

## IMPROVED QR AIDED DETECTION UNDER CHANNEL ESTIMATION ERROR CONDITION

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### ABSTRACT

For future generation wireless techniques, multiple input multiple output (MIMO) systems are gaining an increasing interest due to its higher capacity and diversity gain. MIMO systems provide a linear increase in the channel capacity with respect to the number of antennas. Through multiple transmit antennas, multiple information symbols can be transmitted simultaneously, however, the challenge is to detect these symbols reliably at the receiver end. In this paper, we consider the problem of symbol detection in MIMO systems and present an improved QR aided detector under channel estimation error conditions. The proposed improved QR (IQR) detector is based on the underlying concept of multiple feedback (MF) strategy where multiple constellation points are used in the decision feedback loop of successive interference cancellation (SIC) based MIMO detector. In the proposed work, the detection sequence is ordered based on the signal to noise ratio (SNR) and the threshold radius of MF condition is varied according to the estimation error in the channel. The performance of the proposed algorithm is simulated and compared for different estimation error conditions.

**KEYWORDS:** MIMO, Channel estimation error, Successive Interference Cancellation, BER, Bit Error Rate, QAM, Quadrature Amplitude Modulation.

### I. INTRODUCTION

Due to the demand for higher data traffic in wireless systems, future generation of wireless technology is shifting towards new techniques such as long term evolution (LTE), LTE advanced (LTE-A) and wireless interoperability for microwave access (WiMAX), and a new generation of wireless communications is developed known as 4th generation (4G) and 5th generation (5G). Multiple-input multiple-output (MIMO) is a promising technique to overcome the exponential demand in data traffic [1]. In MIMO systems, the channel capacity depends linear on the minimum number of transmit and receive antennas [2], [3]. Spatial multiplexing (SM) is a key technique using which multiple transmit symbols can be transmitted through the multiple transmit antennas. But, the reliable detection of these symbols is a major challenge in MIMO systems. The optimal bit error rate (BER) performance can be achieved by using a well-known detector called as maximum likelihood (ML) detector. ML detector performs exhaustive search over all the possible transmit vectors and finally select the one which is best amongst all the solutions. In MIMO systems, the number of possible transmit vectors increases exponentially with respect to the number of transmit antennas or the modulation order, and therefore, the computational complexity of ML detector is exponential [4]. Several low complexity MIMO detectors proposed in the literature include linear and non-linear detectors such as zero forcing (ZF) detector, minimum mean squared error (MMSE) detector and successive interference cancellation (SIC) detector [5].

Other well-known detector is sphere decoder (SD) which achieves a near MLD performance with reduced complexity compared to the MLD [6]. Several other algorithms in this context have been proposed in the literature which includes [7]–[18]. Successive interference cancellation (SIC) using the QR decomposition is a well-known technique for sequential detection of symbols where the MIMO channel is decomposed into an orthonormal matrix  $Q$  and an upper triangular matrix  $R$  [7], [8]. Further, SIC is also known as layered detection where symbol corresponding to each transmit antenna is detected

in each layer. But, it is found that the SIC based MIMO detectors suffer from error propagation, and therefore, the BER performance of such detectors degrades comparatively. Therefore, a multiple feedback scheme is proposed in the literature [9] where the concept of shadow region is used in the decision feedback loop of SIC detectors. In multiple feedback strategy, multiple constellation points are used in the decision feedback loop of the SIC detector whenever a shadow condition occurs. The shadow condition occurs if the distance between the estimated value of the decision and the nearest constellation point is above a finite threshold value which means that the decision is unreliable.

In this paper, we consider the problem of symbol detection in MIMO systems and present an improved QR aided detector under channel estimation error conditions. The proposed improved QR (IQR) detector is based on the underlying concept of multiple feedback (MF) strategy where multiple constellation points are used in the decision feedback loop of successive interference cancellation (SIC) based MIMO detector. In the proposed work, the detection sequence is ordered based on the signal to noise ratio (SNR) and the threshold radius of MF condition is varied according to the estimation error in the channel. The simulation results for different channel estimation error situations are performed and compared.

The rest of the paper is organized as follows. In section 2, we present the mathematical model of the point to point MIMO system used in the paper. In section 3, the traditional MIMO detectors such as maximum likelihood detection, zero forcing detectors, minimum mean squared error detector and QR aided SIC detector. The proposed method is discussed in section 4. In section 5, the simulation results on BER performance are discussed for 12×12, 16×16, and 32×32 MIMO systems with 4-QAM signaling. Finally in section 6, we conclude the article.

## II. SYSTEM MODEL

Consider, a point to point MIMO system with N transmit antennas and M receive antennas. Let  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$  represent the transmit vector with symbol  $x_j$  being transmitted from the  $j^{\text{th}}$  transmit antenna. The symbol  $x_i$  is taken from a constellation set represented by C (for e.g. BPSK, QPSK and 4-QAM). The channel through which the symbols are transmitted is given by H, and is assumed to be a Rayleigh flat fading channel. The receiver receives a modified vector (i.e. through channel variations and receiver noise) y as given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where H is an M × N MIMO channel matrix and each element of H i.e.  $h_{kl}$  for  $k = 1, 2, \dots, M$  and  $l = 1, 2, \dots, N$  represents the channel gain between the  $l^{\text{th}}$  transmit antenna and the  $k^{\text{th}}$  receive antenna. The vector n is additive white Gaussian noise vector and each element  $n_i$  for  $i = 1, 2, \dots, N_R$  independent and identically distributed (i.i.d.) and  $\sim CN(0, \sigma^2)$ ,  $\sigma^2$  is the noise variance.

$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{w} \tag{2}$$

where  $\mathbf{r} = \begin{bmatrix} \Re(\bar{\mathbf{y}}) \\ \Im(\bar{\mathbf{y}}) \end{bmatrix}_{2M \times 1}$ ,  $\mathbf{G} = \begin{bmatrix} \Re(\bar{\mathbf{H}}) & -\Im(\bar{\mathbf{H}}) \\ \Im(\bar{\mathbf{H}}) & \Re(\bar{\mathbf{H}}) \end{bmatrix}_{2M \times 2N}$ ,  $\mathbf{s} = \begin{bmatrix} \Re(\bar{\mathbf{x}}) \\ \Im(\bar{\mathbf{x}}) \end{bmatrix}_{2N \times 1}$ , and  $\mathbf{w} = \begin{bmatrix} \Re(\bar{\mathbf{n}}) \\ \Im(\bar{\mathbf{n}}) \end{bmatrix}_{2M \times 1}$ .  $\Re(\cdot)$  and  $\Im(\cdot)$  denote the real and imaginary parts of  $(\cdot)$  respectively. The constellation set with real entries is  $\mathcal{O}$ .

In this paper, we consider the channel estimation error condition where we assume a parameter e which is the measure of error in the channel estimation. Mathematically, the estimation error can be modeled as

$$\mathbf{H}_e = \mathbf{H} + e\hat{\mathbf{H}} \tag{3}$$

where  $\hat{\mathbf{H}}$  follow the distribution similar to that of the H. The value of e is taken to be 0.1 and 0.2 in the paper which means 10% and 20% error respectively.

### III. TRADITIONAL MIMO DETECTORS

In this section, we present overview of traditional MIMO detectors such as maximum likelihood detector (MLD), zero forcing (ZF) detector, minimum mean squared error (MMSE) detector and successive interference cancellation (SIC) detector.

#### A. Maximum Likelihood detector

The maximum likelihood detection is known to achieve the optimal (minimum) bit error rate performance. In ML, an exhaustive search is performed over all the possible transmit vectors. The ML solution is given by

$$s_{ML} = \arg \min_{s \in \mathcal{O}^N} \|\mathbf{r} - \mathbf{G}s\|^2 \quad (4)$$

where  $\mathcal{O}^N$  denotes the set of all possible symbol vectors which could be transmitted by the transmitter. But, the main drawback of MLD is that the number of computations required for performing the exhaustive search grows exponentially with increase in the number of transmit antennas due to which MLD becomes impractical.

#### B. Zero forcing detector

The zero forcing (ZF) detection is a linear detection method for MIMO systems. In ZF, a transformation matrix is used which removes the effect of channel gain matrix from the received vector by multiplying it with the pseudo inverse matrix as

$$\mathbf{T}_{ZF} = (\mathbf{G}^G \mathbf{G})^{-1} \mathbf{G}^H \quad (5)$$

where  $(.)^H$  denote the Hermitian transpose of the matrix and  $(.)^{-1}$  denotes the inverse of a matrix. This matrix is then multiplied with the received vector as

$$\mathbf{z}_{ZF} = \mathbf{T}_{ZF} \mathbf{r} \quad (6)$$

$$\mathbf{z}_{ZF} = \mathbf{s} + \mathbf{T}_{ZF} \mathbf{w} \quad (7)$$

Now every entry of the estimated vector is quantized to the nearest constellation point as

$$s_{ZF}(i) = Q[z_{ZF}(i)] \quad \forall i = 1, 2, \dots, N \quad (8)$$

where  $Q[.]$  is the quantization operator.

#### C. Minimum mean squared error detector

In minimum mean squared error (MMSE) detection technique, the mean squared distance between the transmitted vector and the estimated vector is minimized and the transformation matrix  $\mathbf{T}_{MMSE}$  is found which is given by

$$\mathbf{T}_{MMSE} = (\mathbf{G}^G \mathbf{G} + \sigma^2 \mathbf{I}_N)^{-1} \mathbf{G}^H \quad (9)$$

Similar to the ZF detection, in MMSE also the transformation matrix is multiplied with the received vector followed by the quantization.

#### D. Successive interference cancellation detector

In QR aided successive interference cancellation (SIC), the symbols are detected sequentially for corresponding to each transmit antenna by using the QR decomposition of the channel matrix  $\mathbf{H}$ . After every successful decision about a symbol, its interference from the received vector is canceled and the

decision about next symbol in the sequence is taken accordingly. The steps involved in the SIC detection technique are shown in Algorithm 1.

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**Algorithm 1** QR aided SIC algorithm for MIMO detection
 

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1: input:  $\mathbf{r}$ ,  $\mathbf{G}$ ,  $N$ ,  $M$ ;
2: Compute  $\tilde{\mathbf{r}} = \mathbf{Q}^H \mathbf{r}$  where  $\mathbf{G} = \mathbf{QR}$ 
3: for  $k = 2N : -1 : 1$  do
4:    $\hat{s}_k = \frac{\tilde{r}_k - \sum_{j=k+1}^{2N} R_{k,j} \tilde{s}_j}{R_{k,k}}$ 
5:    $\tilde{s}_k = \mathcal{Q}(\hat{s}_k)$ 
6: end for
7: output:  $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_{2N}]^T$  is output solution vector
  
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#### IV. PROPOSED DETECTION SCHEME

In this section, we present the proposed algorithm for symbol detection in MIMO systems. In the proposed work, the detection sequence is ordered based on the signal to noise ratio (SNR) and the threshold radius of MF condition is varied according to the estimation error in the channel. For different channel estimation error values, different value of the predefined threshold radius values are used in order to enhance the BER performance. The SNR based ordering for ordering the detection sequence of MF SIC algorithm is used. The SNR based ordering is performed while decomposing the channel matrix  $\mathbf{G}$  into an orthonormal matrix  $\mathbf{Q}$  and an upper triangular matrix  $\mathbf{R}$ . The algorithm for SNR ordered QR decomposition is shown in Algorithm 2.

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**Algorithm 2** SNR-based ordering Algorithm for QR Decomposition
 

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1: initialize:  $\mathbf{R} = \mathbf{0}$ ,  $\mathbf{Q} = \mathbf{G}_e$ ,  $\Phi = (1, \dots, 2N)$ ;
2: for  $l = 1, \dots, 2N$  do
3:    $m_l = \arg \min_{i=l, \dots, 2N} (\|\mathbf{q}_i\|)$ ;
4:   where  $\mathbf{q}_i$  is the  $i^{th}$  column of matrix  $\mathbf{Q}$ 
5:   interchange columns  $l$  and  $m_l$  in  $\mathbf{Q}$ ,  $\mathbf{R}$ ,  $\Phi$ ;
6:    $r_{l,l} = \|\mathbf{q}_l\|$ ;
7:    $\mathbf{q}_l = \frac{\mathbf{q}_l}{r_{l,l}}$ ;
8:   for  $i = l + 1, \dots, 2N_i$  do
9:      $r_{l,i} = \mathbf{q}_l^H \mathbf{q}_i$ ;
10:     $\mathbf{q}_i = \mathbf{q}_i - r_{l,i} \mathbf{q}_l$ ;
11:   end for
12: end for
  
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Once the ordered QR decomposition is done, the matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are fed to the SIC based detector for detection. However, the SIC based detector as shown in Algorithm 1 is modified by integrating the concept of multiple feedback strategy. Therefore, after every decision its reliability is checked by comparing the distance between the soft value of the decision and the nearest constellation point. If the distance is more than the predefined threshold value (say  $d$ ) then the decision is said to be unreliable and multiple neighboring constellation points are used in the decision feedback loop followed by the SIC based detector. If the distance is less than  $d$  then the decision is said to be reliable and the detection procedure is continued for the remaining layers. The steps involved in the QR aided ordered multiple feedback SIC algorithm are as below

*Step 1 :* Perform SNR based QR decomposition of the channel matrix  $\mathbf{G}_e$  as shown in Algorithm 2.

*Step 2 :* Initiate the QR aided SIC as shown in Algorithm 1.

*Step 3 :* Compute  $d_{is} = |\tilde{s}_k - \mathcal{Q}(\tilde{s}_k)|$  and check if  $d_{is} < d$  then the decision is reliable and declare  $\tilde{s}_k = \mathcal{Q}(\tilde{s}_k)$  else if  $d_{is} > d$  then the decision falls in the shadow region and is declared unreliable and follow the multiple feedback criteria according to [10].

*Step 4 :* Declare the final result and terminate the procedure.

However, the channel matrix used here is the channel with estimation error  $e$  as shown in Eq. 3. However, the value of the threshold radius are changed to  $d = 0.1, 0.2, 0.5$  for different values of  $e$ .

## V. SIMULATION RESULTS

In this section, the simulation results on bit error rate versus signal to noise ratio are analyzed and compared for the SNR ordered QR decomposition aided SIC detector under different channel estimation error conditions  $e = 0, 0.1, 0.2$  and with different threshold radius values such as  $d = 0.1; 0.2, 0.5$ . The simulations are performed in MATLAB software. For comparing the BER, we considered  $16 \times 16$  and  $32 \times 32$  MIMO systems with 4-QAM signaling.

In figure 1, the BER performance for  $16 \times 16$  MIMO systems is plotted with respect to the SNR for  $e = 0, 0.1, 0.2$  and  $d = 0.1$ . It is observed that the BER performance of the proposed algorithm degrades with increase in the channel estimation error. Similarly, in figure 2 and 3, the BER performance is simulated for  $16 \times 16$  MIMO systems with 4-QAM for  $e = 0, 0.1, 0.2$  and  $d = 0.2, 0.5$ , respectively. The observations are also similar to that of the figure 1. In figure 4, 5 and 6, the BER performance for MIMO systems with  $32 \times 32$  antenna is compared for the proposed algorithm with  $e = 0, 0.1, 0.2$  and  $d = 0.1, 0.2, 0.5$ , respectively. It is observed that the BER performance of MIMO system degrades under the channel estimation error conditions in  $32 \times 32$  MIMO systems as well which can be increasing the threshold radius value.

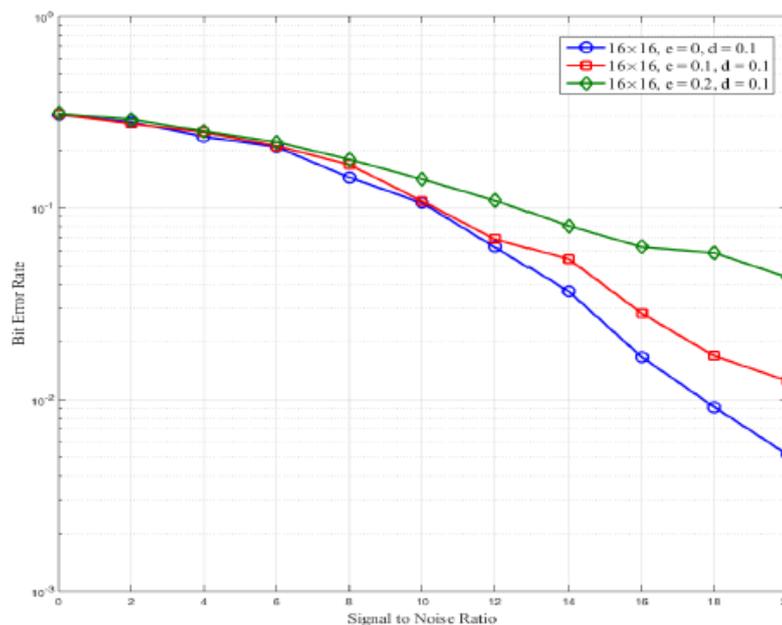


Fig. 1. Bit error rate performance of  $16 \times 16$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.1$

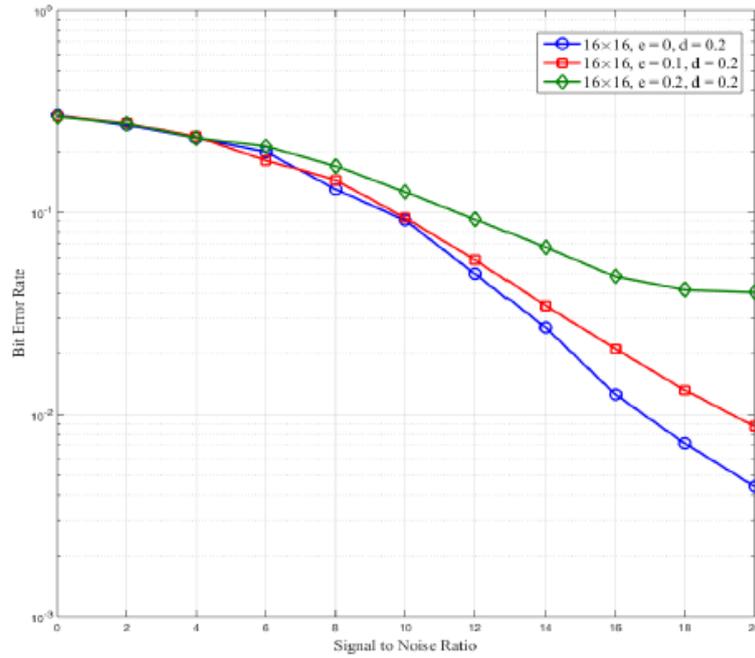


Fig. 2. Bit error rate performance of  $16 \times 16$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.2$

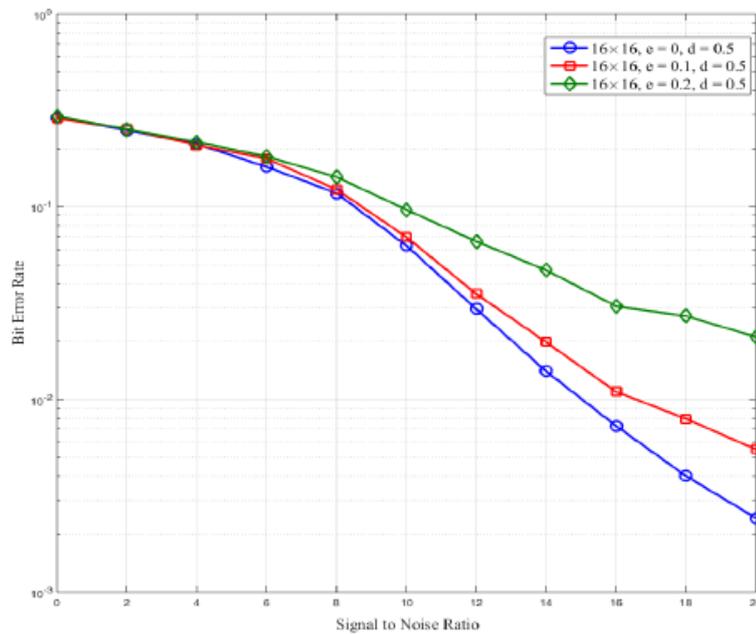


Fig. 3. Bit error rate performance of  $16 \times 16$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.5$

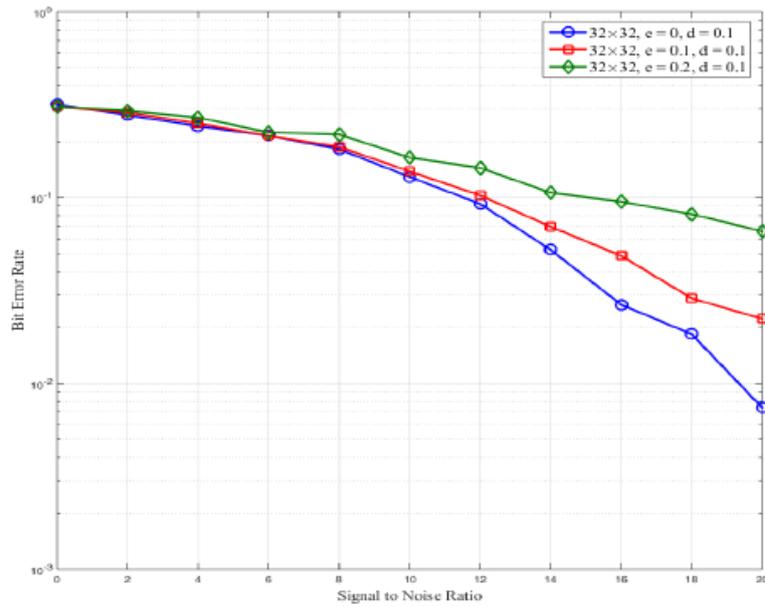


Fig. 4. Bit error rate performance of  $32 \times 32$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.1$

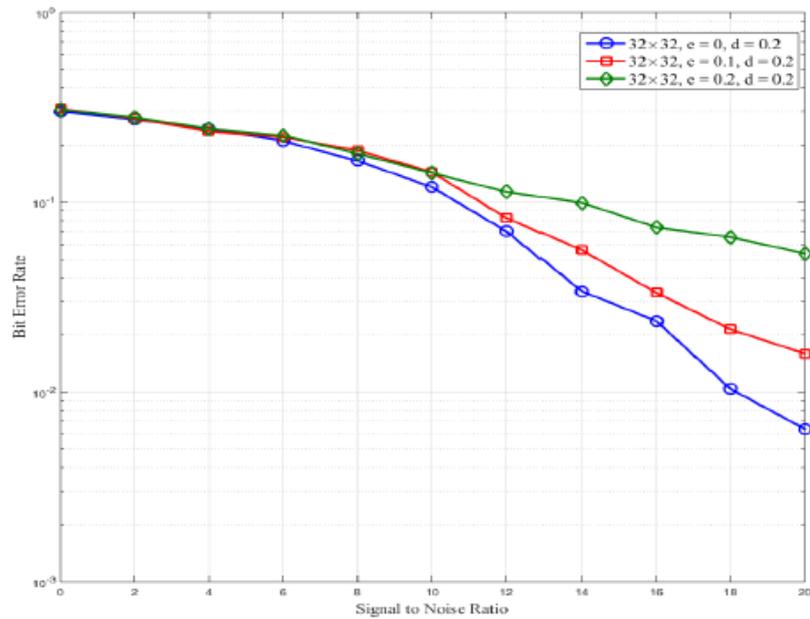


Fig. 5. Bit error rate performance of  $32 \times 32$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.2$

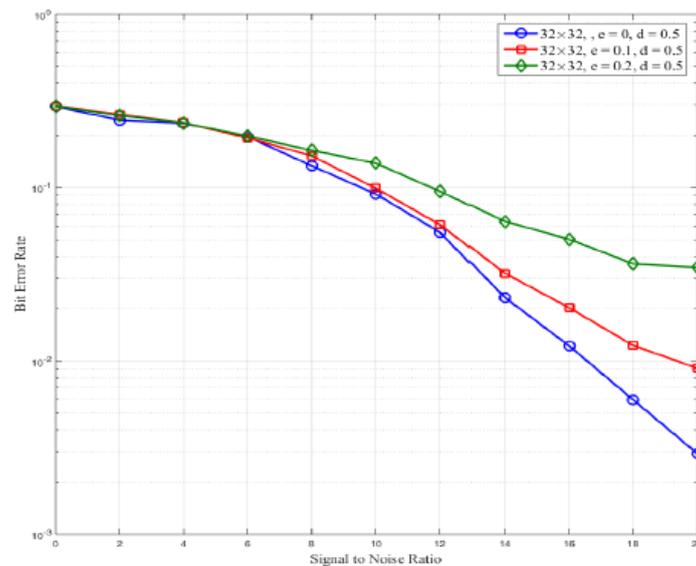


Fig. 6. Bit error rate performance of  $32 \times 32$  multiple-input multiple-output system versus signal to noise ratio (dB) for  $e = 0, 0.1, 0.2$  and  $d = 0.5$

## VI. CONCLUSIONS

In this paper, a QR decomposition aided SNR ordered multiple feedback successive interference cancellation algorithm is proposed for symbol detection in large MIMO systems under channel estimation error conditions. Further, for different estimation error scenarios different threshold radius values are used. For high value of channel estimation error, the increased threshold radius value needs to be increased in order to achieve an enhanced BER performance. Through simulations, the BER performance of the proposed algorithm is analyzed and compared for different MIMO setups.

## VII. FUTURE WORK

In this paper, an Improved QR Aided Detection Under Channel Estimation Error Conditions proposed so as to detect symbols in MIMO systems. As an extension to the work in future one may consider other algorithms to enhance the system performance which are more robust in terms of bit error performance. Furthermore, power allocation to different antennas can also be studied.

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