HALL EFFECTS ON MHD PULSATILE FLOW THROUGH A POROUS MEDIUM IN A FLEXIBLE CHANNEL

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ABSTRACT

In this paper we discuss the hall current effect on the pulsatile flow of a viscous incompressible fluid through a porous medium in a flexible channel under the influence of transverse magnetic field using Brinkman's model. The non-linear equations governing the flow are solved using perturbation technique. Assuming long wavelength approximation, the velocity components and stresses on the wall are calculated up to order in ε *and the behaviour of the axial and transverse velocities as well as the stresses is discussed for different variation in the governing parameters. The shear stresses on the wall are calculated throughout the cycle of oscillation at different points within a wavelength and the flow separation is analyzed.*

KEYWORDS: *MHD flows, flexible channels, hall current effects, porous medium and pulsatile flow*

I. INTRODUCTION

The flow caused by a pulsatile pressure gradient through a porous channel or a porous pipe has been investigated in view of its applications in technological and physiological and physiological problems [1, 2, 3, 10, 11, 17, and 18]. In physiological fluid dynamics this model plays a significant role in explaining the dialysis of blood in artificial kidneys, vasomotor of small blood vessel such as arterioles, venues and capillaries. In most of the above investigations the boundary surface of the channel or pipe is assumed to have uniform cross section and an axial pressure gradient is maintained along the channel, which induces an unidirectional flow. In many biomedical problems one encounters the flow bounded by flexible boundaries and flow through uniform gap is only an approximation. Notable among them is the blood flow through veins. Rudraiah et al [10] discussed the channel flow with permeable walls and through porous medium with non-uniform gap. Mazumdar and Thalassoudis [7] constructed a simple mathematical model of blood flow through a disc-type prosthetic heart valve using computational fluid dynamic methods. They computed stream function, vorticity, velocity and shear stresses by solving the vorticity-transport equation and poisson's equation on a computer using a finite difference method. They reported that the proposed model revealed quite acceptable results for the complex velocity fields and shear stress distributions in the vicinity of the disc valve. The study suggested the possibility of an inexpensive method for the evaluation of existing or future prosthetic heart valve designs. Sud et al [12 & 13] presented an analysis of blood flow in large and small arteries under the influence of externally applied periodic oscillations. They obtained the solutions for velocity, acceleration and shear stress of blood flow and reported that high blood velocity and large shear stress produced in large arteries, but flow is not disturbed large shear stress produced in large arteries, but flow is not disturbed in small arteries. Perktold et al [8] analyzed the pulsatile flow through a model segment of Carotid siphon, which is a multiple curved segment of human internal carotid artery located at the base of the skull. Rappaport et al [9] studied on pulmonary arteries and measurements, Gaehtgens [5] and Integlietta [6] show conclusively that the pulsatile flow exists throughout the microcirculation of blood. Thus the realistic model of the blood flow in the smaller vessels must include pulsatile flow effects in addition to the peristaltic flow created by the motion of the flexible boundary. In this connection, it is very important to understand the phenomena of separation in oscillatory flows. Despard and Miller [4] have established that the steady flow definition of separation which is characterized by the vanishing of shear stress on the wall is inapplicable in oscillatory flows. Separation in oscillating flows can be defined as commencing with the initial occurrence of zero velocity or reversed flow at some point in the velocity profile throughout the entire cycle of oscillation. A suitable criterion for the recognition of separation as stated by Despard and Miller [4] in that the velocity gradient at the wall should be less than or equal to zero throughout the entire cycle of oscillation. This amounts to the existence of a point at which the shear stress at the wall is observed for the first time to be less than or equal to zero throughout the entire cycle of oscillation. Recently Veerakrishna et.al [16] discussed the pulsatile flow of viscous incompressible fluid through a porous medium in a flexible channel under the influence of transverse magnetic field. Syamala Sarojini [14] discussed unsteady MHD flow of a couple stress fluid through a porous medium between parallel plates under the influence of pulsation of pressure gradient. Later Raju [15] investigated unsteady MHD flow of a couple stress fluid through a porous medium between parallel plates under the influence of pulsation of pressure gradient taking hall current into account. The steady hydro magnetic rotating viscous flow through a non- porous or porous medium has drawn attention it the recent years for possible applications in geophysical and cosmical fluid dynamics. For example, the channel flow problems where the flow is maintained by torsional or non-torsional oscillations of one or both the boundaries, throw some light in finding out the growth and development of boundary layers associated with flows occurring in geothermal phenomena. Claire Jacob [19] has studied the transient effects considering the small amplitude torsional oscillations of disks. This problem has been extended to the hydro magnetic case by Murthy [20], who discussed torsional oscillations of the disks maintained at different temperatures. Debnath [21] has considered an unsteady hydrodynamic and hydro magnetic boundary flow in a rotating viscous fluid due to oscillations of plates including the effects of uniform pressure gradients and uniform suction. The structure of the velocity field and the associated Stokes, Ekman and Rayleigh boundary layers on the plates are determined for the resonant and non-resonant cases. Rao *et. al* [22] have made an initial value investigation of the combined free and forced convection effects in an unsteady hydro magnetic viscous incompressible rotating fluid between two disks under a uniform transverse magnetic field. This analysis has been extended to porous boundaries by Sarojamma and Krishna [23] and later by Sivaprasad [24] to include the Hall current effects. In this paper we discuss the hall current effect on the pulsatile flow of a viscous incompressible fluid through a porous medium in a flexible channel under the influence of transverse magnetic field using Brinkman's model. The non-linear equations governing the flow are solved using perturbation technique.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the unsteady fully developed pulsatile flow of viscous incompressible fluid through a porous medium in a flexible channel under the influence of transverse magnetic field of strength H_O. At $t>0$ the fluid is driven by a constant pressure gradient parallel to the channel walls. Choosing the Cartesian coordinate system O(x, y) the upper and lower walls of the channel are given by $y = \pm\, as\left(\frac{x}{\lambda}\right)$ Where, a is the amplitude, λ is the wavelength and s is an arbitrary function of the normalized axial co-ordinate $x^* = \frac{x}{\lambda}$. The entire flow is subjected to strong uniform transverse magnetic field normal to the plate in its own plane. Equation of motion along x-direction the x-component current density $\mu_e J_y H_o$ and the y-component current density $-\mu_e J_x H_o$. The equations governing the two dimensional flow of viscous incompressible fluid through a porous medium under the influence of transverse magnetic field, using Brinkman's model are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \mu_e J_y H_0 - \frac{\nu}{k}u \tag{2.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\mu_e J_x H_0-\frac{\nu}{k}v \tag{2.3}$$

Where (u, v) are the velocity components along O(x, y) directions respectively. ρ is the density of the fluid, p is the fluid pressure, k is the permeability of the porous medium, μ_e the magnetic permeability, ν the coefficient of kinematic viscosity and H_o is the applied magnetic field. Since the plates extends to infinity along x and y directions, all the physical quantities except the pressure depend on z and t alone. Hence u and v are function of z and t alone and hence the respective equations of continuity are trivially satisfied. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current, so that

$$J+\frac{\omega_e\tau_e}{H_0}J\times H=\sigma\,(E+\mu_e\,q\times H) \tag{2.4}$$

Where, q is the velocity vector, H is the magnetic field intensity vector, E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and μ_e is the magnetic permeability. In equation (2.4), the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

$$J_x+m\,J_y=\sigma\mu_e H_0 v \tag{2.5}$$

$$J_y-m\,J_x=-\sigma\mu_e H_0 u \tag{2.6}$$

where $m=\omega_e\tau_e$ is the hall parameter.

On solving equations (2.5) and (2.6) we obtain

$$J_x=\frac{\sigma\mu_e H_0}{1+m^2}(v+mu) \tag{2.7}$$

$$J_y=\frac{\sigma\mu_e H_0}{1+m^2}(mv-u) \tag{2.8}$$

Using the equations (2.7) and (2.8) the equations of the motion with reference to frame are given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho(1+m^2)}(mv-u)-\frac{\nu}{k}u \tag{2.9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho(1+m^2)}(v+mu)-\frac{\nu}{k}v \tag{2.10}$$

Eliminating p from equations (2.9) and (2.10), the governing the flow in terms of appropriate stream function ψ reduces to

$$(\nabla^2\psi)_t-\psi_y\nabla^2\psi_x+\psi_x\nabla^2\psi_y=-\left(\frac{\sigma\,\mu_e^2H_0^2}{\rho(1+m^2)}+\frac{\nu}{k}\right)\nabla^2\psi+\nu\,\nabla^4\psi \tag{2.11}$$

Where ∇^2 is the Laplacian operator

The relevant conditions on ψ are

$$u=-\frac{\partial\psi}{\partial y},v=\frac{\partial\psi}{\partial x} \tag{2.12}$$

The relevant boundary conditions are

$$\psi=0,\ \psi_{yy}=0 \qquad \text{on } y=0 \tag{2.13}$$

$$\psi_y=0,\ \psi=1+k_1e^{it} \qquad \text{on } y=\mathrm{s} \tag{2.14}$$

An oscillatory time dependent flux is imposed on the flow resulting in a pulsatile flow we assume that oscillatory flux across the channel is $\psi_f\left(1+k_1e^{i\omega t}\right)$. Where ψ_f is the characteristic flux, k_1 is its amplitude and ω the frequency of oscillation. We define a characteristic velocity q corresponding to the characteristic flux ψ, so that ψ is $q\,a$.

We introduce the following non-dimensional variables.

$$x^* = \frac{\psi}{\lambda}, y^* = \frac{y}{a}, t^* = \omega t,\ \varepsilon = \frac{a}{\lambda}, \psi^* = \frac{\psi}{q\,a}, \psi^*{}_f = \frac{\psi_f}{q_a}$$

Substituting the above non-dimensional variables into the equation (2.11), the governing equation in terms of non-dimensional parameter ψ (on dropping the asterisks) reduces to

$$R\varepsilon\left(\varepsilon^2(\psi_x\psi_{yxx} - \psi_y\psi_{xxx}) + \psi_x\psi_{yyy} - \psi_y\psi_{xyy}\right) + S\varepsilon^2\psi_{tyy} + S\psi^4\psi_{txx} =$$

$$\varepsilon^4\psi_{xxxx} + \psi_{yyyy} + \varepsilon^2\left[2\psi_{xxyy} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\psi_{xx}\right] - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\psi_{yy} \tag{2.15}$$

Where, $R = \frac{aq}{\nu}$ is the Reynolds number, $S = \frac{\lambda^2\omega}{\nu}$ is the oscillatory parameter, $D^{-1} = \frac{a^2}{k}$ is the inverse Darcy parameter, $M^2 = \frac{\sigma\mu_e^2 H_0^2 a^2}{\rho\nu}$ is the Hartmann number (Magnetic field Parameter), $m = \omega_e\tau_e$ is the hall parameter. Equation (2.15) is highly non-linear and is not amenable for exact solution. However assuming the slope of the flexible channel ε small (<<1). We take ψ may be given asymptotic expansion in the form

$$\psi = \left(\psi_0 k_1 e^{it}\bar{\psi}_0\right) + \varepsilon\left(\psi_1 + k_1 e^{it}\bar{\psi}_1\right) + ------ \tag{2.16}$$

We are making use of transformation

$$\eta = \frac{y}{s(x)} \tag{2.17}$$

And the boundary conditions at $y = s(x)$, Now to be satisfied at $\eta = 1$. Substituting equation (2.16) in the non-dimensional equation (2.15) and equating like powers of ε, the equations corresponding to the zeroth and first order steady and unsteady components are,

Zeroth order,

$$\frac{\partial^4\psi_0}{\partial y^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\frac{\partial^2\psi_0}{\partial y^2} = 0 \tag{2.18}$$

$$\frac{\partial^4\bar{\psi}_0}{\partial y^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\frac{\partial^2\psi_0}{\partial y^2} = 0 \tag{2.19}$$

First Order,

$$\frac{\partial^4\psi_1}{\partial y^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\frac{\partial^2\psi_1}{\partial y^2} = R\left[\frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial y^3} - \frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial x\partial y^2}\right] \tag{2.20}$$

$$\frac{\partial^4\bar{\psi}_1}{\partial y^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)\frac{\partial^2\bar{\psi}_1}{\partial y^2} =$$

$$R\left[\frac{\partial\psi_0}{\partial x}\frac{\partial^3\bar{\psi}_0}{\partial y^3} + \frac{\partial\bar{\psi}_0}{\partial x}\frac{\partial^3\psi_0}{\partial y^3} - \frac{\partial\psi_0}{\partial y}\frac{\partial^3\bar{\psi}_0}{\partial x\partial y^2} - \frac{\partial\bar{\psi}_0}{\partial y}\frac{\partial^3\psi_0}{\partial x\partial y^2}\right] \tag{2.21}$$

Substituting (2.17) in equations (2.18) and (2.20) we obtain

$$\frac{\partial^4\psi_0}{\partial\eta^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)s^2\frac{\partial^2\psi_0}{\partial\eta^2} = 0 \tag{2.22}$$

$$\frac{\partial^4\psi_1}{\partial\eta^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right)s^2\frac{\partial^2\psi_1}{\partial\eta^2} = Rs\left[\frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial\eta^3} - \frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial x\partial\eta^2}\right] \tag{2.23}$$

The corresponding boundary conditions to be satisfied are

$$\psi_0 = 0 = \frac{\partial^2\psi_0}{\partial\eta^2},\ \psi_1 = 0 = \frac{\partial^2\psi_1}{\partial\eta^2} \qquad \text{on} \qquad \eta = 0 \tag{2.24}$$

$$\frac{\partial \psi_0}{\partial \eta} = 0,\ \psi_0 = 1,\ \frac{\partial \psi_1}{\partial \eta} = 0,\ \ \psi_1 = 0 \qquad \text{on} \qquad \eta = 1 \tag{2.25}$$

Again substituting (2.17) in the equations (2.19) and (2.21) we obtain,

$$\frac{\partial^4 \bar{\psi}_0}{\partial \eta^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right) s^2 \frac{\partial^2 \bar{\psi}_0}{\partial \eta^2} = 0 \tag{2.26}$$

$$\frac{\partial^4 \bar{\psi}_1}{\partial \eta^4} - \left(\frac{M^2}{1+m^2} + D^{-1}\right) s^2 \frac{\partial^2 \bar{\psi}_1}{\partial \eta^2} = Rs \left[\frac{\partial \psi_0}{\partial x}\frac{\partial^3 \bar{\psi}_0}{\partial \eta^3} + \frac{\partial \bar{\psi}_0}{\partial x}\frac{\partial^3 \psi_0}{\partial \eta^3} - \frac{\partial \psi_0}{\partial \eta}\frac{\partial^3 \bar{\psi}_0}{\partial x \partial \eta^2} - \frac{\partial \bar{\psi}_0}{\partial \eta}.\frac{\partial^3 \psi_0}{\partial x \partial \eta^2}\right] \tag{2.27}$$

The corresponding boundary conditions to be satisfied.

$$\bar{\psi}_0 = 0 = \frac{\partial^2 \bar{\psi}_0}{\partial \eta^2},\ \bar{\psi}_1 = 0 = \frac{\partial^2 \bar{\psi}_1}{\partial \eta^2} \qquad \text{on} \qquad \eta = 0 \tag{2.28}$$

$$\frac{\partial \bar{\psi}_0}{\partial \eta} = 0,\ \bar{\psi}_0 = 1,\ \frac{\partial \bar{\psi}_1}{\partial \eta} = 0,\ \ \bar{\psi}_1 = 0 \qquad \text{on} \qquad \eta = 1 \tag{2.29}$$

Solving the equations (2.22) and (2.23) subjects to the boundary conditions (2.24) and (2.25) we obtain.

$$\psi_0 = C_1 \sinh(\sigma_1 \eta) + C_2 \eta \tag{2.30}$$

$$\psi_1 = C_3 \sinh(\sigma_1 \eta) + C_4 \eta + C_5 \eta^2 \sinh(\sigma_1 \eta) + C_6 \eta \cosh(\sigma_1 \eta) + C_7 \sinh(2\sigma_1 \eta) \tag{2.31}$$

Similarly solving equations (2.26) and (2.27) subjected to the corresponding boundary conditions (2.28) and (2.29).

$$\bar{\psi}_0 = C_8 \sinh(\sigma_1 \eta) + C_9 \eta \tag{2.32}$$

$$\bar{\psi}_1 = C_{10} \sinh(\sigma_1 \eta) + C_{11} \eta + C_{12} \eta^2 \sinh(\sigma_1 \eta) + C_{13} \eta \cosh(\sigma_1 \eta) + C_{14} \sinh(2\sigma_1 \eta) \tag{2.33}$$

Substituting equations (2.30), (2.31), (2.32) and (2.33) in the equation (2.16), we obtain

$$\psi = [C_1 \sinh(\sigma_1 \eta) + C_2 \eta] + k_1 e^{it}[C_8 \sinh(\sigma_1 \eta) + C_9 \eta] + \varepsilon \{ C_3 \sinh(\sigma_1 \eta) + c_4 \eta + C_5 \eta^2 \sinh(\sigma_1 \eta) + C_6 \eta \cosh(\sigma_1 \eta) + C_7 \sinh(2\sigma_1 \eta) + k_1 e^{it} (C_{10} \sinh(\sigma_1 \eta) + C_{11} \eta + C_{12} \eta^2 \sinh(\sigma_1 \eta) + C_{13} \eta \cosh(\sigma_1 \eta) + C_{14} \sinh(2\sigma_1 \eta)) \} \tag{2.34}$$

Using the equation (2.12), the axial velocity and the transverse velocity are obtained.
Shear stress at the wall $y=s(x)$ is given by

$$\tau = \frac{\sigma_{xy}(1 - s_{xx}) + (\sigma_{yy} - \sigma_{xx}) s_x}{1 + (s_x)^2}$$

Where $\sigma_{xy} = -\mu(\psi_{yy} - \psi_{xx})$, $s(x) = 1 + \delta \sin x$, $\sigma_{yy} - \sigma_{xx} = 1 - 4\mu\psi_{xy}$

III. RESULTS AND DISCUSSION

The flow governed by the non-dimensional parameters *R* the Reynolds number, D^{-1} inverse Darcy parameter, δ the amplitude of the boundary wave, k_1 the amplitude of oscillatory flux, *M* the magnetic parameter (Hartman number), *S* the oscillatory parameter and *m* hall parameter. The axial, transverse velocities and the stresses are evaluated computationally for different variations in the governing parameters *R*, D^{-1}, δ, k_1, *M*, *S* and *m*. For computational purpose we chose the boundary wave $s(x) = 1+\delta \sin x$ in the non-dimensional form. The figures (1-16) represent the velocity components *u* and *v* for different variations of the governing parameters being the other parameters fixed. We observe that for all variations in the governing parameters, the axial velocity *u* attains its maximum on the mid plane of the channel. We notice that the magnitude of the axial velocity *u* enhances and the transverse velocity *v* reduces with increasing the Reynolds parameter *R*. The behavior of transverse velocity *v* is oscillatory with its magnitude decreasing on *R* increases through small values less than 30 and later reduces for further increase in *R*. The resultant velocity also enhances with increasing the Reynolds parameter *R* (Fig 1 and 2). From figures (3 and 4) we concluded that both the velocity components *u* and *v* reduces with increase in the inverse Darcy parameter D^{-1}. Here we observe that higher the permeability of the porous medium larger the axial velocity along the channel and rate of increase is sufficiently high. Similarly, the resultant velocity reduces with increasing in the inverse Darcy parameter D^{-1}. It is evident that the magnitude of *u*, *v* and the resultant velocity increase with increasing the parameters k_1, x and *m* (5, 6, 9, 10, 13 and 14). The magnitude of the axial velocity *u* enhances and the transverse velocity *v* decreases with increasing in the amplitude of the boundary wave δ. The resultant velocity also reduces with increasing the parameter δ (Fig 7 and 8). We notice that the magnitude of the velocity components *u* and *v* reduces with increasing the intensity of the magnetic field *M*. The resultant velocity also decreases with increasing the Hartmann number *M* (Fig 11 and 12). The magnitude of the axial velocity *u* enhances and the transverse velocity *v* reduces with increasing the oscillatory parameter *S*. The behavior of transverse velocity *v* is oscillatory with its magnitude decreasing on *S* increases through small values less than 3 and later reduces for further increase in *S*. The resultant velocity also reduces with increasing the oscillatory parameter *S* (Fig 15 and 16). The shear stress on the upper wall with *S*=1 is evaluated through the entire cycle of oscillation at different points within a wave length for different sets of parameter δ. We choose the boundary wave as $s(x) = 1 + \delta \sin x$ in non-dimensional form. As already stated the criterion for the recognition of separation is the existence of a point at which the shear stresses at the wall less than or equal to zero throughout the entire cycle of oscillation. The influence of porosity in checking separation may be observed from table (1) for arbitrary values of *R*, D^{-1}, k_1, δ, *M* and *m*. This is evident that the shear stress on the upper wall does not vanish or become negative at any point in a wave length range throughout the entire cycle of oscillation. The magnitudes of the stresses enhance with increasing *R*, D^{-1}, k_1, *m* and *R* and reduces with increasing δ and magnetic parameter *M* being fixed *S*.

Table 1: The shear stress at the upper wall with S=1

x	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
0	2.9456	3.0045	3.1453	3.1145	3.2566	3.0835	3.1146	2.8145	2.7116	2.4653	2.0082	3.1566	3.1566
$\pi/4$	2.5675	2.6678	2.7566	2.6334	2.8346	2.6314	2.7366	2.4665	2.2265	2.1458	1.8113	2.8116	2.9958
$\pi/2$	2.1334	2.2469	2.3664	2.2883	2.3483	2.2108	2.3465	2.0065	1.8338	1.6652	1.2115	2.5665	2.8115
$3\pi/4$	3.4356	3.6678	3.6834	3.6836	3.7637	3.5289	3.6336	3.2115	3.0065	3.1148	2.8156	3.8314	4.2254
π	5.3114	5.3836	5.4266	5.4669	5.4995	5.4445	5.5333	5.1426	5.0083	5.1083	4.8309	5.9112	6.2318
$3\pi/2$	5.9445	5.9946	6.0053	6.3365	6.4143	6.1145	6.2225	5.7416	5.5783	5.6008	5.1245	6.5663	6.9665
$7\pi/4$	4.7866	4.8368	4.9014	4.9908	5.4652	5.6652	5.9863	4.4624	4.2211	5.5687	5.1833	4.9952	5.2336

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
R	10	30	50	10	10	10	10	10	10	10	10	10	10
D^{-1}	1000	1000	1000	2000	3000	1000	1000	1000	1000	1000	1000	1000	1000
K	1	1	1	1	1	1.5	2	1	1	1	1	1	1
δ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.03	0.01	0.01	0.01	0.01
M	2	2	2	2	2	2	2	2	2	5	8	2	2
m	1	1	1	1	1	1	1	1	1	1	1	2	3

3.1 GRAPHS AND TABLES

$k_1=1$, $R=10$, $S=1$, $D^{-1}=1000$, $M=2$, $m=1$, $x = t = \frac{\pi}{4}$, $\delta = 0.01$

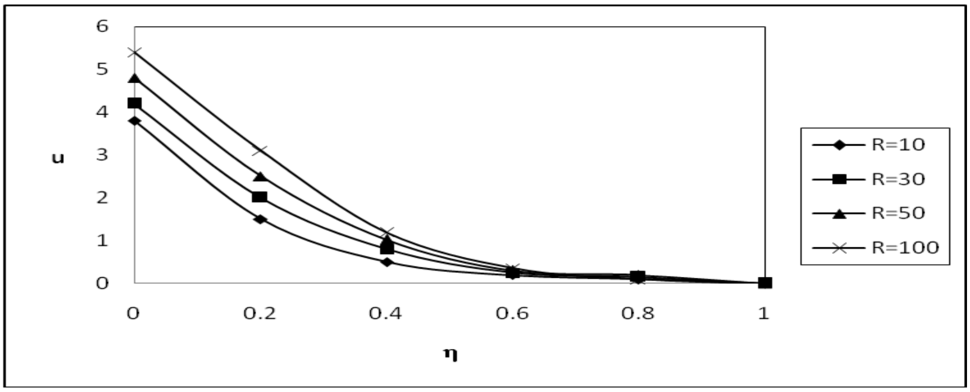

Fig 1: The velocity profile for *u* against *R*

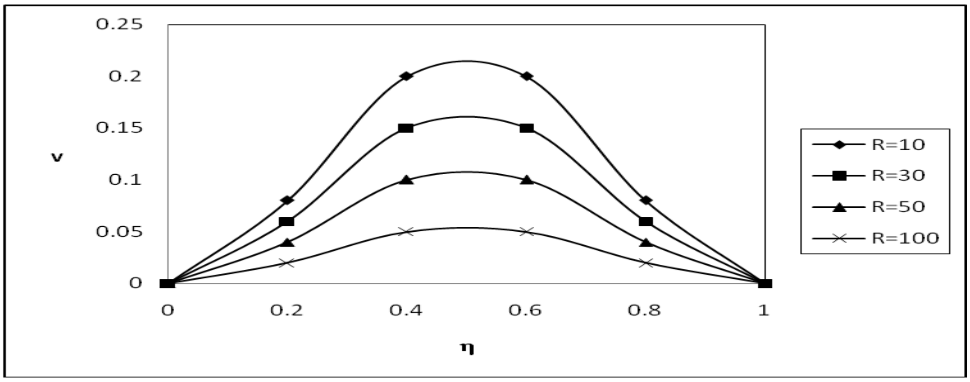

Fig 2: The velocity profile for *v* against *R*

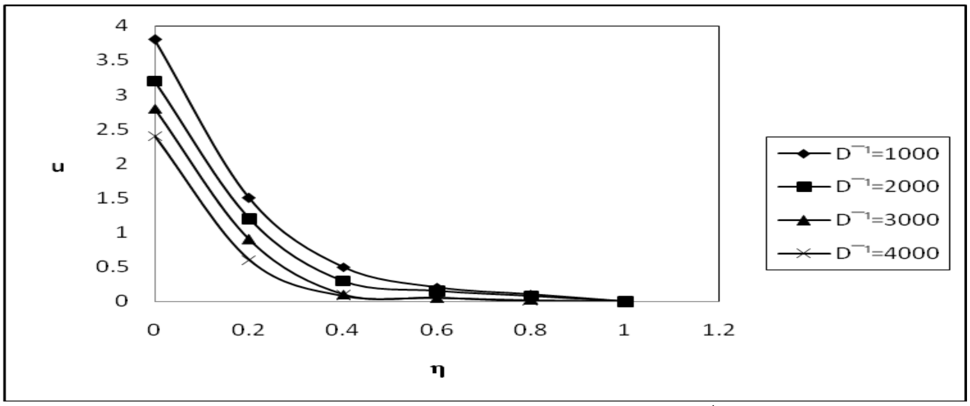

Fig 3: The velocity profile for *u* against D^{-1}

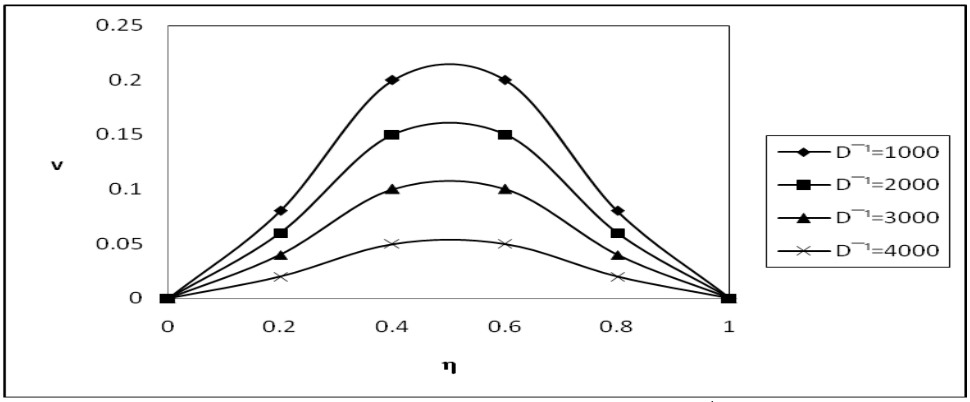

Fig 4: The velocity profile for *v* against D^{-1}

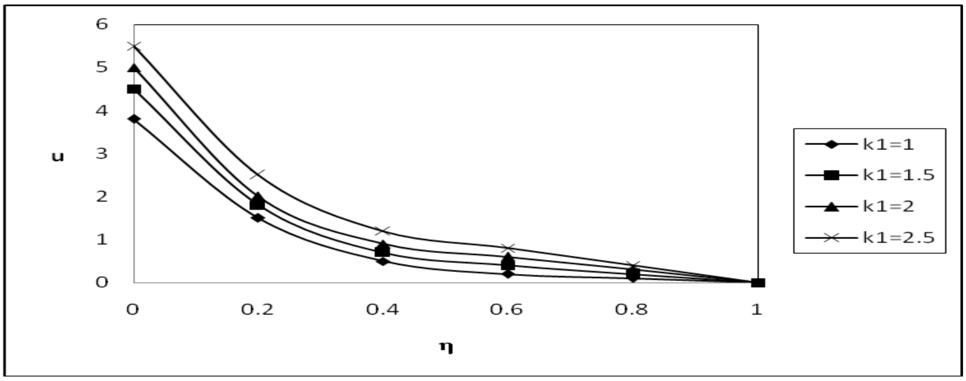

Fig 5: The velocity profile for *u* against k_1

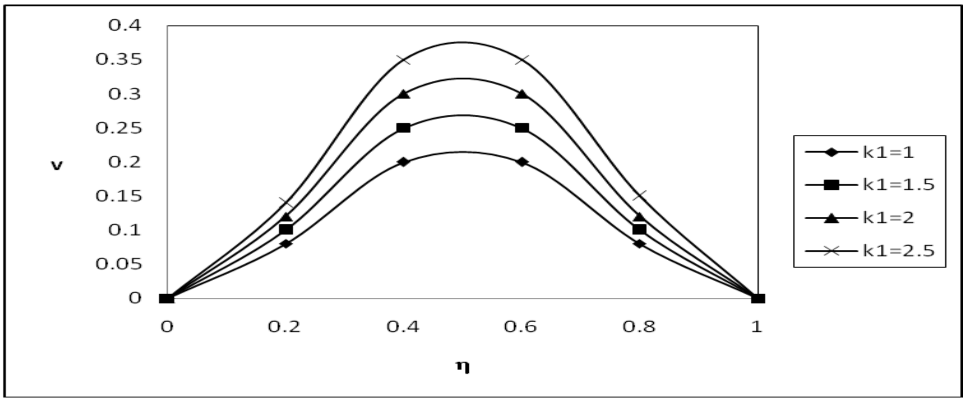

Fig 6: The velocity profile for *v* against k_1

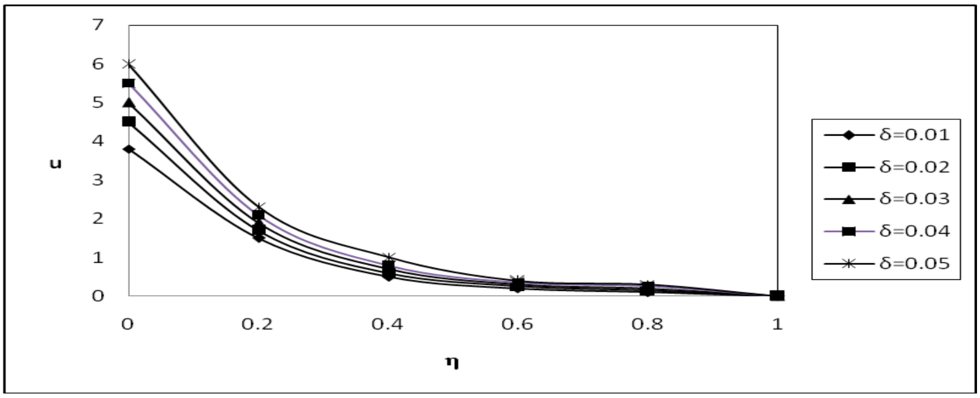

Fig 7: The velocity profile for *u* against δ

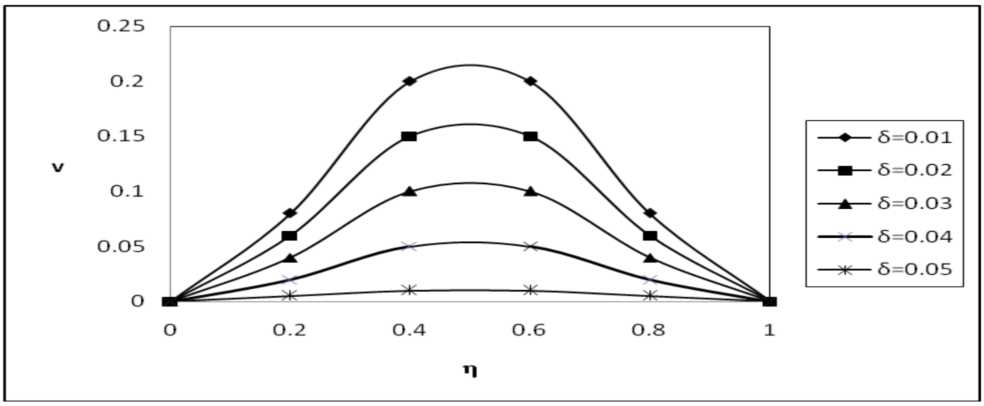

Fig 8: The velocity profile for *v* against δ

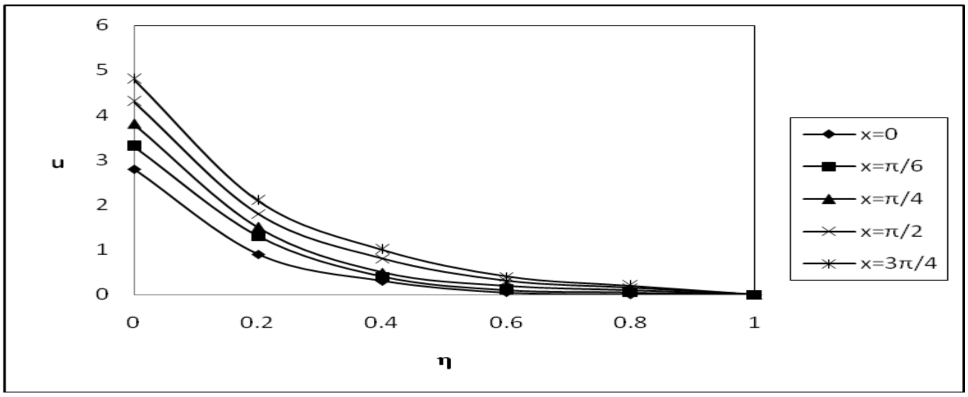

Fig 9: The velocity profile for *u* against *x*

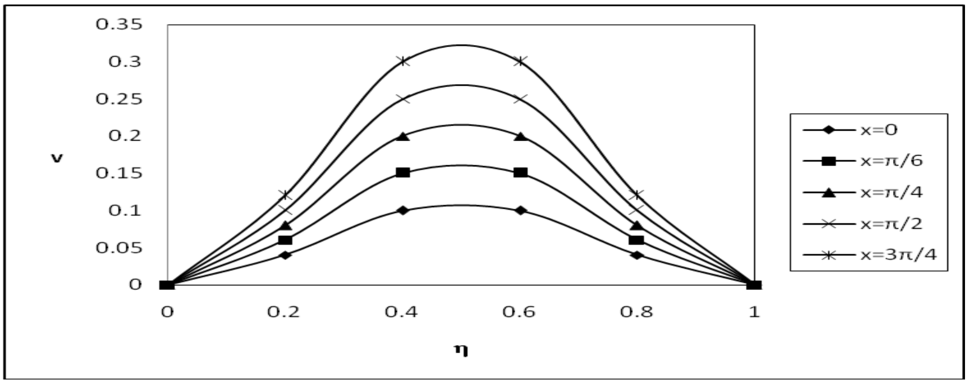

Fig 10: The velocity profile for *v* against *x*

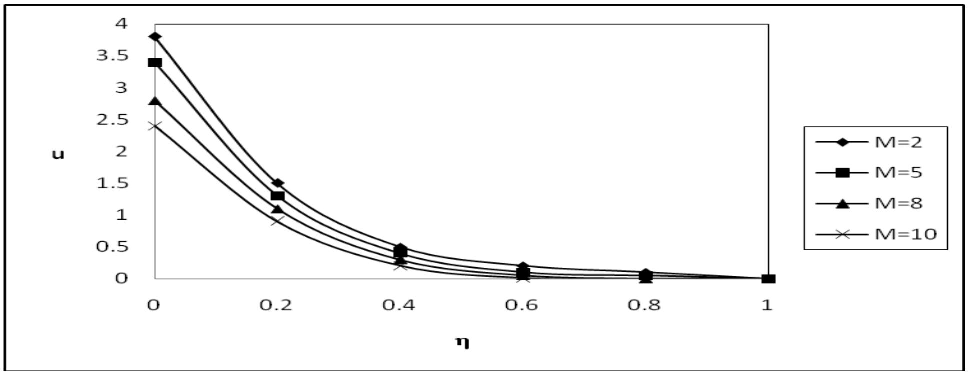

Fig 11: The velocity profile for *u* against M

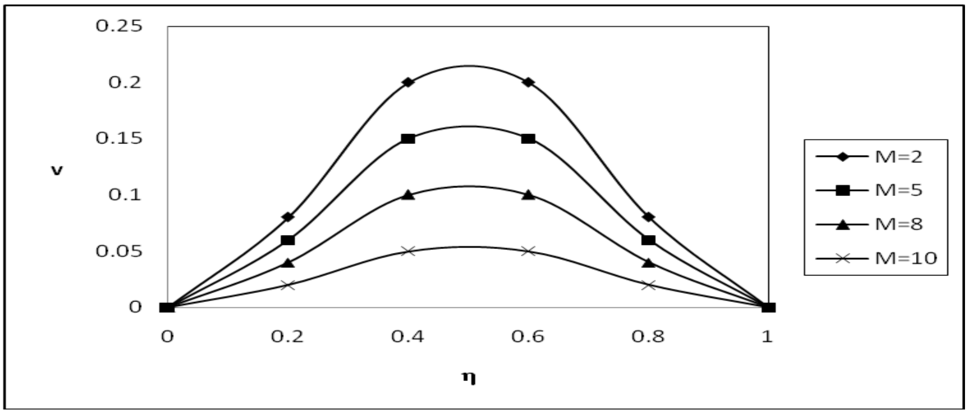

Fig 12: The velocity profile for *v* against M

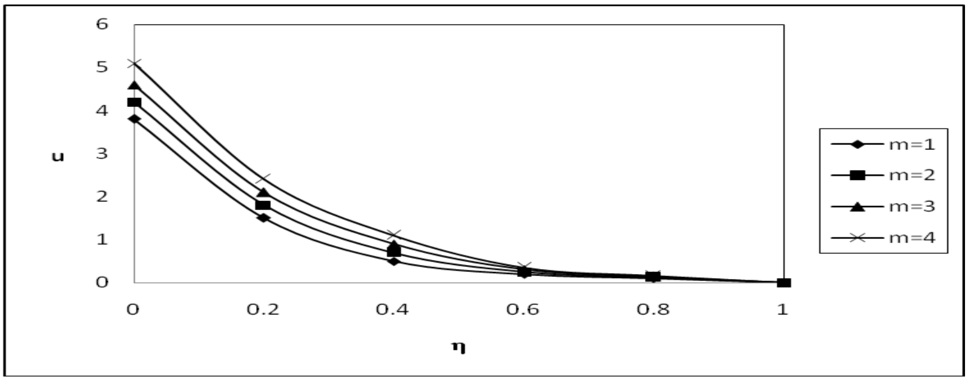

Fig 13: The velocity profile for *u* against *m*

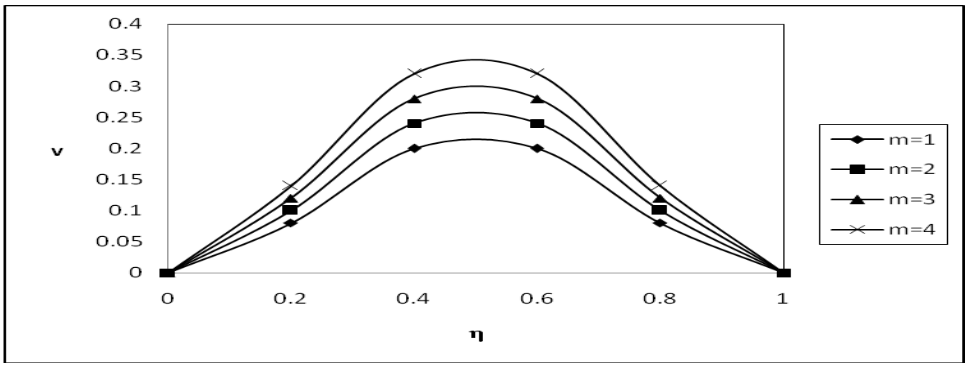

Fig 14: The velocity profile for *v* against *m*

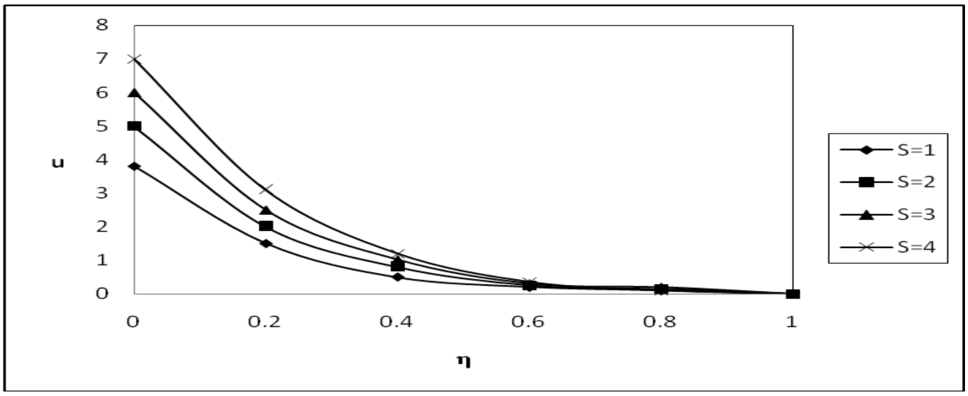

Fig 15: The velocity profile for *u* against *S*

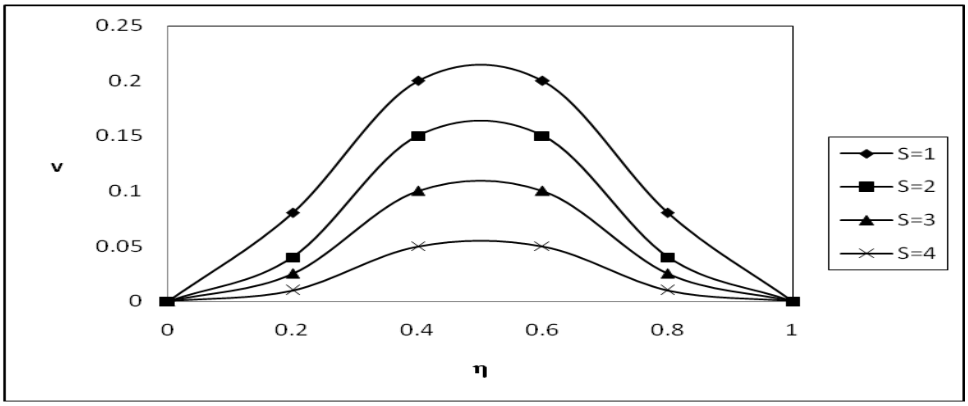

Fig 16: The velocity profile for *v* against *S*

IV. CONCLUSIONS

1. The resultant velocity also enhances with increasing the Reynolds parameter *R*.
2. Higher the permeability of the porous medium larger the axial velocity along the channel and rate of increase is sufficiently high. The resultant velocity reduces with increasing in the inverse Darcy parameter D^{-1}.
3. The resultant velocity increase with increasing the parameters k_1, x and *m*.
4. The resultant velocity also reduces with increasing the parameter δ
5. The resultant velocity also decreases with increasing the Hartmann number *M*.
6. The magnitude of the axial velocity *u* enhances and the transverse velocity *v* reduces with increasing the oscillatory parameter *S*. The resultant velocity also enhances with increasing the oscillatory parameter *S*.
7. The shear stresses at the wall less than or equal to zero throughout the entire cycle of oscillation.
8. The influence of porosity in checking separation may be observed.
9. The shear stress on the upper wall does not vanish or become negative at any point in a wave length range throughout the entire cycle of oscillation.
10. The magnitude of the stresses enhances with increasing *R*, D^{-1}, k_1 , *x* and *m* and reduces with increasing δ and magnetic parameter *M* being fixed *S*.

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REFERENCES

[1]. Berman, A.S.,j. App. Phy., Vol. 24, pp. 1232-1236 (1953).
[2]. Bestman, A.R., Acta Mech. Vol. 45, No. 1, pp. 17-21(1982).
[3]. Bestman, A.R., Int. J. Heat & Mass Transfer. Vol. 25, No. 5, pp. 675-682 (1982).
[4]. Despard, R.A. and Miller, J.A J. Flued Mech, Vol. 47, p. 21 (1971).
[5]. Gaehtgens, P.A, Pflugers Arch. Ges. Physiol. Vol. 316, p. 14 (1970).
[6]. Intaglietta, M, Proc. VI Conf.on microcirculation, Karger, Basel (1971).
[7]. Mazumdar J and K. Thalassodis, Med & Biol. Engg and Computing, Vol. 21, p. 400 (1983).
[8]. Perktold K, H.Florian & D. Hilbert. J.Bio Med. Engg, Vol. 9, p. 46 (1987).
[9]. Rappaport, M.B.Block, E.H. and Irwin, J.W, J.Appl.Physiol, Vol. 14, p. 651 (1959).
[10]. Rudraiah. N, Veerabhadraiah. R. Chandra Sekara. B.C, and Nagaraju,S.T., "Some flow problems in porous media", P.G.S.A.M. Series, Bangalore Publishers, Bangalore, India, Ch.5, pp. 115-151 (1979).
[11]. Sellers, J.R., J. App. Phy. Vol. 26, pp. 485-489 (1958).
[12]. Sud. V.K.,Suri. P.K. and Mishra. R.K.,Studia Bio Physica, Vol. 46, p. 163 (1974).
[13]. Sud. V.K.,Suri. P.K. and Mishra. R.K.,Studia Bio Physica, Vol. 60, p, 175 (1978).
[14]. Syamala Sarojini.M, M.Veera Krishna and C.Uma Shankar, "Unsteady MHD flow of a couple stress fluid through a porous medium between parallel plates under the influence of pulsation of pressure gradient", International Journal of Scientific and Engineering Research, Vol. 3, No. 4, pp. 1-6 (2012).
[15]. Raju.G., M.Veera Krishna and R.Siva Prasad, "Hall current effects on unsteady MHD three dimensional flow of a couple stress fluid through a porous medium in a parallel plate channel", International Journal of Physics and Mathematical Sciences, Vol. 3(1), pp. 18-31 (2013).
[16]. Veera Krishna.M, S.V.Suneetha and K.Nagamani, "Unsteady Magneto hydro dynamic palsatile flow of a viscous incompressible fluid through a porous medium in a flexible channel", Acta Ciencia Indica, Vol. XXXVI M, No. 3, pp. 397-409 (2010).
[17]. Wang, Y.C., J. App. Mech. Vol. 38, pp. 553-555 (1971).
[18]. Yuan, S.W., J. App. Phy. Vol. 27, pp. 267-273 (1956).
[19]. Claire Jacobs, *QJMM*, 24 (1971), pp. 221-230.
[20]. Murthy.K.N.V., *Appl. Sci. Res.* 35 (1979), pp. 111-115.
[21]. Debnath.L., *ZAMM*, 65 (1975), pp. 431-135.
[22]. Rao. D.R.V.P., D.V. Krishna and L. Debnath, *Acta Mechanica*, 34 (1982), pp. 225-240.
[23]. Sarojamma.G. and D.V. Krishna, *Acta Mech.* 40 (1981), pp. 277-288.
[24]. SivaPrasad. R., Convection Flows in Magneto hydro dynamics, Ph.D., thesis, S.K.U., Anantapur, India (1985).

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