, 'S' plays an important role in $\mathbf{p_0}$ generation in such design applications

- u_0 ie Radial Expansion / Ton load (Fges) shows a very good overall matching of the values both @ d_m as well as distribution over δ
- p_0 ie Radial Pressure / Ton load (Fges) shows a very good overall matching of the values @ d_m but the distribution over δ shows that the leading edge chamfer $Sx45^0$ plays a role. . Fig 11 & 12 show plotted summary of the FINEL analysis. This calculation was done with Fges = 4905 daN, E = $20.6x10^4$ N/mm2 , μ = 0.1 and ν = 0.3

IV. CONCLUSIONS

4.1 Mathematical Model

The comparative results show a good overall relationship and the mathematical model derived under Ch 2.0 can be used for a direct evaluation of radial expansion and radial pressure generated under different external conditions as summarised:

$$P_{eff} = \frac{F_{ges}\ \tan\alpha}{2\pi r_m}\ .\ [(1-\mu^2-2\ \mu\ \cot\alpha)/(1+\ \mu\ \tan\alpha)] \tag{25}$$

[where by $\alpha \geq \ \arctan\ (2\mu\ /1-\ \mu^2)$]

$$\vec{u}_{ra} = \nu \cdot \frac{r_a}{\pi\ E}\ .\ \frac{F_{ges}}{r_a^2 - r_i^2} \tag{26}$$

and

$$r_m = (r_a+r_i)\ /\ 2 \tag{27}$$

- By unhindered i.e. free expansion the radial expansion $u_{ra}^{\frown}$ $(=u_{ra}^{*})$

$$u_{ra}^{\frown} = \frac{P_{eff}}{\delta}\ .\frac{r_a}{E}\ .\ \frac{(1-\nu)\ r_m^2+(1+\nu)\ r_i^2}{(r_a^2-r_i^2)} + \vec{u}_{ra} \tag{28}$$

- For a given radial play u_{ra}^{*} (where by $u_{ra}^{\frown} > u_{ra}^{*}$) the maximum possible radial contact pressure p_{ra}^{*}

$$p_{ra}^{*} = \frac{P_{eff}}{\delta}\ .\frac{[(1-\nu)\ r_m^2+(1+\nu)\ r_i^2]}{[(1-\nu)r_a^2+(1+\nu)r_i^2]} + (\vec{u}_{ra} - u_{ra}^{*})\ .\frac{E}{r_a}.\ \frac{[(r_a^2-r_i^2)]}{[(1-\nu)r_a^2+(1+\nu)r_i^2]} \tag{29}$$

4.2 Future Direction

Based on FEM evaluation it is seen that leading edge base chamfer 'S" , plays an important role in pressure distribution profile. Also the definition of δ *and* d_m can be further refined for its sensitivity. Thus further work that could be carried out encompasses:

- Effect Of Base Chamfer and its cross correlation with the FEM results
- Sensitivity analysis of δ *and* d_m on the results $u_{ra}^{\frown}$ & p_{ra}^{*}
- Experimental measurement of radial deformation and pressure generated.

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Symbols used

Symbol	*Units*	*Nomenclature*
b	mm	Sealing ring width
$d_{a,}$	mm	outer diameter
d_m	mm	median diameter
d_i	mm	inner diameter
E	N/mm^2	modulus of elasticity
$F_{ges\ 1}$	N	clamp force , expansion
$F_{ges\ 2}$	N	clamp force , contact pressure
F_{ges}	N	$F_{ges1}+F_{ges\ 2}$ Total Clamp force

Symbol	*Units*	*Nomenclature*
s	mm	ring base chamfer
S_f	-	safety factor
u_{ρ}	mm	radial displacement @ r = ρ
u_m	mm	radial displacement @ r = m
$u_{1,2}$	mm	radial displacement @ ring 1,2
$\vec{u}_{ra}$	mm	lateral expansion
$u_{ra}^{\frown}$	mm	Radial displacement @ $\vec{u}_{ra} = 0$
$u_{ra}^{\frown} =$	$\vec{u}_{ra} + u_{ra}^{\frown}$	Radial displacement, total

Fu	N	circumferential force	u_{ra} *	mm	Radial assembly play
F_z, F_{ges}	N	axial force component	u_o	mm/N	load based displacement
Fr, Fr'	N	Radial force component	α	deg	ring wedge angle
h	mm	ring height, total	δ	mm	ring height @ d_m
h_o	mm	ring height, effective	δp	bar	seal pressure difference
l	mm	cylinder height	ν	-	Poisson's ratio
$P_{1,2}$	N/mm	Line load ; per unit length	μ	-	friction coefficient
P_{eff}	N/mm	Effective line load	σ_B	N /mm^2	material tensile stress
p_a	N /mm^2	axial contact pressure	$\sigma_{0,2}$	N /mm^2	material yield stress
p_r	N /mm^2	radial contact pressure @ u=0	σ_r	N /mm^2	radial stress
p_{ra*}	N /mm^2	radial contact pressure @ u=u_{ra}*	$\sigma_\o$	N /mm^2	circumfrencial stress
$\hat{p}_{ra}$	N /mm^2	dach	σ_z	N /mm^2	axial stress
p_b	bar	cylinder pressure	ε_r	0/00	radial strain
p_o	bar/N	Load based contact pressure	$\varepsilon_\O$	0/00	circumfrencial strain
r_a	mm	outer radius	ε_z	0/00	axial strain
r_m	mm	median radius	r, ρ	-	radial axias / direction
r_i	mm	inner radius	z	-	vertical axis / direction

AUTHORS BIOGRAPHY

Umesh Wazir was born in India in 1949. He received his bachelor's degree from National Institute of Technology Warangal (India) in 1969-70 and his Masters from the Technical University Stuttgart (Germany) in the year 1977-78. Mr Wazir has an experience of 42 yrs; 22 yrs in R&D with Mercedes Benz (Ger), Escorts Ltd (India); 12 yrs in Operations with Piaggio (India) as Director Manufacturing, and Head Product Support Escorts (India); and 8 yrs in Academics. His research interests include Sealing Technology, Gear Design Optimization, New Material application and Vehicle Design Engineering