GRAPHICAL SCHEME FOR DETERMINATION OF MARKET CLEARING PRICE USING QUADRATIC BID FUNCTIONS

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ABSTRACT

Market clearing price (mcp) calculation is one of the most important functions of a power pool operator. Many existing power pools use a linear bid function received from the generators and the consumers for the computation of market clearing price. Although quadratic bid functions are more informative and accurate, they are not generally used, as the process of computing the market clearing price becomes more complicated. In this paper the final market clearing price and the schedules of available generators and loads are calculated using quadratic bid functions with the help of a simple iterative scheme coded in MATLAB.

KEYWORDS

Market clearing price (mcp), bid functions, economy of scale, social welfare function, incremental cost.

1. INTRODUCTION

One of the most important factors for the computation of the market clearing price in a deregulated market is the format of the bids and the cost function. There are four types of cost functions: linear, quadratic, translog and loglog. Most of the electricity market literatures use a piecewise linear or quadratic functions to compute the market clearing price. The effect of choosing different type of bids in electricity auctions and the difference between continuous and discrete bids are discussed by N. Fabra et al. [1]. According to economy of scale a firm in a competitive market maximises its profit when the sale is at marginal cost. But the generating companies have to submit the bids according to the electricity market. The piecewise linear bids are suitable for small or compact markets but for generating companies with multiple units, there would be a sudden change in cost when one unit is turned on or off. This jump would make calculus analysis difficult and would thus result in inconsistencies. Quadratic curves result in smooth revenue, profit curves when constraints like transmission congestion and generator capacities are not active.

The aim of this paper is to develop a method for determining the market clearing price in the case of pool markets neglecting the effect of congestion. In order to do so, bids from both the generating side and consumer side have been considered. Both these bids are considered to be quadratic functions of Real Power instead of linear functions for an accurate approach [2], [3], [4],[5]. Different cases considering violation of maximum and minimum limits of individual generators and loads have been discussed as well. The proposed method has been developed as an extension of an earlier method [2] used for determining mcp. Extending the concept developed in [2] a graphical approach has been developed in order to determine final clearing price. The results obtained are in concurrence with the results as given by [2]. Section 2 discusses the algorithm developed using the formulae for incremental cost as derived by [2], in a minimum cost approach [6] so as to maximize the social welfare function in an elastic demand market. A detailed illustration of algorithm is given thereby. Section 3 gives a detail look at various cases and the comparative results. The final illustration discusses a 30 bus system with a large number of generators and static as well as dynamic loads.

2. FORMULAE AND ALGORITHM

The objective of a power pool operator is to maximise the social welfare function which is defined as the difference between the cumulative consumer and generator bids. In the following subsections we have discussed the formulae used in the basic closed loop solution followed by the algorithm used to determine the final clearing price and the optimal schedules of generators and loads present in the system.

2.1 Formulae used in the closed loop solution

The following formulae used for the calculation of the clearing price have been discussed in detail by Bijuna Kunju K* and P S Nagendra Rao [2] and the proposed algorithm is an extension of their work.

Consider a system with N generators and M consumers. Let the generator bid function for the i^{th} generator be

$$C_i(Pg_i) = a_i Pg^2_i + b_i Pg_i + c_i \quad (1)$$

and the consumer benefit function for the jth load be

$$Bf_j(Pd_j) = \alpha_j Pd^2_j + \beta_j Pd_j + \gamma_j \quad (2)$$

Now, the objective of the pool market operator is to maximize the social welfare function

$$\sum_{j=1}^{M} Bf_j(Pd_j) - \sum_{i=1}^{N} C_i(Pg_i) \quad (3)$$

Subject to the power balance constraint

$$\sum_{i=1}^{N} (Pg_i) = \sum_{j=1}^{M} (Pd_j) \quad (4)$$

The schedules for each of the generators and the demand of each consumer that can be met is obtained as,

$$Pg_i = \frac{\lambda - b_i}{2a_i} \quad (5)$$

$$Pd_i = \frac{\lambda - \beta_j}{2\alpha_j} \quad (6)$$

2.2 Flow chart to determine final schedules and clearing price

The given formulae are valid only if the generators and consumers are within their specified limits. In case of a violation the violating generator's or load's schedule is held constant at its limit and the iterative process is repeated till we get an optimal solution for the clearing price and schedules of each generator or load. These steps have been implemented in form of a flow chart as shown in figure 1.

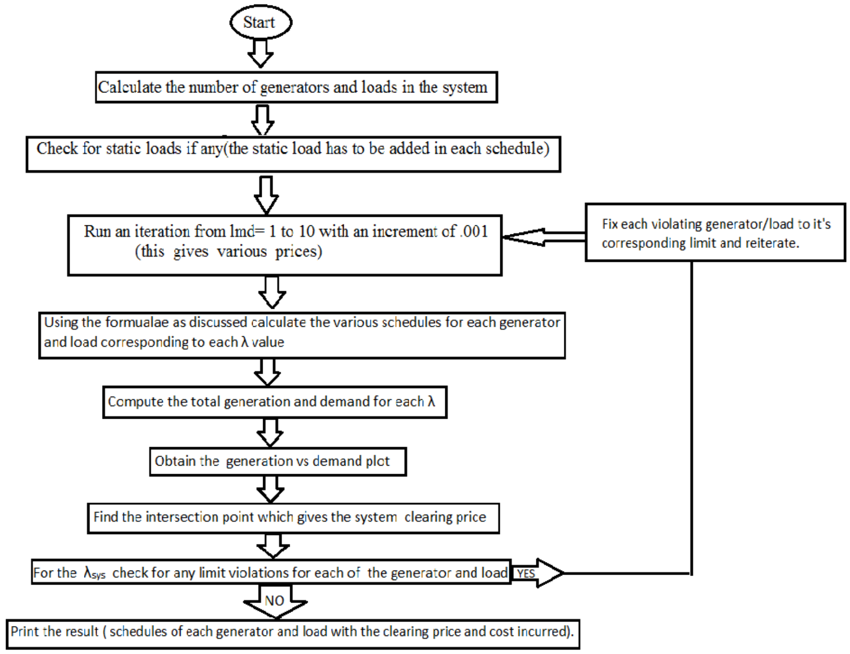

Figure 1: Flow Chart to calculate schedules and clearing price

3. ILLUSTRATION OF THE METHOD

Consider a system consisting of three generators and two consumers with variable demands. The generator cost functions of the three units are,

$C1 = 0.003Pg^2_1 + 2Pg_1 + 80$ Rs/h,
$C2 = 0.015Pg^2_2 + 1.45Pg_2 + 100$ Rs/h,
$C3 = 0.01Pg^2_3 + 0.95Pg_3 + 120$ Rs/h.
The offer functions of the two consumers are,
$Bf1 = -0.002Pd^2_1 + 5Pd_1 + 150$ Rs/h,
$Bf2 = -0.001Pd^2_2 + 6Pd_2 + 200$ Rs/h.

A. *Case 1: No Limit Constraints*

This case discusses the scheduling of the three generators and two loads when they operate without any constraints. The mcp obtained in this case is the intersection point of the aggregate supply and demand curves as shown in figure 2.

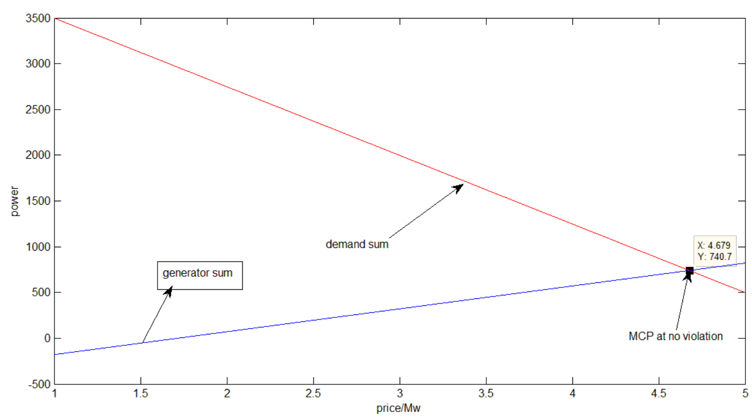

Figure 2. Demand and Generation vs Incremental Cost

The final clearing price is : λ = 4.679 Rs/MW

The schedules obtained are: $\mathbf{Pg_1}$ = 446.5MW, $\mathbf{Pg_2}$ =107.64MW, $\mathbf{Pg_3}$ =186.46MW, $\mathbf{Pd_1}$=80.2MW, $\mathbf{Pd_2}$ =660.4MW

***B**. Case 2: Maximum Limit on Generator Output*

This case discusses the effect of constraining a generator to a certain maximum limit on the clearing price and the individual schedules of the generators and loads.

In addition to the conditions in case 1, suppose that generator 1 has a maximum limit of 400 MW. Hence, the generation of this unit must be constrained to 400 MW, instead of 446.5 MW as calculated above in the previous case. The new clearing price obtained is shown in figure 3.

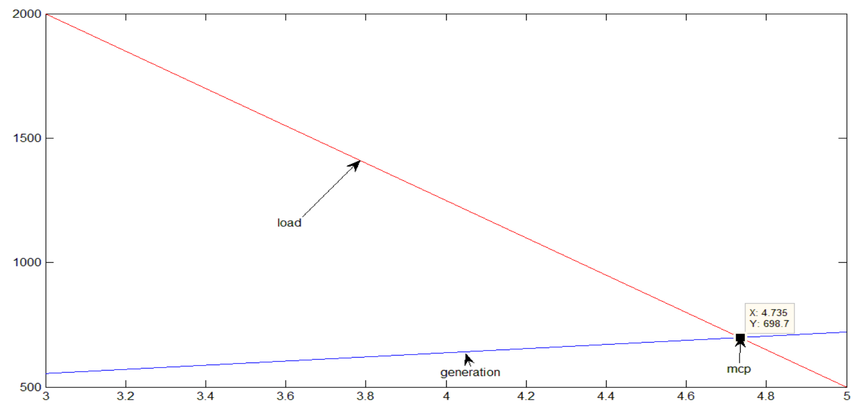

Figure 3. Demand and Generation vs Incremental Cost

The final clearing price is : λ = 4.735 Rs/MW

The schedules obtained after accounting for the violation in generator limits are:

$\mathbf{Pg_1}$ = 400.0 MW, $\mathbf{Pg_2}$ =109.5 MW, $\mathbf{Pg_3}$ =189.25 MW

$\mathbf{Pd_1}$ = 66.25 MW, $\mathbf{Pd_2}$ =632.5MW

***C.** Case 3: Maximum Limit on Load*

In another case one of the loads is constrained to a maximum limit of 600MW. This implies a constraint violation as seen by the schedules of case 1 and the load must be fixed at 600 MW instead of 660.4 MW. The new clearing price obtained after adjusting the schedules is shown in figure 4.

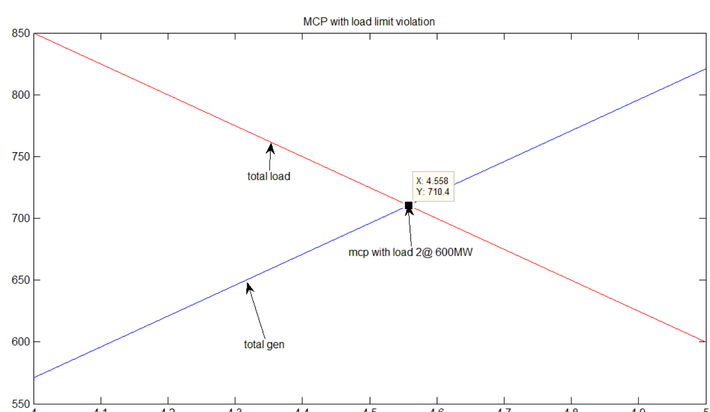

Figure 4.Demand and Generation vs Incremental Cost

The final clearing price is : $\lambda = 4.558$ Rs/MW

The schedules obtained after accounting for the violation in load limits are:

$\mathbf{Pg_1}$ = 426.33 MW, $\mathbf{Pg_2}$ =103.6 MW, $\mathbf{Pg_3}$ =180.4MW

$\mathbf{Pd_1}$ =110.5 MW, $\mathbf{Pd_2}$ =600.0 MW

D. Case 4: Maximum Limits on Both Generators and Consumer Loads

This case discusses the condition when both generator and load limit gets violated. Generator1 has a maximum limit of 400MW and the maximum demand of consumer 2 is 600 MW. From the optimal schedule of case 1 it is seen that both limits get violated. Hence their schedules are readjusted to their respective limits and a new clearing price is calculated as shown in figure 5.

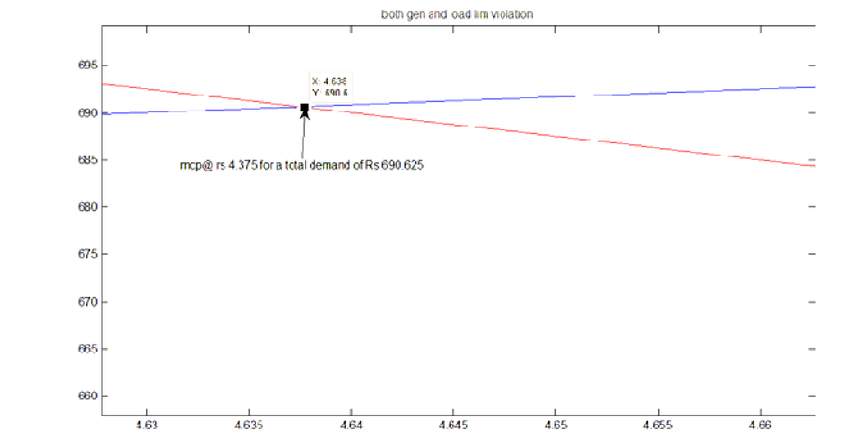

Figure 5. Demand and Generation vs. Incremental Cost

The final clearing price is : $\lambda = 4.637$ Rs/MW

The schedules obtained after accounting for the violation in generator and load limits are:
$\mathbf{Pg_1}$ = 400.00MW, $\mathbf{Pg_2}$ =106.233MW, $\mathbf{Pg_3}$ =184.35MW

$\mathbf{Pd_1}$ = 90.75 MW, $\mathbf{Pd_2}$ =600MW

E. Case 5: Mixed Load - (No Limit violation)

This example is provided to illustrate the method when the system has both fixed loads and price sensitive loads. A 30 bus system is used for this illustration. The bids and offers parameters of the system used here are given Table 1. There are some loads which do not submit offer function implying that they are ready to accept power at any cost. In this system the total fixed demand by those consumers is 75.4 MW. MCP of the system is obtained at the intersection point of aggregate demand and supply curves as shown in figure 6.

Table 1. The Bids of 30 Bus System

Gen No.	a_i	b_i	Dem No.	α_i	β_i
1	0.02	2.00	2	-0.02	6.0
2	0.0175	1.75	7	-0.02	4.4
22	0.0625	1.00	8	-0.02	4.8
27	0.0083	3.25	12	-0.02	4.2
23	0.025	3.00	21	-0.02	4.8
13	0.025	3.00	30	-0.02	4.0

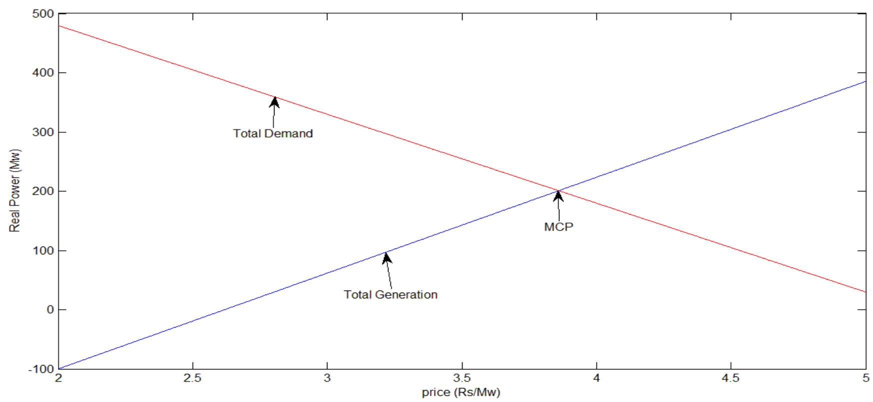

Figure 6. Demand and Generation vs Incremental Cost

The final clearing price is: λ= 3.861
The schedules obtained as per the clearing price have been shown in table 2.

Table 2. The Schedules Obtained For 30 Bus System

Gen No.	P_{gi}	**Dem No.**	P_{di}
1	46.550	2	53.45
2	60.343	7	13.45
22	22.8960	8	23.45
27	36.867	12	8.45
23	17.2400	21	23.45
13	17.2400	30	3.45
∑Gen.	200.9745	**∑Var.Dem**	125.85

***F.** Case 6: Mixed Load with Limit Violation*

Consider the system as in case 5, and suppose that in addition there is a maximum limit of 50 MW on the demand of consumer 2, Pd_2^{max} = 50MW.

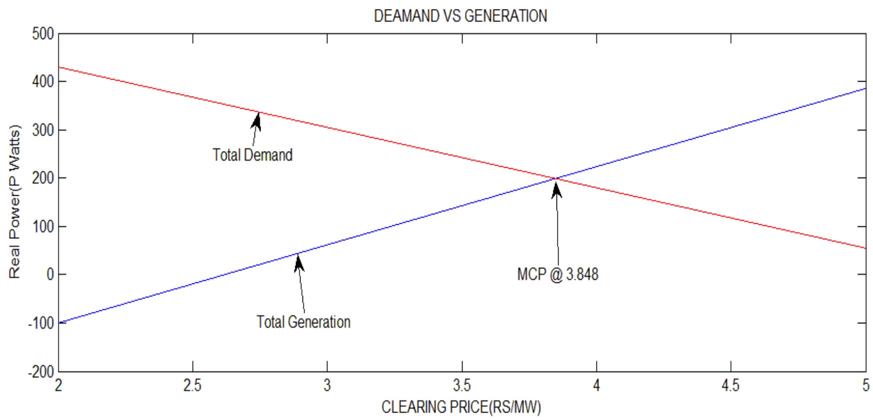

Figure 7. Demand and Generation vs Incremental Cost

The final clearing price is : **λ** = 3.85 Rs/MW

The schedules obtained as per the new clearing price are:

$\mathbf{P_{g1}}$ = 46.25MW, $\mathbf{P_{g2}}$ = 60.00MW, $\mathbf{P_{g22}}$ = 22.8MW, $\mathbf{P_{g27}}$ = 36.1446MW, $\mathbf{P_{g23}}$ = 17.00MW,
$\mathbf{P_{g13}}$ = 17.00MW
$\mathbf{P_{d2}}$ = 50.00MW, $\mathbf{P_{d7}}$ = 13.75MW, $\mathbf{P_{d8}}$ = 23.75MW, $\mathbf{P_{d12}}$ = 8.75MW, $\mathbf{P_{d21}}$ = 23.75MW,
$\mathbf{P_{d30}}$ = 3.75MW

4. CONCLUSION

A graphical scheme for determining the market clearing price using quadratic bid functions is proposed in this paper. The suggested algorithm uses a closed loop solution for the calculation of the incremental cost (λ) which is used iteratively for the calculation of the clearing price and the schedules corresponding to each generator and consumer bid.

The results obtained from the proposed algorithm are congruent with those obtained in [2] as shown in Table 3. Using the iterative approach the need for calculating the intermediate values of incremental cost has been eliminated hence increasing the system efficiency and reducing the computation time to a large extent.

Table 3: Comparison of the results

Case no.	Clearing Price as per [2]	Clearing Price as per proposed Algorithm
1	4.6792	4.679
2	4.735	4.735
3	4.5584	4.558
4	4.6375	4.637
5	3.862	3.862
6	3.8499	3.85

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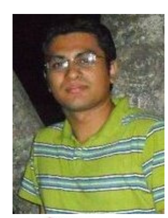

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