MODELING OF VEGETATION EFFECT ON ATMOSPHERIC CO_2 AND O_2

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ABSTRACT

Owing to the need of estimation and control of purity of air with respect to different gases, various types of mathematical models can prove to be highly useful. To be tune with this, an attempt in the form of development of a mathematical model to account for vegetation effect on atmospheric carbon dioxide (CO_2) and oxygen (O_2) has been presented in the paper.

The model uses the numerical solution of various differential equations such as Poisson's equation, drift-diffusion equation, convection- diffusion equation etc. MATLAB based software has been developed to simulate the effect on CO_2 and O_2 contents at a point due to number of sources and sinks of these two gases. The model developed can be highly useful to account for the various effects of tree canopy on atmospheric purity.

KEYWORDS: *Purity function, Poisson's equation, Drift–diffusion equation, Convection-diffusion equation.*

I. INTRODUCTION

Needless to mention the fact that estimation and control of purity of the environment is a topic of vital importance regarding pollution control in the current century. Attempts have been made by the researchers to account for the effect on purity of the air due to various types of gases released from different sources such as industries, automobiles etc. The effects have been modeled in case of gases like SO_2, NOx, Ozone etc. Besides these gases, some medical plants are assumed to have high absorption power of CO_2 and high emissivity of O_2 in the atmosphere. In the environment there are various shrubs/ plants - Tulsi for example- available around us which release oxygen by absorbing carbon dioxide. As plants release remarkable amount of oxygen, the reduction in the air pollution is considerably large. However, due to other agencies, industries for example, the CO_2 contents are increasing remarkably. Due to this, even with high degree of natural contents of oxygen level in the atmosphere it has been a need of generation and transport of oxygen with adequate level so as to enhance the reduction of air pollution in the atmosphere. Hence it is important to measure the CO_2 and O_2contents in the atmosphere.

In this contest, advance techniques and instrumentation related with data acquisition and analysis of $O_2$and CO_2 contents based on theoretical and computational aspects of emission of $O_2$and absorption of CO_2 can be a good step for practical implementation. Any attempt to develop a prototype model can add new dimension in measuring environmental parameters. A mathematical model based on such measurement can be highly useful for analyzing the purity of air with respect to CO_2 and O_2. Beside the models based on Gaussian, Lagrangian, Euler's methods [1]which deal with the discussion on open space air pollution, a model based on Numerical solution of differential equations related with the processes like drift- diffusion [3], convection- diffusion [4] in closed chamber can find applications at various places such as research laboratories, offices, museums etc.

Theoretical considerations while developing such new model, obviously involve fundamental equation of physics namely Poisson's equation which relates the volume charge density 'ρ' to the corresponding potential $'\Phi'$through the differential equation as given below.

$$\nabla^2\Phi = -4\pi\rho \tag{1}$$

Generally in the Poisson's equation (when applied to electromagnetic), ρ is the electric charge density at a point and Φ is the electrostatic potential at a point. However, we can extend the concept of potential and charge density to other fields using analogies, if needed. Thus for example, in magneto-statics ρ and Φ form the pair of magnetic charge density and magnetic scalar potential respectively. In gravitational force field problem, ρ is gravitational charge density i. e mass per unit volume while Φ represents the gravitational potential. The constant -4π may be replaced by an appropriate constant involving the type of force field. For example, in electromagnetics, in three dimensions the equation in S. I system takes the form,

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0} \tag{2}$$

As an extension based on use of such analogy, we can use the same equation with analogous definitions of ρ andΦ. For example, when viewed in the context of air pollution problem, the quantities Φ and ρ in the equation can be taken to represent the physical quantities related with environmental parameters. For example the charge density can be related with CO_2 and O_2 contents at a point in the environment. This automatically suggests that, the potential function,Φwhich in turn can be taken to represent environmental purity function.
The purity function $\Phi(x, y, z)$assumed here can, in general, be a function of densities of many types of gases present in the atmosphere. However, for the present paper, O_2 and CO_2 densities are supposed to have prominence as regards the purity of air. The environmental analogs of properties like permittivity of air as a medium with respect to CO_2 and O_2 absorption and emission can be modeled mostly using the diffusion coefficient. The charge density function ρ will obviously corresponds to the differential changes in O_2 and CO_2contents with the help of which the purity function is to be evaluated. In the absence of any external source of CO_2 and O_2, the purity function Φ will be obtained as a solution of Laplace equation.
The plants can be treated as source of O_2 and sink of CO_2. Using the concept similar to that of center of mass, each plant can be treated as a point source of O_2 or sink of CO_2 for mathematical treatment. A mathematical model in terms of Poisson's equation with these new definitions of charge density and potential can be useful for numerical analysis of the emission and absorption of O_2 and CO_2 respectively.
A software using MATLAB can be developed to solve the Poisson's equation. The necessary data for the analysis using the software so developed can be obtained using an electronic hardware based on wireless sensor network. The few sections which follow describe the attempt regarding theoretical and computational aspects described above with brief mention of the applied aspects useful for development of pollution indicator devices.
The next section of the paper is organized as follows:
Section II - takes into account the theoretical aspects of the work which explains different differential equations in Physics used for formulation of the phenomenon of control of air purity with respect to CO_2 and O_2 contents in a closed chamber.
Section III- Explains the programming strategy for development of software and which involves numerical solution of differential equations.
Section IV- The results of mathematical model developed has been discussed.
Section V- Concludes the paper with comments on the work and described the scope for feature work.
The references are given at the end of the paper.

II. THEORETICAL ASPECTS

For theoretical formulation of the phenomenon of control of air pollution with respect to CO_2 and O_2 contents, we have to consider mainly the following differential equation in physics.

1. Equation of continuity $\nabla.\vec{J} = \frac{\partial\rho}{\partial t}$
2. Poisson's equation $\nabla^2\,\Phi = -4\pi\rho$
3. Drift and Diffusion equation $\frac{\partial n}{\partial t} + \nabla.(nV_n) = 0$
4. Convection and diffusion equation $\frac{\partial C}{\partial t} = \nabla.(D\nabla C) - \nabla.(\vec{v}c) + R$

Where, the symbols have their conventional meaning well documented in the literature.In above equation the various symbols conventionally represent the physical quantities as described below,
j- electric current density, ρ – electric charge density, Φ - electrostatic potential, n – concentration of particles i. e particles per volume, V_n- drift velocity of the particles, C – quantity of interest,$\vec{v}c$– convection velocity, R – constant related with source or sink.
In the present paper, these equations have been related with the pollution of air in terms of new definitions using the concept of analogues physical quantities as described below.

1. The electric potential Φ is treated being analogues to purity function of air with respect to CO_2 and O_2 contents in the air.
2. The electric charge density ρ is analogues to the source densities of carbon dioxide and oxygen designated asρ_{CO2} and $\rho_{O2.}$
3. The plants are supposed to play the dual role of being a source of O_2 and sink for CO_2.
4. For simplicity the charge densities of the two gases have been assumed to have a spatial and time dependence governed by an exponential function. Such exponential variation can be seen to have a firm support of the equation of continuity.
5. Under still conditions of atmosphere surrounding the plants, the drift velocity of the two gas molecules can be assumed to be governed by simple application of motion under gravity in the viscous medium.
6. To account for the transport of the emitted and absorbed gas molecules, the concept of diffusion equation has been incorporated assuming homogeneous nature of the surroundings.

Some important points related with differential equations used for numerical computations of the physical quantities needed for discussion of air pollution problem are briefly described in sequence as given below.

2.1Poisson's Equation:
The solution of Poisson's equation namely

$$\nabla^2\ \Phi(\mathrm{x,y,z}) = -4\pi\rho(x,y,z) \quad (3)$$

can be obtained with mathematical rigor using Green's function technique as discussed by several authors. Here, ρ is the net charge density comprising of three types namely due to drift, diffusion and source emission respectively. However, the equation can also be solved numerically using finite difference approach by incorporating the appropriate correction term at various points in the solution of Laplace's equation for a closed chamber having rectangular parallelepiped shape. The process of iteration for computing the values of $\Phi(\mathrm{x,y,z})$ with appropriate boundary conditions has been adopted for the present work it being a simple and useful approach for development of computer program for the evaluations.
For a cubical enclosure with source and sink inside as shown in Fig.( ), the potential at an interior point *(x + h, y + h, z + h)*can be obtained using expression,

$$\Phi(x+h, y+h, z+h) = \frac{1}{6}[\Phi(x+h) + \Phi(y+h) + \Phi(z+h)] + c(x,y,z) \quad (4)$$

where, h is a suitably chosen small increment in value of co-ordinates. The correction term c(x, y,z) in the above expression (3) can be estimated using the known expression for the charge density function $\rho(x,y,z)$.

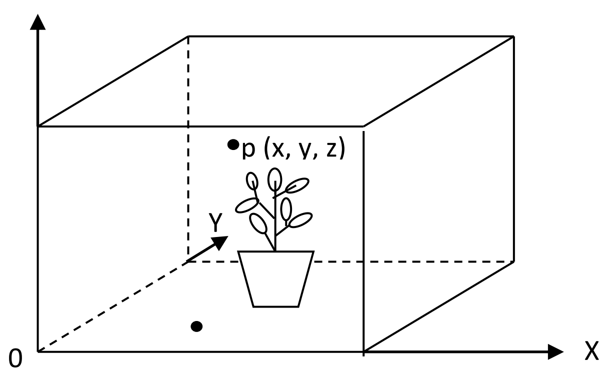

Figure1. Cubical enclosure with a plant

The values of the charge densities at various points in the chamber surrounding the source (plant) can be determined using the experimental measurements. Use of proper sensors can be used for such measurements. The reliability and accuracy of the software developed for computations of $\Phi(\mathrm{x,y,z})$ can be tested initially using known expressions for a pair of $\rho(x,y,z)$ and the corresponding$\Phi(\mathrm{x,y,z})$. Some of the known expressions for $\Phi(\mathrm{x,y,z})$ and the corresponding $\rho(x,y,z)$ are as tabulated below. The expression for $\rho(x,y,z)$ can be obtained by evaluating LHS of equation (2). For example, with$\Phi(\mathrm{x,y,z}) = x^3y + y^2x^2$,we get $\nabla^2\Phi = 6xy + 2y^2 + 2x^2$

$$\rho(x,y,z) = \frac{6xy + 2y^2 + 2x^2}{4\pi}$$

Table1. Expression for $\rho(x,y,z)$ for given $\Phi(\mathrm{x,y,z})$

Sr. No.	$\Phi(\mathrm{x,y,z})$	$\rho(x,y,z)$
1.	$x^3y + y^2x^2$	$\frac{6xy + 2y^2 + 2x^2}{4\pi}$
2.	$e^{-(x^2+y^2+z^2)}$	$\frac{-2(x+y+z)e^{-(x^2+y^2+z^2)}}{4\pi}$

Care is to be taken while selecting the known expression for $\phi(\mathrm{x,y,z})$ not to have any singularity at the source.

2.2Drift - Diffusion equation:

To account for transport of O_2 and CO_2 in the lateral dimensions in a closed chamber, diffusion equation is to be solved while the downward movement of the gas molecule due to gravity can be accounted for using drift equation. The two equations as documented in the literature are as follows.

$$\frac{\partial n}{\partial t} + \nabla.(\mathrm{n}V_{\mathrm{n}}) = 0 \qquad (5)$$

$$\frac{\partial C(x,y,z)}{\partial t} = D(\frac{\partial 2_C}{\partial x^2} + \frac{\partial 2_C}{\partial y^2}) \qquad (6)$$

where, n: concentration of O_2 and CO_2 molecules related with charge density ρ, V_n: velocity of O_2 and CO_2.Solutions of the equations involve consideration of partial numerical solution of first and second order partial differential in two variables namely time and space.

The computation involves the drift velocity which in turn depends upon the geometrical dimensions of the enclosure and physical quantities like mobility, volume density of mass etc. These parameters can be absorbed in a single mathematical constant for programming purpose. While actually implementing the software for real life situation, these quantities and constraints can be estimated on the basis of standard reference tables and system dimensions. In open atmosphere, the wind velocity dominates the effect due to gravity. However, for enclosed chamber, the wind velocity is zero and can be neglected. Similarly, convection effect caused due to gravitational pull on CO_2 or O_2 molecules is also very small as compared to diffusion process and can be ignored. Solution of drift and diffusion equation can yield the respective charge densities.

2.3Convection - diffusion equation:

For prolonged duration of experimental observations, one has to take into consideration the phenomenon of convection along with diffusion. It can be done by solving the convection diffusion equation given by

$$\frac{\partial C}{\partial t} = \nabla.(\mathrm{D}\nabla\mathrm{C}) - \nabla.(\vec{v}\mathrm{c}) + \mathrm{R} \qquad (7)$$

$$\frac{\partial V}{\partial t} = D\Delta V + \rho \qquad (8)$$

Wherein, C: the mass transfer is the variable of interest, D: diffusion coefficient, V: average velocity, R: source or sink of O_2 and CO_2.

All the equations in general are second order in space and first order in time and can be solved numerically using finite difference approach. Accordingly the numerical methods for solution of such differential equations have been developed and used for development of MATLAB based computer program. The section which follows describes the program strategy and algorithmic development pertaining to this aspect.

III. PROGRAMMING STRATEGY

As Mentioned in the earlier section, development of software involves the numerical solution of differential equation in one, two and three dimensions with known boundary conditions. The development of algorithm for such numerical solutions has been done in MATLAB and tested for their reliability.
The programs developed have the following features.

1. Applicability for the cubical enclosure with a facility of arranging the sources in the form of linear or planer array.
2. Programs can determine the potential distribution inside the enclosure due to system of N sources in an array.
3. The term potential distribution in the paper corresponds to purity function of air with the general description of the term charge density ρ comprising of three types as follows,
$$\rho = \rho_{\text{drift}} + \rho_{\text{diffusion}} + \rho_{\text{source}}$$
4. For determining net potential distribution in the enclosure, use of principle of superposition has been adopted. While applying this principle, provision has been made to incorporate appropriate variational parameters depending upon whether the sources are equidistant or not.
5. The effect of variable diffusivity and drift of the medium can be seen separately using the appropriate program developed for the purpose.
6. The result of the program can be seen numerically as well as graphically.

IV. RESULTS AND DISCUSSION

The potential distribution due to N number of sources placed at points (x_i, y_i, z_i) lying in a plane have been tried using the software developed. Results are in good agreement with those predicted theoretically. Some of such graphical results corresponding to some typical values of (x_i, y_i, z_i) due to one, two and three sources are as shown in the figure 1, 2, 3.
Results are in good agreement with those predicted theoretically.

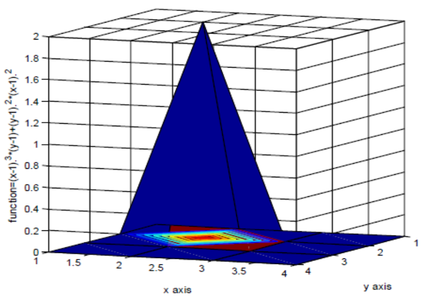

Fig.1. Single source with location (1, 1, 1)

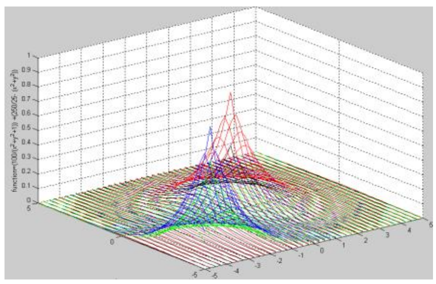

Fig. 2. Two sources with locations (2, 2, 2) and(3,3,3)

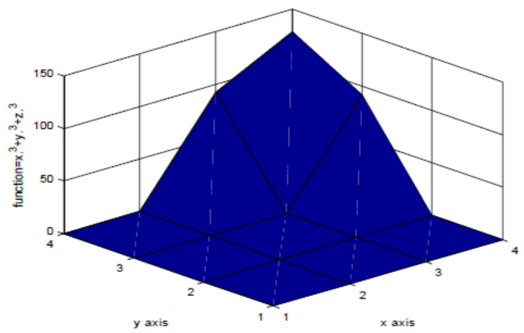

Fig.3.Three sources with locations (2, 2, 2), (1, 1, 1),(3,3,3)

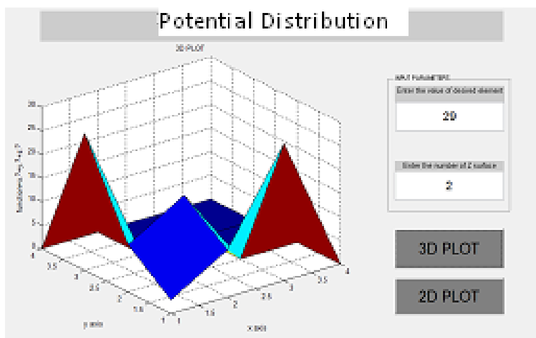

Fig.4. Graphical Display of Potential Distribution

1. The potential distribution due to a system of single source, dual source and triple source located at specified points in various planes are shown in figure 1, 2, 3 respectively. The spread around the source position and the magnitude is in accordance with principle of superposition and the assumption of exponential nature of charge densities ρ_{CO2} and $\rho_{O2.}$.
2. The potential distribution due to a single source gives a peak centered around the position of the source which is quite but obvious as shown in Fig.1.
3. Slight shift and spread around the central position due to two sources can be vividly seen in Fig.2. The spread in the central peak is in consistent with the principle of superposition and the assumption of exponential variation of source and sink charge densities.
4. One can change the number of equidistant sources for visualizing the variation in the potential distribution. The plots correspond to the solution of Poisson's equation assuming low drift and diffusion coefficients. Inclusion of drift and diffusion equations will smoothen the nature of graph as shown in Fig. 3.
5. The software has a facility to display the potential distribution (Fig. 4)due to desired number of sources with provision for variable position of sources. One should note the additional peak in between the sources as shown in figure.
6. Increase in the number of sources can result in increased contribution due to diffusion and drift processes. It has been tried using the software. One can verify this fact using remarkable increase in number of sources. Changes in the nature of the graphs are illustrative of these facts.

V. CONCLUSION:

The programs developed in MATLAB, based on the use of the differential equations in Physics such as Poisson's equation, drift- diffusion equation, convection - diffusion are used to simulate the potential distribution in an enclosed chamber due to a system of N sources located at different points.
Estimation of purity function in case of an enclosed chamber described in this paper can be implemented using medical plants (Tulsi plant for example) as powerful source of oxygen with the help of the Electronic hardware incorporating the use of WSN developed by the authors [].The results have been experimentally verified using system of appropriate sensors of CO_2 and O_2.
The software developed can in general be used to solve any differential equation involving combination of first, second and third order ordinary as well as partial differential equation in one, two and three dimensions to discuss various real life applications. Environmental purity with respect to different gases (CO, SO_2, H_2S etc.) and the corresponding sources can also be analyzed using this software.

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