WAVE PROPAGATION IN A HOMOGENEOUS ISOTROPIC FINITE THERMO-ELASTIC THIN CYLINDRICAL SHELL

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ABSTRACT

In this paper, the wave propagation in a homogeneous isotropic finite thermo-elastic thin cylindrical shell is studied based on the Lord-Schulman (LS) and Green-Lindsay (GL) generalized two dimensional theory of thermo-elasticity. Two displacement potential functions are introduced to uncouple the equations of motion. The frequency equations that include the interaction between the cylindrical shell and foundation are obtained by the traction free boundary conditions using the Bessel function solutions. The numerical calculations are carried out for the material Zinc and the computed non-dimensional frequency and attenuation coefficient are plotted as the dispersion curves for the shell with thermally insulated and isothermal boundaries. The wave characteristics are found to be more stable and realistic in the presence of thermal relaxation times and the foundation parameter. The computed non-dimensional frequencies are plotted in the form of dispersion curves with the support of MATLAB.

KEYWORDS: *wave propagation, isotropic cylindrical shell, modified Bessel function.*

I. INTRODUCTION

Cylindrical thin shell plays a vital role in many engineering fields such as aerospace, civil, chemical, mechanical, naval and nuclear engineering. The dynamical interaction between the cylindrical shell and solid foundation has potential applications in modern engineering fields due to the fact that their static and dynamic behaviors will be affected by the surrounding media. The analysis of thermally induced wave propagation of a cylindrical shell is a problem that may be encountered in the design of structures such as atomic reactors, steam turbines, submarine structures subjected to wave loadings, or for the impact loadings due to superfast trains, or for jets and other devices operating at elevated temperatures. Moreover, it is recognized that the thermal effects on the elastic wave propagation supported by elastic foundations may have implications related to many seismological applications. This study can be potentially used in applications involving non-destructive testing(NDT) and qualitative non-destructive evaluation (QNDE).

Ashida and Tauchert [1] have presented the temperature and stress analysis of an elastic circular cylinder in contact with heated rigid stamps. Later, Ashida [2] has analyzed the thermally induced wave propagation in a piezo-electric plate. Bernhard [3] has studied the buckling frequency for a clamped plate embedded in an elastic medium. Chandrasekharaiah [4] has discussed the thermo-elasticity with second sound. The equations for an isotropic case are obtained by Dhaliwal and Sherief [5]. Erbay and Suhubi [6] have studied the longitudinal wave propagation in a generalized thermo-plastic infinite cylinder and obtained the dispersion relation for the cylinder with a constant surface temperature. The thermal deflection of an inverse thermo-elastic problem in a thin isotropic circular plate has been presented by Gaikward and Deshmukh [7]. Green and Lindsay [8] have obtained an explicit version of the constitutive equations.

A generalization of the inequality was proposed by Green and Laws [9]. Heyliger and Ramirez [10] have analyzed the free vibration characteristics of laminated circular piezo-electric plates and discs by using a discrete layer model of the weak form of the equations of periodic motion. Kamal [11] has discussed a circular plate embedded in an elastic medium, in which the governing differential equation was formulated using the Chebyshev- Lanczos technique. The generalized theory of thermo-elasticity was developed by Lord and Schulman [12], which involves one relaxation time for isotropic homogeneous media, and is called the first generalization to the coupled theory of elasticity. Their equations determine the finite speed of wave propagation of heat and the displacement distributions. The second generalization to the coupled theory of elasticity is known as the theory of thermo-elasticity with two relaxation times, or as the theory of temperature-dependent thermo-electricity.
Mirsky.I, [13] has investigated the three dimensional and Shell-theory analysis of axially symmetric motions of Cylinders. Ponnusamy [14] has studied wave propagations in a generalized thermo-elastic solid cylinder of arbitrary cross sections using the Fourier expansion collocation method. Later, Ponnusamy and Selvamani [15] have obtained mathematical modeling and analysis for a thermo-elastic cylindrical panel using the wave propagation approach. Selvadurai [16] has presented the most general form of a soil model used in practical applications. Sharma and Pathania [17] have investigated the generalized wave propagation in circumferential curved plates. Modeling of circumferential waves in a cylindrical thermo-elastic plate with voids was discussed by Sharma and Kaur [18]. Tso and Hansen [19] have studied the wave propagation through cylinder / plate junctions. Recently, Wang [20] has studied the fundamental frequency of a circular plate supported by a partial elastic foundation using the finite element method.
In this paper, the vibration of a generalized thermo-elastic homogeneous isotropic thin cylindrical shell is studied. The solutions to the equations of motion for an isotropic medium is obtained by using the three dimensional theory of elasticity and Bessel function solutions. The numerical calculations are carried out for the material Zinc. The computed non-dimensional frequency and attenuation coefficient are plotted as dispersion curves for the shell with thermally insulated and isothermal boundaries. The study about a cylindrical shell is important for design of structures such as atomic reactors, steam turbines, submarine structures with wave loads, or for the impact effects due to super-fast train, or for jets and other devices operating at elevated temperatures.

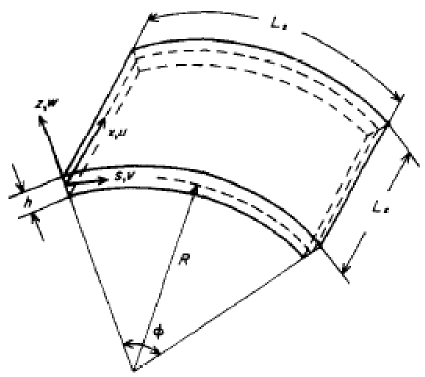

Figure 1. Geometry of the cylindrical shell

II. Formulation Of The Problem

A thin homogeneous, isotropic, thermally conducting elastic cylindrical shell with radius R, uniform thickness d and temperature T_0 in the undisturbed state initially as shown in the figure 1 is considered. The system displacements and stresses are defined in the polar coordinates (r, θ, z) for an arbitrary point inside the shell, with u denoting the displacement in the radial direction of r and ν is the displacement in the tangential direction of θ.

The three dimensional stress equations of motion and heat conduction equation in the absence of body force for a linearly elastic medium are as follows:

$$\frac{\partial}{\partial r}\sigma_{rr}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{r\theta}+\frac{\partial}{\partial z}\sigma_{rz}+\frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta})=\rho\frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial}{\partial r}\sigma_{r\theta}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{\theta\theta}+\frac{\partial}{\partial z}\sigma_{\theta z}+\frac{2}{r}\sigma_{r\theta}=\rho\frac{\partial^2 v}{\partial t^2}$$
$$\frac{\partial}{\partial r}\sigma_{rz}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{\theta z}+\frac{\partial}{\partial z}\sigma_{zz}+\frac{1}{r}\sigma_{rz}=\rho\frac{\partial^2 w}{\partial t^2}$$

The heat conduction equation is given by

$$\mathrm{K}\left[\frac{\partial^2 \mathrm{T}}{\partial t^2}+\frac{1}{r}\frac{\partial \mathrm{T}}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \mathrm{T}}{\partial\theta^2}+\frac{\partial^2 \mathrm{T}}{\partial z^2}\right]$$
$$=\rho C_\gamma\frac{\partial \mathrm{T}}{\partial t}+\rho\tau\frac{\partial^2 \mathrm{T}}{\partial t^2}+\beta \mathrm{T}_o\left[\frac{\partial^2 u}{\partial r\partial t}+\frac{1}{r}\left(\frac{\partial u}{\partial t}+\frac{\partial^2 v}{\partial\theta\,\partial t}\right)+\frac{\partial^2 w}{\partial t\,\partial z}\right] \quad (1)$$

$$\sigma_{rr}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{rr}-\beta \mathrm{T}$$
$$\sigma_{\theta\theta}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{\theta\theta}-\beta \mathrm{T}$$
$$\sigma_{zz}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{zz}-\beta \mathrm{T} \quad (2)$$
$$\sigma_{r\theta}=\mu\gamma_{r\theta}$$
$$\sigma_{\theta z}=\mu\gamma_{\theta z}$$
$$\sigma_{rz}=\mu\gamma_{rz}$$

where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{r\theta},\sigma_{r\theta},\sigma_{\theta z},\sigma_{rz}$ are the stress components, e_{rr}, $e_{\theta\theta}$, e_{zz}, $e_{r\theta}$, $e_{\theta z}$ and e_{rz} are the strain components, T is the change in temperature about the equilibrium temperature T_0, ρ is the mass density, C_γ is the specific heat capacity, β is a coupling factor that couples the heat conduction and elastic field equation, K is the thermal conductivity, t is the time, λ and μ are Lame' constants. The strain e_{ij}, related to the displacements are given by

$$e_{rr}=\frac{\partial u}{\partial r},\ e_{\theta\theta}=\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta};\ e_{zz}=\frac{\partial w}{\partial z}$$
$$\gamma_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta};\ \gamma_{z\theta}=\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial\theta};\ \gamma_{rz}=\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z} \quad (3)$$

In which u, v and w are the three displacement components along radial, circumferential and axial directions respectively.
Substituting the equations (3) and (2) in (1), yields,

$$(\lambda+2\mu)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+\frac{\mu}{r^2}\frac{\partial^2 u}{\partial\theta^2}+\mu\frac{\partial^2 u}{\partial z^2}+\left(\frac{\lambda+\mu}{r}\right)\frac{\partial^2 v}{\partial r\partial\theta}$$
$$-\left(\frac{\lambda+3\mu}{r^2}\right)\frac{\partial v}{\partial\theta}+(\lambda+\mu)\frac{\partial^2 w}{\partial r\partial z}-\beta\frac{\partial \mathrm{T}}{\partial r}=\rho\frac{\partial^2 u}{\partial t^2} \quad (4)$$

$$\left(\frac{\lambda+\mu}{r}\right)\frac{\partial^2 u}{\partial r\,\partial\theta}+\left(\frac{\lambda+3\mu}{r^2}\right)\frac{\partial u}{\partial\theta}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+\left(\frac{\lambda+2\mu}{r^2}\right)\frac{\partial^2 v}{\partial\theta^2}$$
$$+\mu\frac{\partial^2 v}{\partial z^2}+\left(\frac{\lambda+\mu}{r}\right)\frac{\partial^2 w}{\partial\theta\,\partial z}-\beta\frac{\partial T}{\partial\theta}=\rho\frac{\partial^2 v}{\partial t^2} \quad (5)$$

$$(\lambda+\mu)\frac{\partial^2 u}{\partial r\,\partial z}+\left(\frac{\lambda+\mu}{r}\right)\frac{\partial u}{\partial z}+\left(\frac{\lambda+\mu}{r}\right)\frac{\partial^2 v}{\partial\theta\,\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)$$
$$+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\beta\frac{\partial T}{\partial\theta}=\rho\frac{\partial^2 w}{\partial t^2} \quad (6)$$

$$\rho C_\gamma k\left[\frac{\partial^2 T}{\partial t^{2r}}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\gamma z^2}\right]=\rho\tau\frac{\partial^2 T}{\partial t^2}+\rho C_\gamma\frac{\partial T}{\partial t}+$$
$$\beta T_o\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{\partial w}{\partial z}\right] \quad (7)$$

where $k=\dfrac{k}{\rho C_\gamma}$ is the diffusivity.

III. SOLUTION OF THE EQUATIONS

The equations from (4) to (7) have a coupled partial differential equations of the three displacement and heat conduction components. To uncouple the equations (4)-(7), we follow Mirsky [13] and assuming the solution to equation (4) as follows

$$u=\frac{1}{r}\psi_{,\theta}-\phi_{,r};\ v=-\frac{1}{r}\phi_{,\theta}-\psi_{,r};\ w=-\chi_{,z} \quad (8)$$

$$\left[(\lambda+2\mu)\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right]\phi+(\lambda+\mu)\frac{\partial^2\psi}{\partial z^2}-\beta T_{,t}+\mu\left[\nabla_1^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\frac{\partial^2}{\partial t^2}\right]\chi=0 \quad (9)$$

$$\left[(\lambda+2\mu)\nabla_1^2+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right]\phi+(\lambda+\mu)\frac{\partial^2\psi}{\partial z^2}-\beta T_{,t}+\mu\left[\nabla_1^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\frac{\partial^2}{\partial t^2}\right]\chi=0$$

$$\left[\mu\nabla_1^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right]\psi+(\lambda+\mu)\nabla_1^2\phi+\beta T=0 \quad (10)$$

$$\nabla_1^2 T+\frac{\partial^2 T}{\partial z^2}-\frac{\tau}{kC_\gamma}\frac{\partial^2 T}{\partial t^2}-\frac{1}{k}\frac{\partial T}{\partial t}+\frac{\beta T_o}{\rho k C_\gamma}\left[\nabla_1^2\phi+\frac{\partial^2\psi}{\partial z^2}\right]=0 \quad (11)$$

where $\nabla_1^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$

The displacement function and temperature change are given by

$$\phi = \bar{\phi}(r)\,\sin\left(\frac{m\pi z}{L}\right)\sin n\theta\; e^{i\omega t}$$
$$\chi = \bar{\chi}(r)\,\sin\left(\frac{m\pi z}{L}\right)\sin n\theta\; e^{i\omega t}$$
$$T = \bar{T}(r)\,\sin\left(\frac{m\pi z}{L}\right)\sin n\theta\; e^{i\omega t} \qquad (12)$$
$$\psi = \bar{\psi}(r)\,\sin\left(\frac{m\pi z}{L}\right)\cos n\theta\; e^{i\omega t}$$

where $i = \sqrt{-1}$, ω is the angular velocity, p is the angular wave number. Substituting the equation (12) into equation (11) and introducing the following non-dimensional quantities

$$r' = \frac{r}{R};\; z' = \frac{Z}{L};\; \bar{T} = \frac{T}{T_o};\; \Omega^2 = \frac{\rho\omega^2 a^2}{2+\bar{\lambda}}$$
$$t_L = \frac{m\pi R}{L};\; \bar{\lambda} = \frac{\lambda}{\mu};\in_4 = \frac{1}{2+\bar{\lambda}};\; c_1^2 = \frac{\rho}{\lambda+2\mu} \qquad (13)$$

The governing equations can be written as

$$\frac{1}{R^2}\left[\left(\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}-\frac{n^2\theta^2}{r'^2}\right)-\frac{m^2\pi^2R^2}{(2+\bar{\lambda})L^2}+\frac{\rho\omega^2}{\mu(2+\bar{\lambda})}R^2\right]\phi p_1$$
$$-\left(\frac{1+\bar{\lambda}}{2+\bar{\lambda}}\right)\frac{m^2\pi^2R^2}{L^2}\bar{\chi}(r)p_1+\frac{\beta}{\mu}\frac{R^2T_o\bar{T}}{(2+\bar{\lambda})}p_1 \qquad (14)$$
$$+\frac{1}{(2+\bar{\lambda})}\left[\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}-\frac{n^2\theta^2}{r'^2}-\frac{m^2\pi^2R^2}{L^2}+\rho\omega^2R^2\right]\bar{\chi}(r)p_2 = 0$$

Since $\nabla_2{}^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{\delta^2}{r^2}$

$$\left[\left[(\nabla_2{}^2-\in_4 t_L^2+\Omega^2\right]\bar{\phi}-\in_4(1+\bar{\lambda})\,t_L^2\,\bar{\psi}+\frac{\beta T_o a^2\tau\bar{T}}{(\lambda+2\mu)}\right]p_1$$
$$+(\in_4\nabla_2^2-\in_4 t_L^2+\Omega^2)\bar{\psi}\,p_2 = 0$$
$$\Rightarrow\left[\left(\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}-\frac{\delta^2}{r^2}\right)-(2+\bar{\lambda})\frac{m^2\pi^2R^2}{L^2}+\frac{\rho}{\mu}\omega^2R^2\right]\bar{\chi}$$
$$+(1+\bar{\lambda})\left[\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}-\frac{\delta^2}{r'^2}\right]\bar{\phi}+\frac{\beta}{\mu}R^2\bar{T}T_o\tau_1 = 0$$
$$\left[\nabla_2^2-(2+\bar{\lambda})t_L^2+(2+\bar{\lambda})\Omega^2\right]\bar{\psi}+(1+\bar{\lambda})\nabla_2^2\bar{\phi}+(2+\bar{\lambda})\frac{\beta R^2\bar{T}T_o\tau_1}{(2+\bar{\lambda})\mu} = 0 \qquad (15)$$
$$\Rightarrow\left[\nabla_2^2-t_L^2+\frac{\tau\omega^2R^2}{C_\gamma k}-\frac{i\omega R^2}{k}\right]T+\frac{\beta T_o}{\rho C_\gamma k}\left[\nabla_2^2\bar{\phi}-t_L^2\bar{\psi}\right] = 0$$
$$\left[\nabla_2^2-t_L^2+\in^2\Omega^2-i\Omega\in_3\right]\bar{T}+i\in_1\Omega\nabla_2^2\bar{\phi}-i\in_1\Omega t_L^2\bar{\psi} = 0 \qquad (16)$$

From the above equations, ψ gives a purely transverse wave.

$$(\nabla_2^2 - \in_4 t_2^2 + \Omega^2)\bar{\phi} - (1+\bar{\lambda}) \in_4 t_L^2 \bar{\psi} + \left(\frac{\beta T_o R^2 \tau_1}{\lambda + 2\mu}\right)\bar{T} = 0$$

$$\left[\nabla_2^2 - (2+\bar{\lambda})(t_L^2 - \Omega^2)\right]\bar{\psi} + (1+\bar{\lambda})\nabla_2^2 \bar{\phi} + (2+\bar{\lambda})\frac{\beta R^2 \bar{T} T_o \tau_1}{\lambda + 2\mu} = 0$$

$$\left[\nabla_2^2 - t_L^2 + \in_2 \Omega^2 - i\,\Omega \in_3\right]\bar{T} + i \in_1 \Omega \nabla_2^2 \bar{\phi} - i \in_1 \Omega t_L^2 \bar{\psi} = 0$$

Define,

$$g_1 = (2+\bar{\lambda})\,(t_L^2 - \Omega^2)$$

$$g_2 = \left(\frac{1+\bar{\lambda}}{2+\bar{\lambda}}\right)t_L^2 = \in_4 (1+\bar{\lambda}) \in_L^2$$

$$g_3 = \Omega^2 - \in_4 t_L^2$$

$$g_4 = \frac{\beta T_o R^2 \tau_1}{\lambda + 2\mu}$$

$$g_5 = \in_1 \Omega$$

$$(\nabla_2^2 + g_3)\bar{\phi} - g_2 \bar{\chi} + g_4 \bar{T} = 0$$

=> $$(\nabla_2^2 - g_1)\bar{\chi} + (1+\bar{\lambda})\nabla_2^2 \bar{\phi} + (2+\bar{\lambda}) g_4 \bar{T} = 0$$

$$(\nabla_2^2 - t_L^2 + \in_2 \Omega^2 - i\Omega \in_3)\bar{T} + i \in_1 \Omega \nabla_2^2 \bar{\phi} - i \in_1 \Omega\, t_L^2 \bar{\chi} = 0$$

$$\begin{vmatrix} \nabla_2^2 + g_3 & -g_2 & g_4 \\ (1+\bar{\lambda})\nabla_2^2 & \nabla_2^2 - g_1 & (2+\bar{\lambda}) g_4 \\ i g_5 \nabla_2^2 & -i g_5 t_L^2 & \nabla_2^2 - t_L^2 + \in_2 \Omega^2 - i\Omega \in_3 \end{vmatrix} (\bar{\phi}, \bar{\psi}, \bar{T}) = 0$$

$$\left[\nabla_2^2 + g_3 - g_2 di + g_4 e_i\right]\bar{\phi} = 0 \tag{17}$$

$$\left[(1+\bar{\lambda})\nabla_2^2 + (\nabla_2^2 - g_1) d_i + (2+\bar{\lambda}) g_4 e_i\right]\bar{\phi} = 0 \tag{18}$$

Solving (17) and (18), yields,

$$d_i = \frac{\delta_i^2 + g_3(2+\bar{\lambda})}{g_2(2+\bar{\lambda}) + \delta_i^2 + g_1} \quad \text{and}$$

$$e_i = \frac{g_2[g_2(3+\bar{\lambda}) + g_1 - g_3 + \delta_i^2 + g_1 g_3]}{g_4[g_2(2+\bar{\lambda}) + \delta_i^2 - g_1}$$

$$\nabla_2^6 + A\nabla_2^4 + B\nabla_2^2 + C\nabla_2 = 0$$

$$(\nabla_2^2 + \alpha_1^2)\,(\nabla_2^2 + \alpha_2^2)\,(\nabla_2^2 + \alpha_3^2)\bar{\phi} = 0$$

$$\psi = \begin{cases} A_3 J_n\, \alpha_3(ax) + B_3 Y_n\, \alpha_3(ax), & \text{if } \alpha_3 ax > 0 \\ A_3 a^n + B_3 a^{-n}, & \text{if } \alpha_3 ax = 0 \\ A_3 I_n\, \alpha_3(ax) + B_3 K_n\, \alpha_3(ax), & \text{if } \alpha_3 ax < 0 \end{cases} \tag{19}$$

where J_n and Y_n are Bessel functions of the first and second kinds, respectively, while I_n and k_n are modified Bessel functions of first and second kinds, respectively. A_i, and B_i (i = 1,2,3) are arbitrary constants. Since $\alpha_3 ax \neq 0$, thus the condition $\alpha_3 ax \neq 0$ will not be discussed in the following. For a sake of convenience, consider only to the case where $\alpha_3 ax > 0$. The derivation for the case of $\alpha_3 ax < 0$ is similar.

IV. Boundary Conditions And Frequency Equations

In this section the frequency equation for the three dimensional vibration of the cylindrical shell, are derived subjected to stress free boundary conditions at the upper and lower surfaces at r=a,b. Substituting the equations from (1) to (3) into the equation (5), the frequency equation for free vibration as follows:

$|E_{ij}|=0 \quad i,j=1,2,3,4,5,6.$

$E_{11}=(2+\lambda)(nJ_n(\alpha_1 ax)+(\alpha_1 ax)J_{n+1}(\alpha_1 ax))-((\alpha_1 ax)^2R^2-n^2)J_n(\alpha_1 ax))+\lambda(n(n-1)(J_n(\alpha_1 ax)-(\alpha_1 ax)J_{\delta+1}(\alpha_1 ax)))-\beta T(i\omega)\eta_2 d_1(\alpha_1 ax)^2$

$E_{13}=(2+\lambda)(nJ_n(\alpha_2 ax)+(\alpha_2 ax)J_{n+1}(\alpha_2 ax))-((\alpha_2 ax)^2R^2-n^2)J_n(\alpha_2 ax))+\lambda(n(n-1)(J_n(\alpha_2 ax)-(\alpha_2 ax)J_{\delta+1}(\alpha_2 ax)))-\beta T(i\omega)\eta_2 d_2(\alpha_2 ax)^2$

$E_{15}=(2+\lambda)(n(n-1)J_n(\alpha_3 ax)+(\alpha_3 ax)J_{n+1}(\alpha_3 ax))+\lambda(n(n-1)(J_n(\alpha_3 ax)-(\alpha_3 ax)J_{n+1}(\alpha_3 ax))$

$E_{21}=2n(n-1)J_n(\alpha_1 ax)-2n(\alpha_1 ax)J_{n+1}(\alpha_1 ax) \qquad (20)$

$E_{23}=2n(n-1)J_n(\alpha_2 ax)-2n(\alpha_2 ax)J_{n+1}(\alpha_2 ax)$

$E_{25}=2n(n-1)J_n(\alpha_3 ax)-2(\alpha_3 ax)J_{\delta+1}(\alpha_3 ax)+((\alpha_3 ax)^2-n^2)J_n(\alpha_3 ax))$

$E_{31}=d_1(nJ_n(\alpha_1 ax)-(\alpha_1 ax)J_{n+1}(\alpha_1 ax)+hJ_n(\alpha_1 ax))$

$E_{33}=d_2(nJ_n(\alpha_2 ax)-(\alpha_2 ax)J_{n+1}(\alpha_2 ax)+hJ_n(\alpha_2 ax))$

$E_{35}=0$

Obviously Eij (j = 2,4,6) can be obtained by just replacing the Bessel functions of the first kind in Eij (i = 1,3,5) with those of the second kind, respectively, while Eij (i = 4,5,6)can be obtained by just replacing a in Eij (i = 1,2,3) with b. Allowing for the effect of the surrounded elastic medium, which is treated as the Pasternak model, the boundary conditions at the inner and outer surfaces r = a, b can be considered as follows:

$\sigma_{rr} = \sigma_{r\theta} = 0 \qquad T_{,r} = 0 \quad (r=a) \qquad (21)$

$\sigma_{rr} = -Ku + G\Delta u \qquad \sigma_{r\theta} = 0 \quad (r=b) \qquad (22)$

where $\Delta= \partial^2/\partial z^2 + (1/r^2)\partial^2/\partial\theta^2$, K is the foundation modulus and G is the shear modulus of the foundation.

From equations (21) to (22) and the results obtained in the preceding section, we get the coupled free vibration frequency equation as follows:

$|E_{ij}|=0 \quad i,j=1,2,3,4,5,6.$

$E_{41}=p^*(nJ_n(\alpha_1 ax)-(\alpha_1 ax)J_{n+1}(\alpha_1 ax))$

$E_{42}=p^*(nY_n(\alpha_1 ax)-(\alpha_1 ax)Y_{n+1}(\alpha_1 ax))$

$E_{43}=p^*(nJ_n(\alpha_2 ax)-(\alpha_2 ax)J_{n+1}(\alpha_2 ax))$

$E_{44}=p^*(nY_n(\alpha_2 ax)-(\alpha_2 ax)Y_{n+1}(\alpha_2 ax))$

$E_{45}=p^*(nJ_n(\alpha_3 ax)-(\alpha_3 ax)J_{n+1}(\alpha_3 ax))$

$E_{46}=p^*(nY_n(\alpha_3 ax)-(\alpha_3 ax)Y_{n+1}(\alpha_3 ax))$

$p^*= p_1+ p_2(p^2+n^2)$ where $p_1=KR/\mu$ and $p_2=G/R\mu$

V. Numerical Results And Discussion

The coupled free wave propagation in a simply supported homogeneous isotropic thermo-elastic cylindrical shell is numerically solved for the Zinc material. The material properties of Zinc are given as follows:

$\rho = 7.14\times10^3$ kgm $^{-3}$ $\qquad T_0 = 296K$; $\quad K= 1.24\times 10^2$ Wm $^{-1}$deg $^{-1}$

$\mu = 0.508 \times 10^{11}\ Nm^{-2}$ $\beta = 5.75\times 10^{6} Nm^{-2} deg^{-1}$; $\epsilon_1 = 0.0221$
$\lambda = 0.385\times 10^{11}\ Nm^{-2}$ $C\nu = 3.9\times 10^{2} Jkg^{-1} deg^{-1}$

The roots of the algebraic equation in equation (15) were calculated using a combination of the Birge-Vita method and Newton-Raphson method. For the present case, the simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using the Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. Such a combination can overcome the difficulties encountered in finding the roots of the algebraic equations of the governing equations. Here the values of the thermal relaxation times are calculated from Chandrasekharaiah [4] as seconds and $\tau_1 = 0.5 \times 10^{-13}$ sec. Because the algebraic equation (11) contains all the information about the wave speed and angular frequency, and the roots are complex for all considered values of wave number, therefore the waves are attenuated in space.
We can write the attenuation coefficient as $c^{-1} = \nu^{-1} + i\omega^{-1} q$, so that , where $R = \omega / \nu$, ν and q are real numbers. Upon using the above relation in equation (20), the values of the wave speed (ν) and the attenuation coefficient (q) for different modes of wave propagation can be obtained.

Table:1 Comparison of non-dimensional frequencies among the Green-Lindsay Generalized Theory(G.L), Lord-Schulman Theory (L.S) and Classical Theory (C.T) of Thermo-elasticity for clamped and unclamped boundaries of thermally insulated cylindrical shell.

Mode	Un clamped			Clamped		
	LS	**GL**	**CT**	**LS**	**GL**	**CT**
1	0.1672	0.0765	0.0139	0.1508	0.1342	0.1152
2	0.3335	0.2719	0.0541	0.2255	0.1969	0.1564
3	0.5337	0.4977	0.1174	0.5773	0.3248	0.2444
4	0.8292	0.4385	0.1994	0.5941	0.5593	0.3487
5	1.1408	0.6952	0.2964	0.6303	0.8050	0.6584
6	1.4579	0.8714	0.4051	0.7070	0.8512	0.7551
7	1.7707	1.1350	0.6478	1.2007	1.0230	0.9038

Table:2 Comparison of non-dimensional frequencies among the Greeen-Lindsay Generalized Theory(G.L), Lord-Schulman Theory (L.S) and Classical Theory (C.T) of Thermo-elasticity for clamped and unclamped boundaries of thermally insulated cylindrical shell.

Mode	Un clamped			Clamped		
	LS	**GL**	**CT**	**LS**	**GL**	**CT**
1	0.1781	0.1336	0.0532	0.1084	0.1049	0.0259
2	0.5747	0.2736	0.1801	0.2130	0.1213	0.1702
3	0.6492	0.2928	0.2063	0.3220	0.2563	0.2950
4	0.5391	0.3727	0.3967	0.3295	0.3732	0.3837
5	1.7853	0.4036	0.5010	0.4752	0.4831	0.5129
6	1.9288	0.5308	0.6400	0.6349	0.6422	0.8727
7	2.0824	0.7015	0.9025	0.9142	0.8231	0.9308

A comparison is made for the non-dimensional frequencies among the Green-Lindsay generalized theory (G.L), Lord-Schulman Theory (L.S) and Classical Theory (C.T) of thermo-elasticity for the clamped and unclamped boundaries of the thermally insulated and isothermal circular shell in Tables 1 and 2,respectively. From these tables, it is clear that as the sequential number of the vibration modes increases, the non-dimensional frequencies also increases for both the clamped and unclamped cases. Also, it is clear that the non-dimensional frequency exhibits higher amplitudes for the LS theory compared with the GL and CT due to the combined effect of thermal relaxation times and damping of the foundation. In figures 2 and 3, the dispersion of frequencies with the wave number is studied for both the thermally insulated and isothermal boundaries of the cylindrical shell in different modes of vibration. From figure 2, it is observed that the frequency increases exponentially with increasing wave number for thermally insulated modes of vibration. But smaller dispersion exist in the frequency in the current range of wave numbers in figure 3 for the isothermal mode due to the combined effect of damping and insulation. In figure 4, the variation of attenuation coefficients with respect to the wave number of the cylindrical shell is presented for the thermally insulated boundary. The magnitude of the attenuation coefficient increases monotonically, attaining the maximum in for first four modes of vibration, and slashes down to become asymptotically linear in the remaining range of wave $0.1 \le \delta \le 0.8$ number.

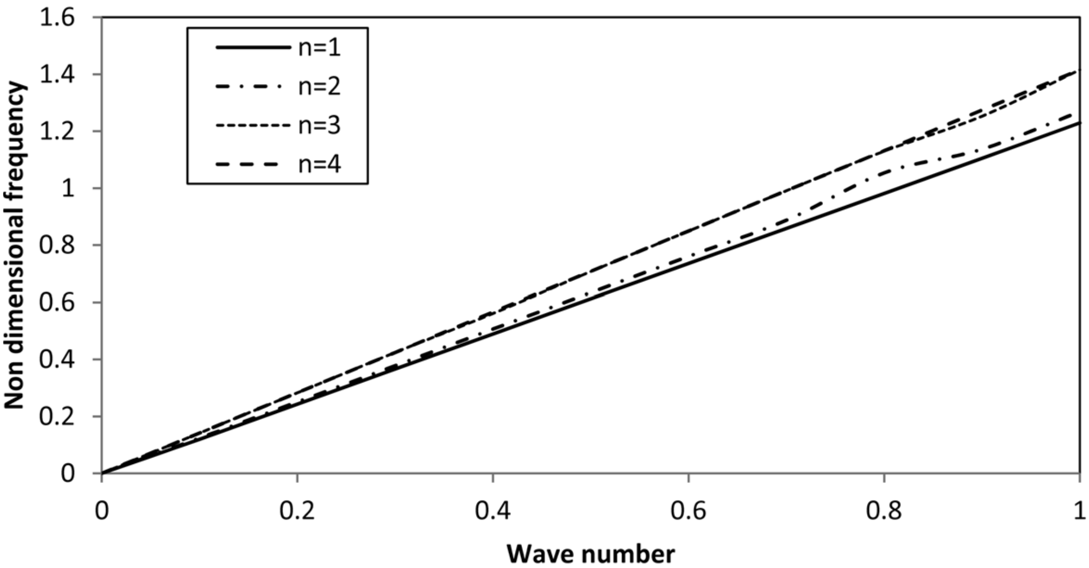

Figure2. Variation of non-dimensional frequency of thermally insulated cylindrical shell with wave number on elastic foundation ($\nu = 0.3$, $K = 1.5 \times 10^7$, $p_2 = 0$)

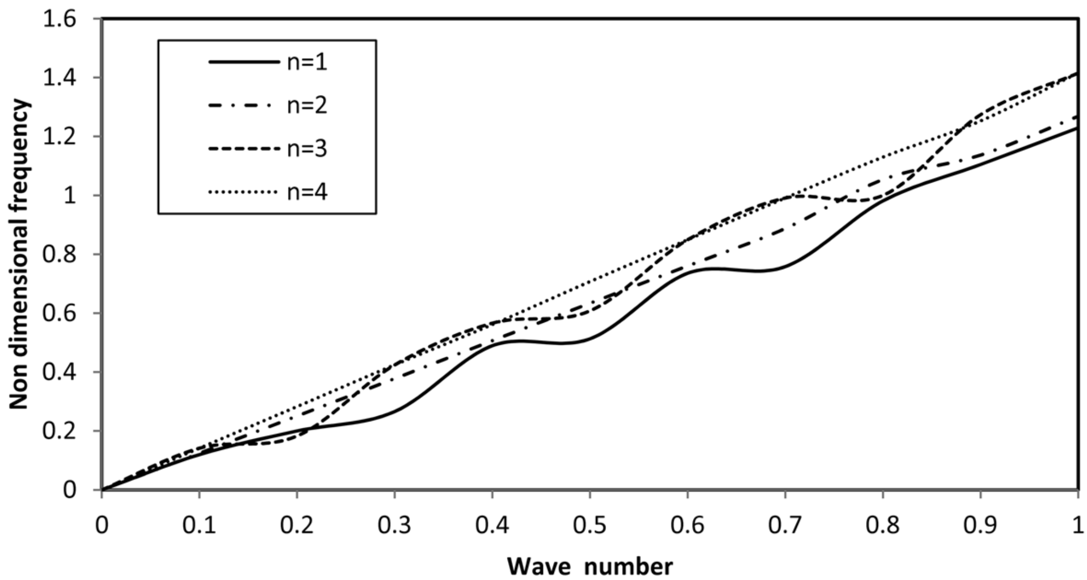

Figure3. Variation of non-dimensional frequency of thermally insulated cylindrical shell with wave number on elastic foundation ($\nu = 0.3$, $K = 1.5 \times 10^7$, $p_2 = 0$)

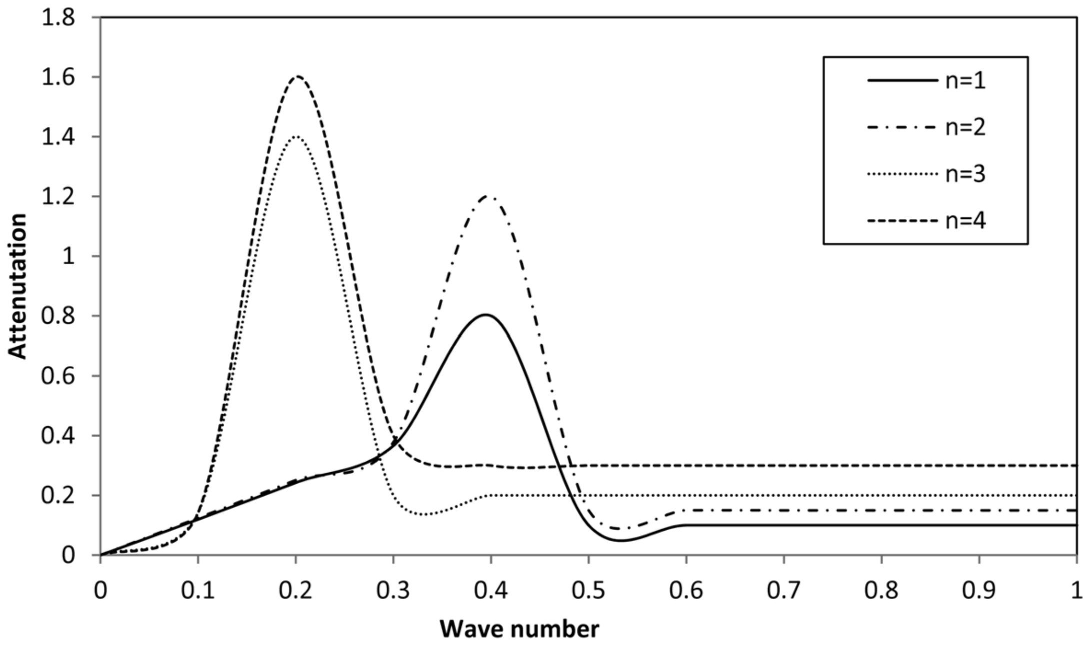

Figure4.Variation of attenuation of thermally insulated cylindrical shell with wave number on elastic foundation ($\nu = 0.3$, $K = 1.5 \times 10^7$, $p_2 = 0$)

VI. CONCLUSIONS

The three dimensional wave propagation of a homogeneous isotropic generalized thermo-elastic cylindrical shell was investigated in this paper. For this problem, the governing equations of three dimensional linear theory of generalized thermo-elasticity have been employed in the context of the Lord and Schulman theory and solved by the modified Bessel function with complex arguments. The effects of the frequency and attenuation coefficient with respect to the wave number and the foundation parameter p_1 on the natural frequencies of a closed Zinc cylindrical shell was investigated, with the results presented as the dispersion curves. A comparative study is made among the LS, GL and CT theories and the frequency change is observed to be highest for the LS theory, followed by the GL and CT theories due to the thermal relaxation effects and damping.

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